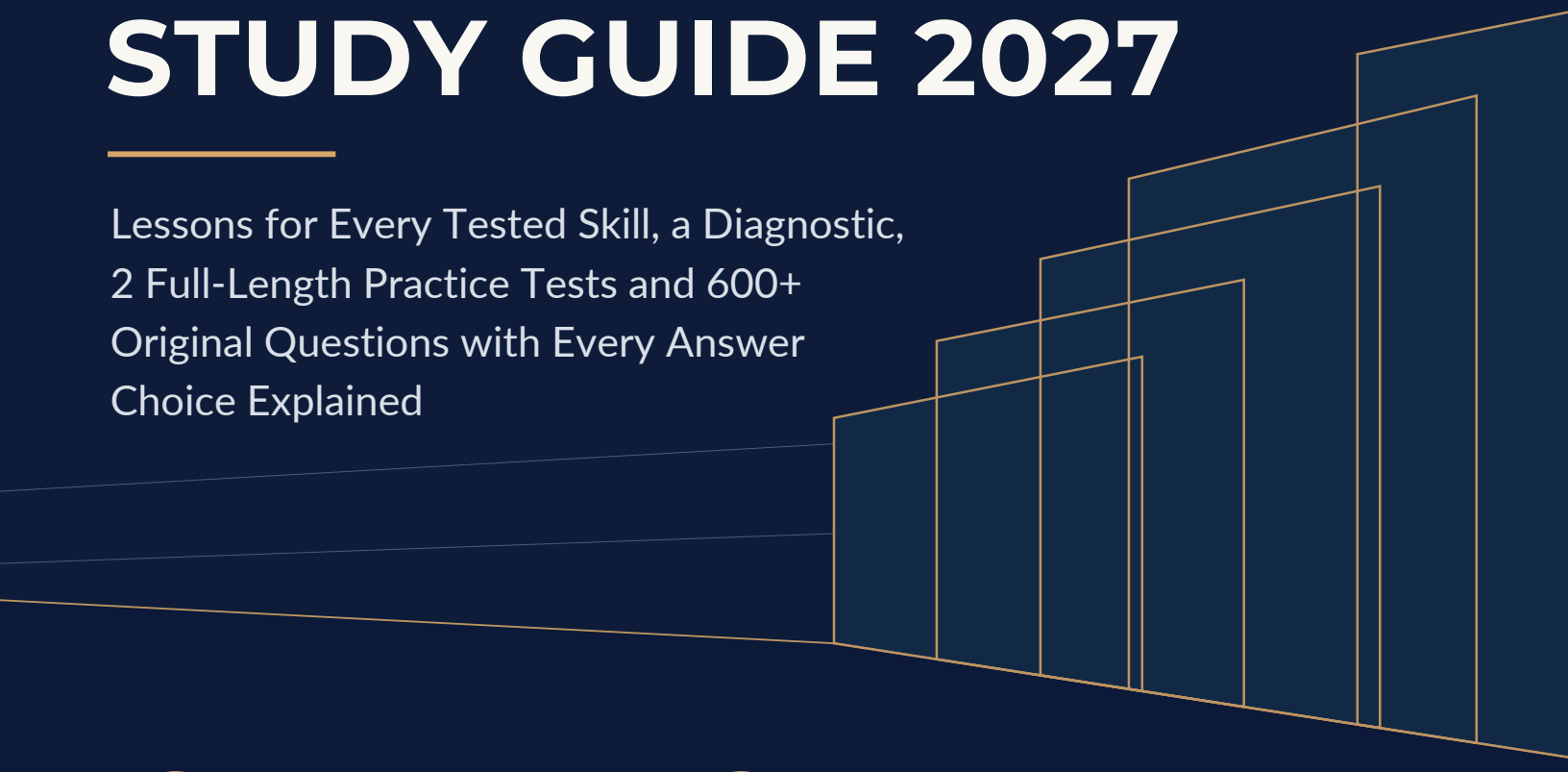
SKILL BY SKILL — 2027 EDITION

READING AND WRITING · MATH

DIGITAL SAT®

STUDY GUIDE 2027

Lessons for Every Tested Skill, a Diagnostic, 2 Full-Length Practice Tests and 600+ Original Questions with Every Answer Choice Explained



615 ORIGINAL QUESTIONS

2 FULL-LENGTH PRACTICE TESTS

MATTHEW D. GIRON, MBA

SKILL BY SKILL

Digital SAT Study Guide 2027

Lessons for Every Tested Skill, a Diagnostic, 2 Full-Length Practice Tests and 600+ Original Questions with Every Answer Choice Explained

Matthew D. Giron, MBA

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Every practice question, worked example, explanation and passage in this book, except the public-domain excerpts, was written for this edition. None is an official, released, retired or recalled test question. Question counts, timing, scoring and policy facts follow the College Board's published test specifications and web pages as reviewed on 29 October 2026. Policies, dates, fees and tools can change, so confirm them on the College Board website before you register and again before test day.

Practice results in this book are correct counts and percent-correct summaries for study planning. These tests have not been statistically calibrated to the SAT and do not predict a scaled SAT score. Difficulty labels are editorial judgments, not statistical calibrations.

Passages that describe real people, places, research or events were checked against the sources listed in the Selected Sources section. Other passages, data sets, people and places are fictional or hypothetical and are presented as such. Excerpts from public-domain literature are identified by author, title and year.

This book is self-contained. Everything you need is printed in these pages; no website, app or subscription is required.

Updates and corrections: skillbyskillprep.com/updates

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How to Use This Book

This book is for students preparing for the digital SAT in 2027 and beyond, especially those who want to raise their score by learning each tested skill one at a time. It also serves parents who want to see what their teen is studying, tutors who want a print bank of skill-based practice, and strong students looking for harder-module practice. Everything you need is printed in these pages.

What is inside

The book has four parts, followed by five appendices.

- **Part I, Getting Started.** Chapters 1 to 3 explain the test, build your study plan and set your pacing. The Diagnostic Test follows Chapter 3.
- **Part II, Reading and Writing.** Chapter 4 explains how the section works. Chapters 5 to 14 teach its 11 skills.
- **Part III, Math.** Chapter 15 covers the calculator, the reference sheet and student-produced responses. Chapters 16 to 28 teach the 19 Math skills.
- **Part IV, Full-Length Practice and Final Preparation.** Practice Tests 1 and 2 (Chapters 29 and 31), their scoring and review (Chapters 30 and 32), and the final two weeks (Chapter 33).
- **Appendices A to E.** Grammar and punctuation, transition words, graphing-calculator strategies, math formulas and facts, and the error log and study tracker.

After the appendices, the Question-Type Finder lists every skill with the chapter that teaches it and the number of times it appears in the diagnostic and in each practice test.

The path through the book

1. Read Chapters 1 to 3.
2. Take the Diagnostic Test and fill in your skill profile.
3. Build your plan with Chapter 2, putting your weakest skills first.
4. Study the skill chapters and do each chapter's Practice Set.
5. Take Practice Test 1 and review it with Chapter 30.
6. Strengthen your weak domains, then take Practice Test 2 and review it with Chapter 32.
7. Follow the 14-day plan in Chapter 33, which includes the free full-length practice tests in the College Board's Bluebook app.

Practice Sets and explanations

Each skill chapter ends with a Practice Set arranged from easier to harder, followed by explanations. Answer every question in a set before you read any explanation.

Each explanation gives the correct answer; the skill, domain and difficulty; why the correct answer works; and why each wrong choice fails. Math explanations add numbered solution steps and, where graphing helps, a calculator option. Every explanation ends by naming the chapter and page to review. For student-produced responses, the answer lists examples of accepted forms, such as 3/4, .75 or 0.75. Equivalent entries that follow Chapter 15's digital entry rules also count as correct.

Question IDs

Every question has an ID printed beside its number. Use it to find the matching explanation and to record misses in your error log. The ID starts with the section, RW for Reading and Writing or M for Math, and a letter for where the question appears: D for the diagnostic, C for a chapter Practice Set and T for a practice test.

ID	Meaning
RW-D08	Reading and Writing, Diagnostic, question 8
M-C1712	Math, Chapter 17 Practice Set, question 12
RW-T2M1-14	Reading and Writing, Practice Test 2, Module 1, question 14
M-T1M2-03	Math, Practice Test 1, Module 2, question 3

Callout boxes

Shaded boxes in the lessons carry four labels.

- **Tip:** a faster or more reliable way to work.
- **Trap:** a wrong-answer pattern to watch for.
- **Check:** a quick test of your understanding.
- **Note:** useful background or a caution.

Difficulty labels

Questions are labeled easier, medium or harder. These labels are the author's editorial judgments, based on the reading and the number of steps involved, not statistical measurements. A question labeled "harder" may feel easy to you, or the reverse.

Answer sheets, copies and dates

The diagnostic and both practice tests have answer sheets with bubbles for multiple-choice answers and labeled answer lines for student-produced responses. You may photocopy the answer sheets and the error log in Appendix E for your own use.

Exam facts in this book follow the College Board's published specifications and web pages as reviewed on 29 October 2026. Dates, fees, tools and rules can change, so confirm them on the College Board website before you register and again before test day.

The book's website (optional)

Everything you need is printed in these pages. If you want more, skillbyskillprep.com, the website for this book, has a free online diagnostic, two video lessons for every skill and practice with a digital study coach. Each skill chapter gives the address of that skill's page between the lesson and the Practice Set, and a few other chapters point to short guides. Corrections to this book are posted at skillbyskillprep.com/updates.

ONLINE (OPTIONAL) Point your phone's camera at a code, or type the address printed under it.

Home page
skillbyskillprep.com

Updates and corrections
skillbyskillprep.com/updates

The website is independent of the College Board, which does not endorse it. This book reports correct counts and percent correct. Any practice estimates on the website are for study planning and are not official scores.

PART I

Getting Started

The Digital SAT at a Glance

Your Study Plan

Pacing, Guessing and Test-Day Strategy

Diagnostic Test

CHAPTER 1

The Digital SAT at a Glance

The digital SAT is a two-section test that you take on a laptop or tablet in the College Board's Bluebook app, at a test center or at a school. It is not taken at home. This chapter maps the whole test: how it is built, how its adaptive modules work, what each section measures, how scoring works and which tools you will have on screen.

The exam facts in this chapter follow the College Board's published specifications and web pages as reviewed on 29 October 2026. Confirm anything time-sensitive on the College Board website before you register and again before test day.

How the test is built

The test has two sections: Reading and Writing first, then Math. Each section is split into two modules, and each module is timed separately. A 10-minute break separates the two sections.

Part of the test	Questions	Time	Average time per question
Reading and Writing, Module 1	27	32 minutes	about 71 seconds
Reading and Writing, Module 2	27	32 minutes	about 71 seconds
Break	—	10 minutes	—
Math, Module 1	22	35 minutes	about 95 seconds
Math, Module 2	22	35 minutes	about 95 seconds
Total	98	134 minutes of testing, plus the break	—

Inside a module you can skip ahead, go back, mark questions for review and change answers until the module's time runs out. When a module ends, it closes for good: you cannot return to Module 1 once Module 2 begins, and you cannot return to Reading and Writing during Math.

Each module also contains 2 pretest questions. These are questions being tried out for future tests. They look exactly like the others, they do not count toward your score, and you cannot tell which ones they are. That means each Reading and Writing module has 25 scored questions and each Math module has 20.

TIP Because you cannot spot a pretest question, treat every question as if it counts. A question that seems strangely hard may be a pretest question, or it may simply be hard. Either way, give it a fair try, choose an answer, mark it if needed and keep going.

How the adaptive modules work

The digital SAT is a *multistage adaptive* test. "Adaptive" means the test adjusts to the student, but it adjusts between modules, not after every question. Here is the sequence for each section.

1. Module 1 contains a broad mix of easier, medium and harder questions. Every student starts with this kind of mix.
2. Your performance on Module 1 decides which Module 2 you receive. One version is, on average, harder than Module 1. The other is, on average, easier.
3. Your section score uses every scored question from both modules.

The College Board does not publish the cutoff that decides which Module 2 a student sees, and this book does not guess at one. Any chart that claims to tell you "how many you can miss" to reach the harder module is not based on published information.

The College Board also states that students are not advantaged or disadvantaged by which Module 2 they receive, and that the college and career readiness benchmarks can be reached on either path. The practical lesson: Module 1 deserves your full care and accuracy, and Module 2 deserves the same steady method. Chapter 3 turns this into a pacing plan.

NOTE If Module 2 feels harder or easier than Module 1, do not try to read meaning into it during the test. You cannot change which module you received, and you cannot identify the unscored pretest questions, so treat every question as if it counts. Keep working one question at a time.

What each section tests

The College Board organizes each section into four content domains. Each domain contains a set of skills, and this book teaches every skill under its official name. Knowing the domains also helps you read this book's diagnostic and practice-test results, which are sorted by skill and domain.

Reading and Writing

Domain	Share of the section	Scored questions per section	Skills
Craft and Structure	about 28%	13–15	Words in Context; Text Structure and Purpose; Cross-Text Connections
Information and Ideas	about 26%	12–14	Central Ideas and Details; Command of Evidence: Textual; Command of Evidence: Quantitative; Inferences
Standard English Conventions	about 26%	11–15	Boundaries; Form, Structure, and Sense
Expression of Ideas	about 20%	8–12	Transitions; Rhetorical Synthesis

Every Reading and Writing question has its own short passage, or a pair of short passages, of 25 to 150 words. One four-option multiple-choice question follows each passage, with a single best answer. Passages come from four subject areas: literature, history/social studies, humanities and science. Some questions include a table, a bar graph or a line graph. Chapter 4 explains the order of the questions and gives you a general method for the whole section.

Math

Domain	Share of the section	Scored questions per section	Skills
Algebra	about 35%	13–15	Linear equations in one variable; Linear functions; Linear equations in two variables; Systems of two linear equations in two variables; Linear inequalities in one or two variables
Advanced Math	about 35%	13–15	Equivalent expressions; Nonlinear equations in one variable and systems of equations in two variables; Nonlinear functions
Problem-Solving and Data Analysis	about 15%	5–7	Ratios, rates, proportional relationships, and units; Percentages; One-variable data: distributions and measures of center and spread; Two-variable data: models and scatterplots; Probability and conditional probability; Inference from sample statistics and margin of error; Evaluating statistical claims: observational studies and experiments
Geometry and Trigonometry	about 15%	5–7	Area and volume; Lines, angles, and triangles; Right triangles and trigonometry; Circles

About 75% of Math questions are four-option multiple choice. The other 25% or so are *student-produced responses* (SPR), where you type your own answer instead of choosing one. About 30% of Math questions are set in a context, such as a science, social studies or everyday situation, and most of those use 50 words or fewer.

Notice where the weight sits. Algebra and Advanced Math together make up about 70% of the Math section, so the chapters on linear equations, functions, expressions and nonlinear equations (Chapters 16–22) carry the most points. In Reading and Writing the four domains are closer in size, so no single domain can be skipped.

How scoring works

Score	Range
Reading and Writing section	200–800
Math section	200–800
Total score	400–1600, the sum of the two section scores

Your score is not a simple count of right answers. The College Board scores each section with *item response theory* (IRT), a statistical method that takes each question's difficulty and other characteristics into account. Only

scored questions count, and they come from both modules. As a result, two students with the same number correct can receive different scaled scores, and no simple chart converts a raw count into an official score.

Four more scoring facts shape how you should test.

- **There is no penalty for wrong answers.** The College Board says that for most students it is better to guess than to leave a question blank. In practice, answer every question, every time.
- **The benchmarks.** As of October 2026, the college and career readiness benchmarks are 480 for Reading and Writing and 530 for Math. These values carried over from the paper SAT, and the College Board says it will re-examine them, so check the current values when you set goals.
- **One scale.** Digital and paper SAT scores are on the same scale. A 1050 on the digital SAT corresponds to a 1050 on the paper SAT, so an older score can be compared directly with a new one.
- **Section scores add.** A 40-point gain in either section raises your total by the same 40 points, so spend your study time where points are easiest for you to gain.

NOTE This book's practice tests are printed and fixed, so they cannot produce official scores. Chapters 30 and 32 show you how to record correct counts and percent correct, identify weak skills and plan your review. These uncalibrated tests cannot provide a defensible estimate on the SAT score scale.

The Science Practices and Foundations score

The College Board has announced that, starting in spring 2027, the SAT will report a science cross-test score. On the score report it may appear as a separate cross-section score called Science Practices and Foundations. It is calculated from questions that are already on the test, in both Reading and Writing and Math, so it adds no testing time and no extra questions.

This score is sent to your school, your district and sometimes your state. It is not sent to colleges. You do not need a separate study plan for it, because it comes from questions you are already preparing to answer.

Tools you will use in Bluebook

Bluebook includes the on-screen tools below. Practice with them on the free full-length practice tests in the College Board's Bluebook app before test day.

Tool	What it does	How to use it well
Testing Timer	Counts down the time left in the module. You can hide it until 5 minutes remain; then an alert appears.	Check the time at planned checkpoints instead of watching it constantly (Chapter 3).
Calculator	The Desmos graphing calculator, built into the Math section. You can move it around the screen, and starting with 2026–27 testing you can also resize it.	Decide what you are solving for first, then graph, compute or check (Chapter 15).
Reference Sheet	Common formulas and facts, available throughout the Math section. Starting with 2026–27 testing, the sheet and images have a new layout with zoom and pan.	Know what is on it, so you look up only what you need (Chapter 15 and Appendix D).

Tool	What it does	How to use it well
Highlights & Notes	Lets you highlight text and add notes on screen.	Highlight a passage's main claim or a key qualifier.
Mark for Review	Flags a question so you can find it again quickly.	Use it with the two-pass method in Chapter 3.
Line Reader	Helps you keep your place as you read.	Use it on dense passages if your eyes tend to skip lines.
Option Eliminator	Lets you cross out answer choices.	Cross out a choice as soon as you can name why it fails.
Question Menu	Lets you jump to any question in the current module.	Use it on your second pass to return to marked questions.
Zoom	Enlarges what is on the screen.	Use it on small tables, graphs and figures.

The calculator

A calculator is allowed on every Math question and is not allowed in Reading and Writing. The built-in calculator can be switched between scientific and graphing modes. You may also bring an approved handheld calculator, but calculators with computer algebra system (CAS) functionality are not allowed. The approved list changes, so confirm your model on the College Board website.

The reference sheet

The reference sheet gives area formulas for a circle, a rectangle and a triangle; the Pythagorean theorem; the two special right triangles (30°-60°-90° and 45°-45°-90°); and volume formulas for a rectangular solid, a cylinder, a sphere, a cone and a pyramid. It also states that a circle has 360 degrees and 2π radians of arc and that the angles of a triangle sum to 180 degrees. It does not include slope, line equations, the quadratic formula, the circle equation, trigonometric ratios, percent change or statistics formulas. Chapter 15 and Appendix D list everything you need to know by heart.

Scratch paper

Testing staff give you scratch paper and collect it at the end of the test. You may not bring your own paper, but you do bring pencils or pens for working on it. Practice with scratch paper at home, because some steps, such as setting up a system of equations or sketching a figure, are faster on paper than on screen.

What this book can and cannot do

What this book does:

- Teaches all 11 Reading and Writing skills and all 19 Math skills, each under its official name, with a repeatable method and fully worked examples.
- Starts with a 49-question diagnostic that produces a skill profile, so you know which chapters to study first.
- Gives you a Practice Set in every skill chapter, arranged from easier to harder, with every answer choice explained.
- Includes two full-length practice tests with the same question counts, section order and timing as the digital SAT.

- Explains graphing-calculator methods in words and gives the exact rules for entering student-produced responses.

What this book cannot do:

- **Adapt.** A printed test cannot change based on your answers. Each practice test's Module 2 is a fixed set that is somewhat harder than Module 1, similar in spirit to the higher-difficulty path.
- **Estimate a scaled SAT score.** These tests have not been statistically calibrated to the SAT. Chapters 30 and 32 report correct counts and percent correct for study planning.
- **Copy the screen.** Paper cannot give you practice with Bluebook's tools, timer and layout.

For all three reasons, use the College Board's free full-length practice tests in the Bluebook app alongside this book. They adapt like the test itself, use the same on-screen tools and report practice scores on the SAT scale. Chapter 2 shows where they fit in your plan.

Where scores fall

A percentile rank compares a score with the scores of a group of other students. A percentile of 75, for example, means the score was equal to or higher than about 75% of that group's scores. The table shows a few total scores with their percentile ranks for the SAT user group.

Total score	User percentile
1500	97
1400	93
1300	85
1200	75
1100	62
1000	47
900	32

These are College Board user percentiles as published in October 2026; they are updated each year. Use them for context only. Your goals should come from the colleges you are considering, which Chapter 2 shows you how to research. Do not convert this book's percent-correct results into SAT scores or percentiles.

What to take from this chapter

- Two sections, four modules, 98 questions and 134 minutes of testing, with a 10-minute break between the sections.
- Adaptation happens between modules. Module 1 accuracy matters, and no routing cutoff is published.
- Scores run from 200 to 800 per section and 400 to 1600 in total, with no penalty for wrong answers.
- A calculator is allowed on every Math question, and a reference sheet is always available in Math.
- This book teaches the skills and gives you fixed, print practice. The free Bluebook practice tests give you the adaptive experience and practice scores on the SAT scale.

Chapter 2 turns these facts into a study plan built around your own strengths and weaknesses.

ONLINE (OPTIONAL) The website's test-format guide puts the sections, modules, timing and tools from this chapter on one page.

skillbyskillprep.com/test-format

CHAPTER 2

Your Study Plan

A good plan spends your time where your points are, builds in review so that what you learn stays learned, and places full-length practice where it tells you the most. This chapter builds that plan in six steps, then covers registration basics, because your test date anchors the whole calendar.

Step 1: Take the diagnostic and read your skill profile

The Diagnostic Test comes right after Chapter 3. It has 27 Reading and Writing questions and 22 Math questions, arranged like Module 1 of each section: a broad mix of easier, medium and harder questions. It covers every Reading and Writing skill and most Math skills. Take it before you study any skill chapter, so the results show where you are now rather than what you reviewed yesterday.

You can take it timed (32 minutes for Reading and Writing, then 35 minutes for Math) or untimed. Timed shows your pacing; untimed shows what you know. A good compromise: time it, note where you are when time runs out, then finish untimed so that your profile covers every skill.

After you check your answers, fill in the skill profile in the Diagnostic Answer Key. Mark every question you missed and every question you got right by guessing. The profile groups the questions by skill and names the chapter that teaches each one. Then sort your skills into the priority levels below.

What your profile shows	Priority	What to do
Two or more misses in one domain	Top	Study every chapter in that domain early in your plan.
One miss in a skill	High	Study that chapter in the first half of your plan.
Correct, but you guessed	Medium	Read the lesson and do the Practice Set; move on if you do well.
Correct and confident	Maintain	Do the Practice Set as a check and spend the saved time elsewhere.

The diagnostic reports skill-level results, not a scaled SAT score. It samples only one module per section and has not been statistically calibrated to the SAT. Its job is to tell you where to start.

TIP Write your three weakest Reading and Writing skills and your three weakest Math skills on a card and keep it in this book. Those six skills get your best study hours.

ONLINE (OPTIONAL) Prefer to start on screen? The website has a free online diagnostic, and its study plan guide sums up the six steps in this chapter. The printed diagnostic after Chapter 3 is all you need.

Free online diagnostic
skillbyskillprep.com/practice?mode=diagnostic

Study plan guide
skillbyskillprep.com/study-plan-guide

Step 2: Set a target score

Your target should come from the colleges you are considering, not from a friend's score or a round number. Many colleges publish a *middle-50% range* for the SAT: the span from the 25th-percentile score to the 75th-percentile score of the students they admit or enroll.

1. List three to six colleges you are seriously considering.
2. Find each college's published middle-50% SAT range on its admission website or in its published admission profile. Note whether the range is for admitted or enrolled students.
3. Check each college's own testing policy, because colleges differ in whether and how they use scores.
4. Set a target at or above the middle of the range for the colleges you care about most.
5. Split the target into a Reading and Writing goal and a Math goal, based on which section has more room to grow.

College	Published middle-50% SAT range	Testing policy	My target
—	—	—	—
—	—	—	—
—	—	—	—
—	—	—	—

The College Board's college and career readiness benchmarks give you a second reference point. As of October 2026 they are 480 for Reading and Writing and 530 for Math. Use them as context alongside your own college goals; they do not guarantee admission or college success.

For a practice score on the SAT scale, use one of the College Board's free full-length practice tests in Bluebook. This book's tests report correct counts and percent correct, not scaled scores. If you want a starting score early, take one Bluebook practice test in your first or second week and save the rest for later.

Step 3: Choose your plan length

Pick the table that matches the weeks between today and your test date. The hours are guides, not rules: move top-priority chapters earlier and give them extra time, and treat strengths as quick checks.

Each table alternates Reading and Writing chapters with Math chapters so that both sections improve together. "Untimed" and "timed" refer to the two rounds of practice in Step 4. Day numbers count down: day 14 means 14 days before your test.

The 12-week plan (about 5 to 7 hours a week)

Week	Chapters	Drills	Full tests	Review
1	1, 2, 3; Diagnostic; 4	—	Diagnostic	Fill in the skill profile; set your target
2	5, 6, 15, 16	Practice Sets 5, 6, 15, 16 (untimed)	—	Start the error log
3	7, 8, 17, 18	Practice Sets 7, 8, 17, 18 (untimed)	—	3-day redos
4	9, 10, 19, 20	Practice Sets 9, 10, 19, 20 (untimed)	—	3-day and 10-day redos
5	11, 12, 21, 22	Practice Sets 11, 12, 21, 22 (untimed)	—	3-day and 10-day redos
6	13, 14, 23, 24	Practice Sets 13, 14, 23, 24 (untimed)	—	3-day and 10-day redos
7	25, 26, 27, 28	Practice Sets 25–28 (untimed)	—	3-day and 10-day redos
8	29, 30	Timed round for Practice Sets 5–14 that had misses	Practice Test 1 (about day 34)	Review every miss with Chapter 30
9	Your three weakest chapters from Test 1	Timed round for Practice Sets 15–28 that had misses	Bluebook practice test (about day 27)	Log and redo every miss
10	31, 32	Timed redos for your weakest skills	Practice Test 2 (about day 20)	Compare Tests 1 and 2 with Chapter 32
11	33 (days 14 to 8)	Timed redos for your weakest skills	Bluebook practice test (day 14)	Full error-log redo
12	33 (days 7 to 1)	Short timed sets	Bluebook practice test (day 7)	Final error-log pass; rest and logistics

The 8-week plan (about 6 to 9 hours a week)

Week	Chapters	Drills	Full tests	Review
1	1, 2, 3; Diagnostic; 4	—	Diagnostic	Fill in the skill profile; set your target
2	5, 6, 7, 15, 16	Practice Sets 5, 6, 7, 15, 16 (untimed)	—	Start the error log
3	8, 9, 10, 17, 18	Practice Sets 8, 9, 10, 17, 18 (untimed)	—	3-day redos
4	11, 12, 19, 20, 21	Practice Sets 11, 12, 19, 20, 21 (untimed)	—	3-day and 10-day redos

Week	Chapters	Drills	Full tests	Review
5	13, 14, 22, 23, 24	Practice Sets 13, 14, 22, 23, 24 (untimed)	—	3-day and 10-day redos
6	25, 26; 29, 30; then 27, 28	Practice Sets 25–28 (untimed)	Practice Test 1 midweek (about day 18)	Review every miss with Chapter 30 before day 14
7	33 (days 14 to 8); your weakest chapters	Timed round for your weakest Practice Sets	Bluebook practice test (day 14)	Log and redo every miss
8	31, 32; 33 (days 7 to 1)	Short timed sets	Practice Test 2 (day 7)	Compare Tests 1 and 2; rest and logistics

Week 6 is the heaviest week in this plan. Taking Practice Test 1 midweek leaves time to review it before the day-14 test and to finish Chapters 27 and 28.

The 4-week plan (about 10 to 12 hours a week)

With four weeks, you cannot study every chapter in depth. Let the skill profile choose your chapters, and use Chapter 15 early because it affects every Math question.

Week	Chapters	Drills	Full tests	Review
1	1, 2, 3; Diagnostic; 4, 15; your top three priority chapters	Practice Sets for those chapters (untimed)	Diagnostic	Skill profile; list your priority chapters in order
2	Your next five or six priority chapters	Their Practice Sets, untimed, then timed	Practice Test 1 midweek (about day 18)	Review every miss with Chapter 30 before day 14
3	33 (days 14 to 8); chapters for your weakest Test 1 domains	Timed round for those Practice Sets	Bluebook practice test (day 14)	Log and redo every miss
4	31, 32; 33 (days 7 to 1)	Short timed sets	Practice Test 2 (day 7)	Compare Tests 1 and 2; rest and logistics

The 2-week plan (about 12 to 15 hours a week)

Two weeks is enough to fix a few high-value skills and to rehearse the test's timing, but not to relearn a whole section.

Week	Chapters	Drills	Full tests	Review
1	1 and 3; Diagnostic; 15; your two weakest chapters in each section	Their Practice Sets, timed	Diagnostic; Bluebook practice test (about day 10)	Log every miss and guess
2	33 (days 7 to 1); one more weak chapter if time allows	Timed error-log redos	Practice Test 1 or a second Bluebook practice test (day 7)	Final error-log pass; rest and logistics

Step 4: Work the Practice Sets in two rounds

Every skill chapter ends with a Practice Set arranged from easier to harder. Do each set twice, for two different purposes.

Round 1: untimed, to learn the method. Do this right after you read the lesson.

1. Work through each worked example with the solution covered by a card. Try it first, then compare.
2. Do the whole Practice Set without a clock. Beside each answer, write S if you are sure or U if you are unsure.
3. Check your answers with the explanations that follow the set. Read the explanation for every miss and every U, including the reason each wrong choice fails.
4. Within 24 hours, log every miss and every lucky guess in the error log (Step 5).

Round 2: timed, to build pace. At least a week later if your plan allows, redo the set with a clock running at the test's average pace: 71 seconds per Reading and Writing question and 95 seconds per Math question. Do this round for every set in which round 1 produced a miss or a U; a set you finished with every answer sure and correct has already done its job. Any question you miss again goes into the error log with a star.

Questions in the set	Reading and Writing (71 seconds each)	Math (95 seconds each)
10	—	about 16 minutes
12	about 14 minutes	19 minutes
15	about 18 minutes	about 24 minutes
18	—	about 29 minutes
20	about 24 minutes	—

You will remember some answers in round 2. That is fine: the point is to apply the method at test speed. If you finish with time to spare and every answer correct, try an unfamiliar question on the same skill. A remembered answer alone does not show that the method transfers.

TRAP Reading an explanation and thinking "I knew that" is not the same as being able to do it. A redo without looking checks whether you can recall the method; an unfamiliar question checks whether you can apply it.

Step 5: Keep an error log and redo on schedule

Mistakes tend to repeat until you name them, and you cannot fix a mistake you have forgotten. Appendix E contains a ready-to-copy error log with a column for each part of the routine below.

The routine has four parts.

1. **Log within 24 hours.** Log every miss and every correct answer you guessed.
2. **Name the miss type.** Use one of five types: content gap, trap, misread, careless or time. Appendix E explains each one.
3. **Redo after 3 days and again after 10 days.** Cover the explanation and solve the question again. A redo after a few days forces you to recall the method, which is exactly what test day asks you to do.
4. **Retire or repeat.** If you get it right on both redos, retire the entry. If you miss it again, reread that part of the chapter and schedule a new pair of redos.

Once a week, count your miss types. The most common type tells you what to change next.

Most common miss type	What to change
Content gap	Reread the lesson and redo the worked examples before more practice.
Trap	Study the trap table in the chapter and name the trap before you choose.
Misread	Name the task word in each question before solving; in Math, write what you are solving for on scratch paper.
Careless	Add a five-second check to your method: reread the question, then confirm the answer.
Time	Practice the two-pass method and the checkpoints in Chapter 3.

Step 6: Schedule your full-length tests

Full-length tests do two jobs that Practice Sets cannot: they build stamina for a test of more than two hours, and they show how your pacing holds up when the questions are mixed.

Test	When to take it	What it tells you
Diagnostic	Right after Chapter 3, before any skill chapter	Your starting skill profile
Practice Test 1	After most skill chapters	Your first full-length rehearsal, correct counts and domain results
Practice Test 2	At least one week after you finish reviewing Test 1	Whether your weak domains have improved
Bluebook practice tests	At least one in the final two weeks, more if time allows	Practice scores on the SAT scale, with the adaptive design and the on-screen tools

Take every full-length test in one sitting under test conditions: the real time limits, the 10-minute break between sections, scratch paper, no phone and no pausing. If you can, start at the same time of day as your test. The College Board also offers free paper versions, but use the Bluebook versions when you can, because only they adapt and use the on-screen tools.

Registration basics

SAT Weekend is given at test centers on Saturdays, and you register yourself. SAT School Day is given at schools during set windows (for spring 2027, 1 March to 30 April), and your school or state chooses the day. Ask your school counselor whether your school offers it.

For SAT Weekend, keep these points in mind.

- **Fee.** The U.S. registration fee is $68 as of October 2026; students testing internationally pay an additional regional fee. Fee waivers are available to eligible students; your school counselor can tell you whether you qualify.
- **Deadlines.** Each test date has a registration deadline and a later deadline for changes and regular cancellation. Late registration is available for an extra fee, and a waitlist is available in the U.S.
- **Sunday testing.** It is offered only to students who cannot test on Saturday for religious reasons.

- **Devices.** If you need to borrow a testing device, request one at least 30 days before your test.

As of October 2026, the College Board lists these spring 2027 SAT Weekend dates.

Test date	Registration deadline	Deadline for changes, regular cancellation and late registration
6 March 2027	19 February 2027	23 February 2027
1 May 2027	16 April 2027	20 April 2027
5 June 2027	21 May 2027	25 May 2027

For the 2027–28 school year, the College Board lists these *anticipated* SAT Weekend dates. Check the College Board website for their registration and change deadlines.

Season	Anticipated 2027–28 test dates
Fall 2027	28 August, 18 September, 9 October, 6 November, 4 December
Spring 2028	4 March, 6 May, 3 June

NOTE Dates, deadlines and fees can change. Confirm them on the College Board website before you register.

Choose a test date that gives you the plan length you need and still leaves a later date before your college application deadlines, in case you decide to retake. Chapter 33 helps you make that decision after your first test.

CHAPTER 3

Pacing, Guessing and Test-Day Strategy

Knowing the material is only part of a good score. You also have to show what you know inside fixed time limits, on a screen, on one particular day. This chapter gives you a time budget, a two-pass method for each module, a clear rule about guessing, and a plan for the break and for test day itself. Practice these habits on every full-length test so that on test day they run on their own.

Your time budget

Module	Questions	Time	Average per question
Reading and Writing (each module)	27	32 minutes	about 71 seconds
Math (each module)	22	35 minutes	about 95 seconds

An average is not a speed limit. Some questions should take 30 seconds, and a few will deserve more than two minutes. What matters is reaching the end of the module with every question answered and a few minutes left to revisit the ones you marked.

Checkpoints

The Bluebook timer counts down, so set your checkpoints by time remaining. The plan below finishes the first pass with about 3 minutes to spare. In Math it saves time early, because Math questions get harder as the module goes on.

Checkpoint	Reading and Writing (27 questions, 32 minutes)	Math (22 questions, 35 minutes)
First checkpoint	22 minutes left: about 9 questions done	28 minutes left: about 7 questions done
Second checkpoint	12 minutes left: about 18 questions done	17 minutes left: about 15 questions done
End of the first pass	3 minutes left: every question answered	3 minutes left: every question answered

In Reading and Writing, this works out to a little over a minute per question on the first pass. In Math, it means about 1 minute for each of the first 7 questions, about 80 seconds for each of the next 8 and about 2 minutes for each of the last 7.

If you are behind at a checkpoint, do not rush the next few questions; apply the stuck rule below more strictly until you are back on pace. Rushing easier questions is how careless misses happen.

TIP Write your checkpoints at the top of your scratch paper after the proctor tells you to begin: "R&W 22 / 12 / 3" and "Math 28 / 17 / 3". Then you only need to glance at the timer when you reach those numbers.

The two-pass method

The test lets you move freely within a module, mark questions for review and change answers until time runs out. The two-pass method uses that freedom so that one hard question never costs you three easy ones.

Pass 1: answer everything once.

1. Work through the module in order.
2. Use the stuck rule. If you cannot see a way to start after about 30 seconds, or a question has taken about 2 minutes and the end is not in sight, stop.
3. Eliminate what you can, choose your best answer, click Mark for Review and move on.

Pass 2: spend the time you saved.

1. Use the Question Menu to return to your marked questions. Start with the ones where you narrowed the choices to two, because they are the most likely to turn into points.
2. Change an answer only for a reason you can name: you found new evidence in the passage or an error in your work. A vague feeling is not a reason.
3. In the final minute, confirm that every question has an answer.

TRAP Marking a question and leaving it blank "for later". If later never comes, a blank cannot earn a point. Always choose an answer first, then mark it.

Never leave a blank

There is no penalty for a wrong answer on the digital SAT. The College Board says that for most students it is better to guess than to leave a question blank. A guess has a chance of being right; a blank has none.

Make your guesses smart guesses.

- On multiple-choice questions, use the Option Eliminator to cross out choices you can rule out, then choose among the rest.
- If you have no basis at all, pick any choice and move on without agonizing.
- On student-produced responses, enter your best attempt. If you have a partial result or a reasonable estimate, type it in.

How difficulty is arranged

Knowing the order helps you decide where to spend time and keeps one hard question from shaking you.

Reading and Writing: difficulty restarts

Each Reading and Writing module follows the same domain order: Craft and Structure, then Information and Ideas, then Standard English Conventions, then Expression of Ideas. In the reading domains and in Expression of Ideas, questions that test the same skill are grouped together, and each group runs from easiest to hardest. In Standard English Conventions, Boundaries questions and Form, Structure, and Sense questions are mixed together and arranged from easiest to hardest.

So difficulty does not climb steadily through a Reading and Writing module. It rises within a group and then drops back at the start of the next one, like the teeth of a saw. The last question in a skill group may be among

the hardest in the module, and the first question of the next group is usually easier. The last reading question before the conventions questions is often one of the harder ones, and the conventions questions then start easier again.

Three habits follow from this.

- **Do not let the end of a group drag you down.** If the last question in a group is hard, use the stuck rule. The next group starts easier.
- **Bank time at the start of each group.** Easier questions at the start of a group are where you rebuild time, as long as you still read carefully.
- **Expect the conventions questions to move faster.** They are often quicker than the reading questions, so they are a good place to catch up if you are behind.

The skills inside each domain usually appear in the order the chapters of this book follow. Chapter 4 maps a whole module for you.

Math: difficulty climbs

Within each Math module, questions run from easiest to hardest, with the four domains mixed together. Student-produced responses are mixed in with the multiple-choice questions rather than grouped, and pretest questions can appear anywhere.

- **Early questions:** these are the questions you should not miss. Work efficiently, but read every word of the question. A careless slip on an early question gives away a point you had already earned.
- **Middle questions:** expect two or three steps and a context to translate. Write the setup on scratch paper.
- **Late questions:** expect parameters, unfamiliar forms and models to interpret. These deserve the time you saved early. If one is beyond you after a fair try, guess, mark it and give your time to the next one.

Why Module 1 accuracy matters

Your performance on Module 1 decides whether your Module 2 is, on average, harder or easier. The College Board does not publish the cutoff, so there is no number of misses you can safely plan for. Your section score then uses every scored question from both modules.

The College Board states that students are not advantaged or disadvantaged by which Module 2 they receive. Still, Module 1 both counts toward your score and sets your Module 2 path, so treat it with special care.

- In Module 1, protect the points on easier and medium questions. Read each one fully, check the questions you were unsure about and use your pass-2 time.
- Do not speed through Module 1 to "save energy" for Module 2. Both modules count toward your score.
- In Module 2, use the same method. Do not spend time guessing which version you received; it changes nothing about the next question.

Using the timer

The Bluebook timer counts down the time left in the current module. You can hide it until 5 minutes remain; then an alert appears.

Some students work better with the timer always visible; others find a constant countdown distracting. Try both on your practice tests and choose one. If you hide it, check the time at your checkpoints. A watch without an audible alarm is allowed and recommended as a backup.

When the 5-minute alert appears, finish the question you are on, then make sure every question in the module has an answer before you return to anything you marked.

Using the 10-minute break

The 10-minute break comes between the Reading and Writing section and the Math section. Use it to reset your body and your attention.

1. Stand up and stretch. Sitting still for over an hour makes concentration harder.
2. Drink water and eat the snack you brought. Choose something you have eaten before a practice test.
3. Do not replay the Reading and Writing section in your head or discuss it. It is finished.
4. Prepare for Math with one sentence: "The first questions are the easiest; I will get them right." Picture your checkpoints: 28, 17, 3.
5. Return on time.

Phones, smartwatches and other wearables or communication devices cannot be used during the break either.

SAT Weekend test-day checklist

Pack your bag the night before. This checklist is for SAT Weekend. SAT School Day students should follow their school's instructions and use the school-issued Bluebook sign-in ticket; their exam setup and identification requirements can differ. Confirm the current lists on the College Board website, because rules can change.

Item	Status	Notes
Testing device, fully charged, with Bluebook installed and exam setup completed	Required	The device needs a battery life of at least 3 hours. A phone cannot be your testing device.
Admission ticket from Bluebook	Required	Available after exam setup, starting 5 days before the test. A printed copy is preferred.
Acceptable photo ID	Required	It must be a physical document. Check the ID requirements on the College Board website.
Pencils or pens	Required	For scratch work. Staff provide the scratch paper.
College Board username and password	Required	You need them to sign in.
Acceptable calculator	Optional	No calculators with computer algebra system (CAS) functionality. The built-in calculator is available on every Math question.
Watch without an audible alarm	Recommended	A backup for checking your checkpoints.
Charging cable or portable charger	Recommended	Insurance against a low battery.
Drink or snacks for the break	Recommended	Something familiar.
Backup testing device	Recommended	Useful if your main device has a problem.

Some items are not allowed. Phones, smartwatches, smart glasses, fitness trackers and other wearables or communication devices may not be used during the test or the breaks. Headphones and earbuds are not allowed

unless an approved accommodation requires them. You may not bring your own scratch paper; testing staff provide scratch paper and collect it at the end.

Accommodations

If you have a documented disability, you may be eligible for testing accommodations. Approved accommodations include extended time (such as time and one-half or double time), extra and extended breaks, and assistive technology such as Text-to-Speech. Requests take time, so ask your school counselor or check the College Board website well before you register. If you are approved for extended time, plan checkpoints using the actual module times and breaks in your approved setup, and practice with those settings. Approval may apply to one section or the whole test.

Device rules

Bluebook's device requirements changed for fall 2026. As listed by the College Board in October 2026, Windows devices need a supported Windows 11 release, and Chromebooks must be school-managed with verified mode enabled. Technology rules change often, so confirm the current requirements on the College Board website and test your device in Bluebook well before test day. If you do not have a suitable device, you can request to borrow one at least 30 days before your test.

Managing nerves

Some nervousness is normal, and a little of it can sharpen your focus. The goal is not to feel nothing; it is to keep nerves from changing how you work. These habits help.

1. **Rehearse.** Take your full-length practice tests in one sitting, with real timing, at the same time of day as your test. Familiar conditions feel less threatening.
2. **Have a first-minute routine.** At the start of each module, read the first question slowly and completely. A calm first question sets the pace for the rest.
3. **Use slow breathing.** When you feel tense, breathe in for a count of four and out for a count of six, three times. It takes about 30 seconds.
4. **Reset after a hard question.** Use three words: mark, breathe, move. A hard question tells you something about that question, not about you, and every student meets questions they cannot solve.
5. **Keep a bad module in perspective.** Your section score uses both modules. A rough stretch is not the end of the section, so give the next question your full attention.
6. **Protect the night before.** Do no new studying. Pack your bag, charge your device, eat a normal dinner and go to bed at your usual time.
7. **Plan the morning.** Eat a breakfast you are used to, leave early and arrive with time to spare.

If anxiety about tests feels overwhelming or affects your daily life, talk with a school counselor or another trusted adult. That is a sensible step, not a sign of weakness.

Your pacing plan in one view

- Checkpoints: Reading and Writing 22 / 12 / 3 minutes left; Math 28 / 17 / 3 minutes left.
- Pass 1: answer every question once, using the stuck rule. Pass 2: return to marked questions.
- Never leave a blank, and never mark a question without choosing an answer.

- Reading and Writing difficulty restarts with each skill group; Math difficulty climbs.
- Module 1 deserves full care; Module 2 deserves the same method.

Now take the Diagnostic Test that follows this chapter. Use these habits on it, and you will learn as much about your pacing as about your skills.

ONLINE (OPTIONAL) The website's test-day guide puts a time budget for every module, the checklist and the break on one page.

skillbyskillprep.com/test-day

Diagnostic Test

Before you begin

This diagnostic gives you one module's worth of each section of the digital SAT: 27 Reading and Writing questions and 22 Math questions, arranged like a first module on test day. Take it before you study Chapters 4–28. Its job is to show you which skills to study first, so answer the way you would today, without looking anything up.

How to take it

1. Gather a pencil, scratch paper and a calculator for the Math questions (a graphing calculator if you have one).
2. Mark your answers on the Diagnostic Answer Sheet that follows these instructions. For a student-produced response (a Math question with no answer choices), write your number, fraction or decimal on the labeled answer line. Chapter 15 explains the entry rules for the digital test.
3. Time yourself if you can: 32 minutes for Reading and Writing, a 10-minute break, then 35 minutes for Math, the module times on test day. If time runs out, note the last question you reached, then finish untimed so that you have a response to every question. Mark answers completed after the time limit so that you can separate pacing issues from skill gaps.
4. Answer every question. The SAT does not penalize wrong answers, so guess when you must, but write a small "g" beside any guess on your answer sheet.
5. Use the Math reference sheet printed before the Math questions, just as you would on test day.

How to score it and read your skill profile

After the Math questions you will find the answer key, followed by a table titled "Your skill profile." It lists each skill tested, the diagnostic questions that test it and the chapter that teaches it.

This short diagnostic samples 26 of the 30 tested skills. It has no standalone questions on two-variable data (Chapter 24), sampling and margin of error or evaluating statistical claims (Chapter 25), or circles (Chapter 28). Include those lessons in your study plan even if you answer every diagnostic question correctly.

1. Check each answer against the key. For a student-produced response, compare your value with the key. Equivalent fractions or decimals are also correct when they follow Chapter 15's entry rules; the key does not list every possible form.
2. In the Missed column of the skill profile, write the number of every question you missed or marked "g." A lucky guess is not yet a skill you own.
3. Read the explanation for each of those questions. Every explanation names the chapter to review.

Then read the profile this way. **Any skill with a miss is a priority. Two or more misses in one domain is a red flag:** plan extra time for that whole group of chapters. Use this table to group your misses by domain.

Domain	Chapters	Diagnostic questions
Craft and Structure	5–7	Reading and Writing 1–8
Information and Ideas	8–10	Reading and Writing 9–15

Domain	Chapters	Diagnostic questions
Standard English Conventions	11–12	Reading and Writing 16–22
Expression of Ideas	13–14	Reading and Writing 23–27
Algebra	16–19	Math 1, 4, 5, 8, 11, 12, 13, 19
Advanced Math	20–22	Math 6, 10, 15, 17, 18, 21, 22
Problem-Solving and Data Analysis	23–25	Math 2, 7, 9, 16
Geometry and Trigonometry	26–28	Math 3, 14, 20

Why there is no score

The diagnostic produces a skill profile, not an SAT-scale score. The questions in this book have not been statistically calibrated to the SAT scale, and its printed tests do not adapt. Chapters 30 and 32 show you how to track your number correct, percent correct and domain results after each full-length practice test.

What to do next

Turn to Chapter 2, "Your Study Plan," and build your schedule around the red flags and priority skills in your profile. Log every miss in the error log (Appendix E) so you can redo those questions later.

Diagnostic Answer Sheet

Choose one bubble for multiple-choice questions. On rows labeled Answer, write a number, fraction or decimal. Follow the entry rules in Chapter 15.

Reading and Writing

1 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**19** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**21** Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	**13** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**23** Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Ⓐ Ⓑ Ⓒ Ⓓ	**24** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**16** Ⓐ Ⓑ Ⓒ Ⓓ	**25** Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	**17** Ⓐ Ⓑ Ⓒ Ⓓ	**26** Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ	**18** Ⓐ Ⓑ Ⓒ Ⓓ	**27** Ⓐ Ⓑ Ⓒ Ⓓ

Math

1 Ⓐ Ⓑ Ⓒ Ⓓ	**9** Answer: _____	**17** Answer: _____
2 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**18** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Ⓐ Ⓑ Ⓒ Ⓓ	**19** Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
5 Answer: _____	**13** Answer: _____	**21** Answer: _____
6 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Ⓐ Ⓑ Ⓒ Ⓓ	
8 Ⓐ Ⓑ Ⓒ Ⓓ	**16** Ⓐ Ⓑ Ⓒ Ⓓ	

Diagnostic: Reading and Writing

Suggested time: 32 minutes for 27 questions.

1 RW-D01

> When the night train finally pulled into Harlow Junction, Inés pressed her face to the window. She had expected a busy station, but the platform was nearly _____: a single lamp, one empty bench, and a porter asleep on a stack of mailbags. She wondered whether anyone would be waiting for her at all.

Which choice completes the text with the most logical and precise word or phrase?

A crowded **B** deserted **C** colorful **D** dangerous

---

2 RW-D02

> During a chemistry lab, a student tested a cleaning solution with an indicator, a dye whose color depends on how acidic or basic a liquid is. This indicator stays colorless in pure water but turns bright pink in a basic solution. When the student added one drop to the cleaning solution, the liquid turned pink at once, a _____ sign that the cleaner was basic.

Which choice completes the text with the most logical and precise word or phrase?

A faint **B** hidden **C** delayed **D** definite

---

3 RW-D03

> Folklorists at a fictional regional archive have found that no single version of a story is the "original." A tale about a clever fox may be told one way in a mountain village and quite differently in a coastal town fifty miles away, and the archive treats both tellings as equally authentic. For this reason, its collectors _____ the urge to correct or combine versions, preserving each teller's words exactly as spoken.

Which choice completes the text with the most logical and precise word or phrase?

A resist **B** indulge **C** announce **D** inherit

---

4 RW-D04

> Before standardized time zones were widely adopted in North America, communities often kept local time, setting their clocks by the sun. As railways spread, this old custom became a source of confusion: travelers had to juggle different local and railway time standards. In the United States and Canada, where a single long-distance route could cross places several hours apart in local time, the need for a shared standard was especially _____.

Which choice completes the text with the most logical and precise word or phrase?

A remote **B** trivial **C** pressing **D** gradual

---

5 RW-D05

> The old ferryman spoke rarely, and when he did, his remarks were so _____ that passengers often puzzled over them for the rest of the crossing, long after the far bank had come into view. Asked whether the river was deep, he said only, "Deep enough for the questions people bring to it." Asked how long he had worked the ferry, he replied, "Longer than the river has worked me." The schoolteacher who rode each morning decided that he was either a philosopher or a man who simply enjoyed being misunderstood.

Which choice completes the text with the most logical and precise word or phrase?

A enigmatic **B** tedious **C** boastful **D** candid

6 RW-D06

> Each autumn the swallows gather on the wire,
> a row of dark commas along the road.
> They chatter, lift, and settle, lift again,
> as if rehearsing something they already know.
> Then one gray morning the wire is bare,
> no note, no sign, just the long field and the wind,
> and I, who stay, still leave the porch light burning
> for travelers who will not need it.

Which choice best states the main purpose of the text?

A To explain how swallows prepare their bodies for long migrations

B To argue that people should protect the fields where swallows gather

C To compare the speaker's own travels with the journeys of the swallows

D To describe the departure of a flock of birds and the speaker's response to their absence

7 RW-D07

> A team of materials chemists in a hypothetical study set out to create a plastic that breaks down safely in seawater. Their first version dissolved within days, far too quickly to be useful for fishing nets. The team then adjusted the recipe, adding a small amount of a slower-reacting ingredient that extended the plastic's life in seawater to about two years. Finally, the chemists checked whether the modified plastic released substances harmful to fish as it broke down, and they found none in their trials.

Which choice best describes the function of the underlined portion in the text as a whole?

A It states the main goal that guided the team's research.

B It describes how the team responded to a problem with the first version of the plastic.

C It presents evidence that the plastic is safe for fish.

D It explains why fishing nets need to last for about two years.

8 RW-D08

> **Text 1**
> Oral history, the recording of interviews in which people describe their own pasts, has an obvious weakness: memories fade and shift. A person interviewed decades after an event may misremember dates, merge separate incidents, or borrow details from later accounts. Oral testimony is therefore most useful where written records are missing, and even then it should be checked carefully against documents.
>
> **Text 2**
> Interviewees certainly make factual errors. Imagine a former streetcar conductor who insists that a strike lasted "all winter," when newspapers show that it lasted three weeks. The exaggeration is still valuable. It reveals how long and hard the strike felt to the workers and why it mattered so much in their community. Such distortions are themselves evidence, of a kind that documents rarely provide.

Based on the texts, how would the author of Text 2 most likely respond to the view of oral testimony presented in Text 1?

A By denying that people interviewed long after an event misremember facts such as dates

B By agreeing that oral testimony should be used mainly where written records are missing

C By acknowledging that oral testimony contains errors but arguing that some errors are informative in their own right

D By claiming that newspapers and other documents are generally less accurate than interviews

---

9 RW-D09

> Every Sunday, Tomás sat with his great-aunt Lucía while she told stories about the village where she grew up. At first he only half listened, glancing at his phone. Then one week she described the bakery her father had run, and Tomás realized that no one else alive remembered it. He found a notebook and began writing down her stories, asking her to repeat names and spell them out so that he could get every detail right.

Which choice best states the main idea of the text?

A Tomás comes to value his great-aunt's stories and begins working to preserve them.

B Tomás's great-aunt is reluctant to share details about the village where she grew up.

C Tomás decides to learn the recipes used in the bakery that his great-aunt's father ran.

D Tomás prefers looking at his phone to listening to stories about the past.

---

10 RW-D10

> To learn how young sparrows develop their songs, a team of biologists in a hypothetical study raised two groups of chicks in quiet indoor rooms. One group heard daily recordings of adult sparrow song during their first months of life; the other group heard no sparrow song at all. When the birds matured, those in the first group sang songs closely resembling the recordings, while those in the second group produced only simple, unstructured songs. The team concluded that exposure to adult song helped these birds develop typical songs under the study conditions.

Which choice best states the main idea of the text?

A Sparrows raised indoors sing more complex songs than sparrows raised in the wild.

B Young sparrows can learn their songs equally well at any point in their lives.

C Recordings of adult song are more effective teachers than live adult sparrows.

D A study suggests that hearing adult song helps young sparrows develop typical songs.

---

11 RW-D11

> For a science fair project, a student timed how long identical sugar cubes took to dissolve in stirred water at four different temperatures. The student used the same amount of water and the same stirring speed in every trial. The student concluded that sugar dissolves more quickly in warmer water, noting that a sugar cube dissolved in _____

Time for a Sugar Cube to Dissolve (hypothetical data)

Water temperature (°C)	Time to dissolve (seconds)
10	210
25	150
40	95
60	55

Which choice most effectively uses data from the table to complete the statement?

A 55 seconds in 10°C water but took 210 seconds in 60°C water.

B 55 seconds in 60°C water but took 210 seconds in 10°C water.

C 150 seconds in 25°C water and also took 150 seconds in 40°C water.

D less than 250 seconds at every temperature the student tested.

---

12 RW-D12

> A student is writing a short story about Hollis, a former railway signalman who spent forty years in a small signal box beside a busy line. The student wants to show that Hollis takes comfort in keeping the same daily routines he followed during his working years, even though he no longer has a signal box to run.

Which quotation from the student's draft most effectively illustrates this portrayal of Hollis?

A "Hollis kept a shoebox of photographs from his years on the railway, though he rarely opened it."

B "When his granddaughter suggested a spur-of-the-moment trip to the coast, Hollis agreed at once and packed that evening."

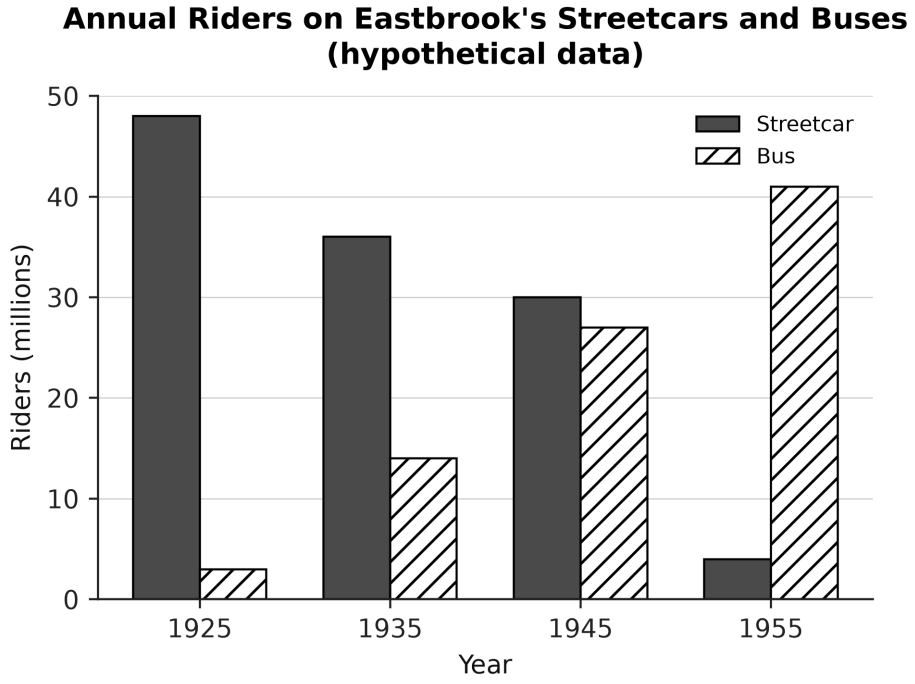
C "At 6:14 each morning, the minute the old express had once passed his signal box, Hollis set down his coffee and checked his watch, and only then did his shoulders relax."

D "The signal box where Hollis had worked for forty years was torn down the spring after his final shift."

---

13 RW-D13

> The fictional city of Eastbrook once relied on streetcars, but over several decades it shifted to buses, which needed no tracks or overhead wires and could be rerouted as new neighborhoods grew. A historian studying Eastbrook's transit records argues that the change happened in stages rather than through a single decision. For a period, the historian says, buses and streetcars each carried a large number of the city's riders, and only later, as the last lines closed, did streetcar ridership collapse.



Which choice most effectively uses data from the graph to support the historian's argument?

A Bus ridership rose in each year shown, from 3 million riders in 1925 to 41 million in 1955.

B Streetcars carried more riders than buses did in 1925, 1935, and 1945.

C In 1945, streetcars and buses each carried more than 25 million riders, but by 1955 streetcars carried fewer than 5 million.

D In 1955, streetcars and buses carried about the same number of riders.

14 RW-D14

> Can a plant-based coating slow the rusting of steel? In a hypothetical experiment, a team of chemists placed coated and uncoated steel nails in two sealed chambers, one filled with dry air and the other with humid air. After a month, none of the nails in the dry chamber had rusted. In the humid chamber, the uncoated nails were covered in rust, while the coated nails showed none. The chemists noted that the results from the dry chamber, considered on their own, _____

Which choice most logically completes the text?

A could not show whether the coating slows rusting, since the uncoated nails in that chamber did not rust either.

B proved that the coating prevents rust in every kind of environment.

C showed that dry air makes coated nails rust faster than uncoated nails.

D suggested that the coating works only when the air is dry.

15 RW-D15

> When stories with nearly identical plots turn up in societies far apart from one another, scholars of folklore must explain the resemblance. One possibility is that a single early story spread from place to place along routes of trade and migration. Another is that similar plots arose independently, because people in different societies faced similar hopes and problems. Consider a hypothetical tale, found in several such societies, in which a youngest child succeeds after older siblings fail. If, like many folktales, this story circulated orally for generations before anyone wrote it down, then _____

Which choice most logically completes the text?

A scholars could prove that the tale arose independently in each society.

B the tale probably originated in the place where it was first written down.

C the tale could not have spread along routes of trade and migration.

D written records alone might be unable to show which explanation is correct.

16 RW-D16

> In a hypothetical survey, volunteers counted 214 heron nests along a ten-kilometer stretch of river in _____ same stretch had held only 61. The volunteers credit the increase to new rules that protect the tall trees in which herons build their nests, and they plan to repeat the count every spring.

Which choice completes the text so that it conforms to the conventions of Standard English?

A 2024, ten years earlier, the

B 2024. Ten years earlier, the

C 2024 ten years earlier. The

D 2024, ten years earlier. The

17 RW-D17

> A fictional county museum has assembled bus timetables, ticket stubs, and route maps from the 1930s, most of them donated by the families of former drivers. The collection of old documents _____ visitors a clear picture of how residents once traveled between the county's farm towns and its one small city.

Which choice completes the text so that it conforms to the conventions of Standard English?

A give **B** gives **C** have given **D** are giving

18 RW-D18

> In the folklore of the Akan people of West Africa, Ananse—a trickster who is usually depicted as a _____ moral and social lessons through puzzles and surprising turns. One well-known tale tells how he won the rights to all stories from Nyame, the Sky God, and other tales credit him with bringing writing, agriculture, and hunting to humans. In the Caribbean, where enslaved Black laborers spread the tales, Ananse remains a famous figure.

Which choice completes the text so that it conforms to the conventions of Standard English?

A spider, teaches **B** spider teaches **C** spider; teaches **D** spider—teaches

19 RW-D19

> Ornithologists in a hypothetical tracking study fitted twenty-four warblers with backpack-style tags, each weighing less than a paper clip. The team chose birds that had already been banded, so that every tag could be matched to a known individual. The warblers carried _____ tags for a full year, allowing the team to map the birds' routes between their summer and winter ranges and to record where they stopped to rest.

Which choice completes the text so that it conforms to the conventions of Standard English?

A their **B** its **C** it's **D** they're

20 RW-D20

> Before recording equipment came to the fictional island of Corvel, its folktales reached print in a roundabout way. Visiting collectors wrote down the tales they heard from skilled _____ the printed versions lost the pauses, gestures, and audience responses that had shaped each live telling. The island's folklore society therefore now pairs every written transcript with an audio recording.

Which choice completes the text so that it conforms to the conventions of Standard English?

A storytellers, **B** storytellers; but, **C** storytellers, but **D** storytellers, but,

21 RW-D21

> For decades, ferries were the only link between the fictional island town of Calder and the mainland, carrying everything from schoolchildren to sacks of flour. The best remembered of them was the Marisol, whose deep horn could be heard across the whole harbor. Built in 1910 to carry passengers and mail across the sound, _____ before being taken out of service in 1968. Islanders still keep its brass bell in the town library.

Which choice completes the text so that it conforms to the conventions of Standard English?

A the Marisol's steam engines were replaced twice

B engineers replaced the Marisol's steam engines twice

C twice the steam engines of the Marisol were replaced

D the Marisol had its steam engines replaced twice

22 RW-D22

> The fictional Harbor Line Oral History Project recorded interviews with people who had worked on the city's ferries and streetcars. Among those interviewed were Rosa Delgado, a ticket agent who worked the night shift for thirty _____ Samuel Obi, a mechanic who rebuilt engines; and Lena Park, a dispatcher who coordinated schedules in the 1950s. Together, their recollections describe working days that began long before the first passengers arrived and ended long after the last had gone home.

Which choice completes the text so that it conforms to the conventions of Standard English?

A years; **B** years, **C** years: **D** years

23 RW-D23

> Storytellers in many oral traditions use fixed phrases to signal that a tale is about to begin. _____ an English-speaking teller might open with "Once upon a time," a phrase that tells listeners to expect a story set in no particular place or year. Formulas like this one prepare an audience to listen in a particular way.

Which choice completes the text with the most logical transition?

A However, **B** For example, **C** As a result, **D** In contrast,

24 RW-D24

> A hypothetical study compared two populations of a fictional finch species living on neighboring islands. On one island the main food is a hard-shelled seed that must be cracked open; on the other, the birds eat mostly soft seeds and insects. Finches from the hard-seed island had noticeably deeper and stronger beaks, a difference visible even to casual observers. _____ the researchers cautioned against concluding that the two populations had inherited different beak shapes, since the size of a bird's beak can also be influenced by what the bird eats while it is growing.

Which choice completes the text with the most logical transition?

A Indeed, **B** Thus, **C** Even so, **D** Specifically,

25 RW-D25

> While researching a topic, a student has taken the following notes:
>
> - Horsecars were streetcars pulled along rails by horses.
> - Cable cars were pulled along rails by gripping a steel cable that moved continuously beneath the street.
> - In the fictional city of Port Alder, horsecars began carrying passengers in 1868.
> - Port Alder's first cable car line opened in 1884.
> - Both kinds of cars carried passengers on Port Alder's main streets.

The student wants to emphasize a difference between how horsecars and cable cars were powered. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Horsecars were pulled by horses, while cable cars moved by gripping a steel cable that ran beneath the street.

B Port Alder's horsecars began carrying passengers in 1868, and its first cable car line opened in 1884.

C Both horsecars and cable cars carried passengers on Port Alder's main streets.

D Cable cars moved by gripping a steel cable that ran continuously beneath the street.

---

26 RW-D26

> While researching a topic, a student has taken the following notes:
>
> - Zora Neale Hurston (1891–1960) was an American folklorist and writer associated with the Harlem Renaissance.
> - She studied anthropology at Barnard College under the anthropologist Franz Boas.
> - Hurston grew up in Eatonville, Florida, the first incorporated all-Black town in the United States.
> - Her book *Mules and Men* (1935) is a study of the folkways of African Americans in Florida and Louisiana.
> - Her most celebrated novel, *Their Eyes Were Watching God*, was published in 1937.

The student wants to introduce *Mules and Men* to an audience unfamiliar with Zora Neale Hurston. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Hurston, who studied anthropology under Franz Boas, published *Mules and Men* in 1935.

B *Mules and Men*, published in 1935, appeared two years before *Their Eyes Were Watching God*.

C Hurston grew up in Eatonville, Florida, the first incorporated all-Black town in the United States.

D *Mules and Men* (1935), by the American folklorist and writer Zora Neale Hurston, is a study of the folkways of African Americans in Florida and Louisiana.

---

27 RW-D27

While researching a topic, a student has taken the following notes:

- Stephanie Kwolek (1923–2014) was a chemist who worked for decades at the DuPont company.
- In the mid-1960s, DuPont researchers began searching for lightweight fibers of unusual strength.
- In 1965, Kwolek prepared a thin, cloudy polymer solution unlike any her laboratory had made before.
- The thin, cloudy appearance of the solution made a colleague reluctant to spin it, but the resulting fibers surprised the team with their high strength and stiffness.
- One fiber developed from this research, sold as Kevlar, is used in many products, including protective vests, boats and ropes.
- Kwolek received the National Medal of Technology in 1996.

The student wants to emphasize that Kwolek's discovery was unexpected and to explain its practical importance. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Kwolek, a DuPont chemist who received the National Medal of Technology in 1996, prepared an unusual polymer solution in 1965.

B Although its thin, cloudy appearance caused concern, Kwolek's solution surprisingly yielded very strong fibers, leading to Kevlar, which is used in protective vests, boats and ropes.

C In the mid-1960s, DuPont researchers searched for lightweight fibers of unusual strength, and Kwolek's 1965 solution spun into fibers of very high strength and stiffness.

D Kevlar, a fiber developed at DuPont, is used in many products, including protective vests, boats and ropes.

Math Reference Sheet

Use these facts during both Math modules. Your test-day reference sheet may present them differently.

Topic	Formula or fact
Circle	$A = \pi r^2$ $C = 2\pi r$
Rectangle	$A = \ell w$
Triangle	$A = (1/2)bh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Special right triangles	30°-60°-90°: sides x, $x\sqrt{3}$, 2x 45°-45°-90°: sides s, s, $s\sqrt{2}$
Rectangular prism	$V = \ell wh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = (4/3)\pi r^3$
Cone	$V = (1/3)\pi r^2 h$
Pyramid	$V = (1/3)\ell wh$
Angles and arcs	A circle has 360° of arc, or 2π radians. The angles of a triangle sum to 180°.

Diagnostic: Math

Suggested time: 35 minutes for 22 questions. A calculator is allowed on every question.

1 M-D01

What value of x is the solution to the equation $4(x - 3) = 2x + 10$?

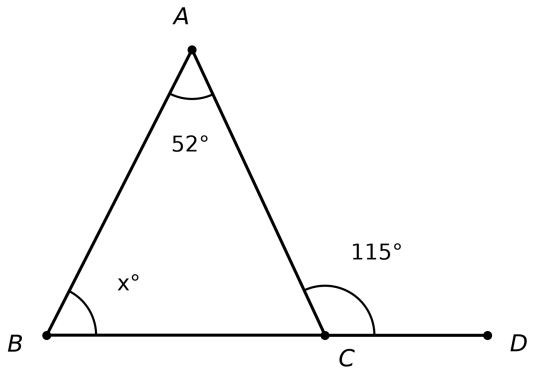
A −1 **B** 6.5 **C** 11 **D** 22

2 M-D02

Of the 320 students who volunteered at a school's spring fair, 15% worked at the used-book booth. How many of the volunteers worked at the used-book booth?

A 48 **B** 272 **C** 335 **D** 368

3 M-D03



In the figure, side BC of triangle ABC is extended to point D. The measure of angle BAC is 52°, the measure of angle ACD is 115°, and the measure of angle ABC is x°. What is the value of x?

A 63 **B** 65 **C** 128 **D** 167

4 M-D04

x	f(x)
0	−4
1	−1
2	2
3	5

The table shows four values of x and the corresponding values of f(x) for the linear function f. Which equation defines f?

A $f(x) = -4x + 3$ **B** $f(x) = x - 4$ **C** $f(x) = 3x + 4$ **D** $f(x) = 3x - 4$

5 M-D05

What is the solution to the equation $3(2x + 1) = 2x + 5$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

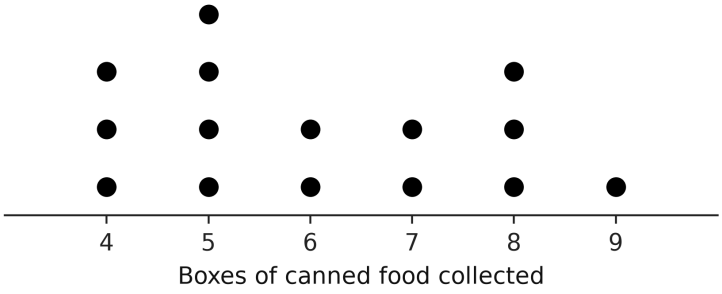
6 M-D06

Which expression is equivalent to $(3x^2 - 5x + 2) - (x^2 + 4x - 6)$?

A $2x^2 - x - 4$ **B** $2x^2 - 9x + 8$ **C** $2x^2 - 9x - 4$ **D** $4x^2 - x - 4$

7 M-D07

Boxes Collected per Homeroom (hypothetical data)



For a canned-food drive, each of the 15 homerooms at a school recorded the number of boxes of food it collected. The dot plot shows the results. What is the median number of boxes collected per homeroom?

A 2.5 **B** 5 **C** 6 **D** 6.5

8 M-D08

In the xy-plane, line k is defined by $3x - 4y = 12$. Line j is perpendicular to line k and passes through the point (6, 1). Which equation defines line j?

A $y = (3/4)x - 7/2$ **B** $y = -(4/3)x + 1$ **C** $y = (4/3)x - 7$ **D** $y = -(4/3)x + 9$

9 M-D09

A light-rail train travels at an average speed of 45 kilometers per hour. At this speed, how many meters does the train travel each second? (1 kilometer = 1,000 meters)

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

10 M-D10

What is the positive solution to the equation $2x^2 + 3x - 20 = 0$?

A 2/5 **B** 5/2 **C** 4 **D** 5

11 M-D11

The function B(n) = 40 − 2.75n gives the balance, in dollars, on a transit card after n bus rides. What is the best interpretation of B(8) = 18 in this context?

A After 8 bus rides, the balance on the card is \$18.

B After 18 bus rides, the balance on the card is \$8.

C Each bus ride costs \$8, and the card started with \$18.

D The balance decreases by \$18 for every 8 bus rides.

12 M-D12

Which point (x, y) is a solution to the system of inequalities $y > 2x - 3$ and $x + y < 6$?

A (0, −4) **B** (1, 5) **C** (3, 4) **D** (2, 3)

13 M-D13

The solution to the system of equations $5x + 3y = 31$ and $3x + 5y = 33$ is (x, y). What is the value of $x + y$?

Student-produced response. Write your answer on the answer sheet or below.

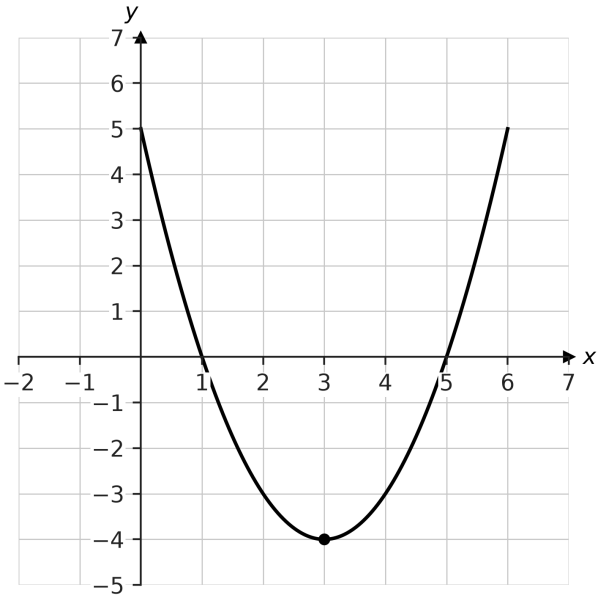
Answer: _____

14 M-D14

For a school event, the student council made a rectangular banner that is 4 feet tall and 6 feet wide. Next year, it plans to make a banner whose height and width are each 1.5 times those of this banner. What will be the area, in square feet, of the new banner?

A 36 **B** 41.25 **C** 54 **D** 81

15 M-D15



The graph of y = f(x) is shown, where f is a quadratic function. Which equation could define f?

A $f(x) = (x + 3)^2 - 4$ **B** $f(x) = (x - 3)^2 + 4$ **C** $f(x) = -(x - 3)^2 - 4$ **D** $f(x) = (x - 3)^2 - 4$

16 M-D16

Bus Trips by Route Type and Arrival (hypothetical data)

Route type	On time	Late	Total
Express	132	18	150
Local	204	96	300
Total	336	114	450

The table summarizes one week of trips on a fictional city's bus routes, classified by route type and by whether each trip arrived on time. If one of the late trips is selected at random, what is the probability that it was an express trip?

A 3/19 **B** 3/16 **C** 19/75 **D** 1/3

17 M-D17

In the equation $(2x + p)(x - 3) = 2x^2 + 5x + q$, p and q are constants. If the equation is true for all values of x, what is the value of q?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

18 M-D18

In the xy-plane, the graph of $y = 2x + k$ intersects the graph of $y = x^2 - 4x + 12$ at exactly one point, where k is a constant. What is the value of k?

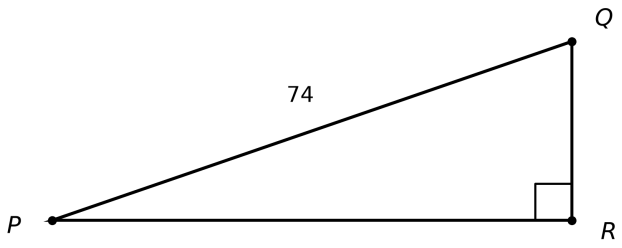
A −3 **B** 3 **C** 4 **D** 8

19 M-D19

For the spring concert, a school's two choirs have 84 members in all. If 6 members moved from the senior choir to the junior choir, the junior choir would have exactly twice as many members as the senior choir. How many members does the junior choir have now?

A 28 **B** 34 **C** 50 **D** 56

20 M-D20



In the figure, triangle PQR has a right angle at R, and $\tan P = 12/35$. If PQ = 74, what is the length of QR?

A 12 **B** 24 **C** 37 **D** 70

21 M-D21

The function f is defined by $f(x) = a(2)^x + b$, where a and b are constants. If $f(0) = 7$ and $f(3) = 35$, what is the value of $f(1)$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

22 M-D22

The function f is defined by $f(x) = a(x - 3)^2 + k$, where a and k are constants. In the xy-plane, the graph of $y = f(x)$ passes through the points (1, 0) and (0, −10). What is the value of k?

A −10 **B** −8 **C** −2 **D** 8

Diagnostic Answer Key and Skill Profile

Reading and Writing

#	Answer	Skill	Review
1	B	Words in Context	Ch. 5, p. 69
2	D	Words in Context	Ch. 5, p. 69
3	A	Words in Context	Ch. 5, p. 69
4	C	Words in Context	Ch. 5, p. 69
5	A	Words in Context	Ch. 5, p. 69
6	D	Text Structure and Purpose	Ch. 6, p. 82
7	B	Text Structure and Purpose	Ch. 6, p. 82
8	C	Cross-Text Connections	Ch. 7, p. 98
9	A	Central Ideas and Details	Ch. 8, p. 112
10	D	Central Ideas and Details	Ch. 8, p. 112
11	B	Command of Evidence: Quantitative	Ch. 9, p. 128
12	C	Command of Evidence: Textual	Ch. 9, p. 128
13	C	Command of Evidence: Quantitative	Ch. 9, p. 128
14	A	Inferences	Ch. 10, p. 150
15	D	Inferences	Ch. 10, p. 150
16	B	Boundaries	Ch. 11, p. 164
17	B	Form, Structure, and Sense	Ch. 12, p. 180
18	D	Boundaries	Ch. 11, p. 164
19	A	Form, Structure, and Sense	Ch. 12, p. 180
20	C	Boundaries	Ch. 11, p. 164
21	D	Form, Structure, and Sense	Ch. 12, p. 180
22	A	Boundaries	Ch. 11, p. 164
23	B	Transitions	Ch. 13, p. 195
24	C	Transitions	Ch. 13, p. 195
25	A	Rhetorical Synthesis	Ch. 14, p. 208
26	D	Rhetorical Synthesis	Ch. 14, p. 208
27	B	Rhetorical Synthesis	Ch. 14, p. 208

Math

#	Answer	Skill	Review
1	C	Linear equations in one variable	Ch. 16, p. 239
2	A	Percentages	Ch. 23, p. 328
3	A	Lines, angles, and triangles	Ch. 26, p. 372
4	D	Linear functions	Ch. 17, p. 251
5	1/2 or .5 or 0.5	Linear equations in one variable	Ch. 16, p. 239
6	B	Equivalent expressions	Ch. 20, p. 289
7	C	One-variable data: distributions and measures of center and spread	Ch. 24, p. 340
8	D	Linear equations in two variables	Ch. 17, p. 251
9	12.5 or 25/2	Ratios, rates, proportional relationships, and units	Ch. 23, p. 328
10	B	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
11	A	Linear functions	Ch. 17, p. 251
12	D	Linear inequalities in one or two variables	Ch. 19, p. 278
13	8	Systems of two linear equations in two variables	Ch. 18, p. 264
14	C	Area and volume	Ch. 26, p. 372
15	D	Nonlinear functions	Ch. 22, p. 313
16	A	Probability and conditional probability	Ch. 25, p. 357
17	-33	Equivalent expressions	Ch. 20, p. 289
18	B	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
19	C	Systems of two linear equations in two variables	Ch. 18, p. 264
20	B	Right triangles and trigonometry	Ch. 27, p. 389
21	11	Nonlinear functions	Ch. 22, p. 313
22	D	Nonlinear functions	Ch. 22, p. 313

Your skill profile

Mark each question you missed or guessed. Start your study with the skills where you missed the most.

Skill	Questions	Chapter	Missed
Words in Context	1, 2, 3, 4, 5	5 (p. 69)	
Text Structure and Purpose	6, 7	6 (p. 82)	

Skill	Questions	Chapter	Missed
Cross-Text Connections	8	7 (p. 98)	
Central Ideas and Details	9, 10	8 (p. 112)	
Command of Evidence: Quantitative	11, 13	9 (p. 128)	
Command of Evidence: Textual	12	9 (p. 128)	
Inferences	14, 15	10 (p. 150)	
Boundaries	16, 18, 20, 22	11 (p. 164)	
Form, Structure, and Sense	17, 19, 21	12 (p. 180)	
Transitions	23, 24	13 (p. 195)	
Rhetorical Synthesis	25, 26, 27	14 (p. 208)	
Linear equations in one variable	1, 5	16 (p. 239)	
Linear functions	4, 11	17 (p. 251)	
Linear equations in two variables	8	17 (p. 251)	
Systems of two linear equations in two variables	13, 19	18 (p. 264)	
Linear inequalities in one or two variables	12	19 (p. 278)	
Equivalent expressions	6, 17	20 (p. 289)	
Nonlinear equations in one variable and systems of equations in two variables	10, 18	21 (p. 300)	
Nonlinear functions	15, 21, 22	22 (p. 313)	
Percentages	2	23 (p. 328)	
Ratios, rates, proportional relationships, and units	9	23 (p. 328)	
One-variable data: distributions and measures of center and spread	7	24 (p. 340)	
Probability and conditional probability	16	25 (p. 357)	
Lines, angles, and triangles	3	26 (p. 372)	
Area and volume	14	26 (p. 372)	
Right triangles and trigonometry	20	27 (p. 389)	

Diagnostic Explanations: Reading and Writing

1. Correct answer: B RW-D01

Words in Context · Craft and Structure · Easier

B *deserted*

The word "but" contrasts what Inés "had expected," a busy station, with what she finds. The details that follow (a single lamp, one empty bench, and a sleeping porter) describe a place with almost no one in it, so "deserted" is the precise word.

A "Crowded" is the opposite of the scene. The contrast word "but" tells you the platform is not the busy place Inés expected.

C Nothing in the details suggests bright or varied colors; a single lamp and an empty bench describe emptiness, not color.

D The scene is quiet and empty, but nothing in it threatens Inés; a porter asleep on mailbags suggests calm, not danger.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: D RW-D02

Words in Context · Craft and Structure · Easier

D *definite*

The described indicator turns bright pink in a basic solution, and the color appeared "at once." In this experiment, that strong, immediate color change was a "definite" sign, meaning a clear indication, that the cleaner was basic.

A "Faint" conflicts with "bright pink." A faint sign would be weak or hard to see, but the passage describes a strong color.

B "Hidden" contradicts the passage: the pink color appeared immediately and visibly, so the sign was obvious rather than concealed.

C "Delayed" conflicts with "at once." The color change happened immediately, not after a wait.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: A RW-D03

Words in Context · Craft and Structure · Medium

A *resist*

Because the archive treats every telling as "equally authentic," its collectors have a reason not to change any version, and the sentence ends by saying they preserve "each teller's words exactly as spoken." To "resist" an urge is to hold back from acting on it, which matches that careful practice.

B "Indulge" means to give in to an urge, which would mean correcting or combining versions, the opposite of preserving each teller's exact words.

C Collectors do not "announce" an urge here; the sentence describes something they hold back from doing, not something they declare.

D "Inherit" means to receive something from an earlier generation, which does not fit collectors choosing not to act on an urge.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: C RW-D04

Words in Context · Craft and Structure · Medium

C *pressing*

The text says that local times caused "confusion" for railway timetables and that in the United States and Canada one route could cross places "several hours apart in local time," an extreme form of the problem. The need for a standard there was therefore "pressing," or urgent.

A "Remote" plays on the long distances mentioned, but a need is not far away; the sentence describes how urgent the need was.

B "Trivial" means unimportant, which contradicts the point that differences of several hours made the problem especially serious.

D "Gradual" describes slow change, but the sentence is about how strongly the need was felt, not how slowly it developed.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: A RW-D05

Words in Context · Craft and Structure · Harder

A *enigmatic*

Passengers "puzzled over" the ferryman's remarks, and both sample answers are riddles rather than plain information. The teacher even suspects he "enjoyed being misunderstood." "Enigmatic," meaning mysterious and hard to interpret, fits remarks that leave listeners puzzling.

B "Tedious" means dull and tiresome, but the passengers are intrigued enough to keep thinking about his remarks, not bored by them.

C Both replies avoid plain answers. The passage emphasizes the listeners' puzzlement, not the ferryman's pride in his own abilities.

D "Candid" means frank and direct, the opposite of replies that avoid plain answers and leave passengers puzzling.

Review: Chapter 5, Words in Context (page 69)

6. Correct answer: D RW-D06

Text Structure and Purpose · Craft and Structure · Easier

D *To describe the departure of a flock of birds and the speaker's response to their absence*

The first four lines show the swallows gathering "as if rehearsing" to leave, the next lines show the wire "bare" after they go, and the last lines show the speaker still leaving a light on for them. The poem describes the birds' departure and the speaker's response.

A The poem observes the birds' behavior but never explains how their bodies prepare to migrate; "rehearsing" is a comparison, not a scientific account.

B The poem makes no argument and never mentions protecting fields; the field appears only as part of the empty scene after the birds leave.

C The speaker does not travel. The speaker is the one "who stay[s]," so there is no comparison of the speaker's journeys with the birds'.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: B RW-D07

Text Structure and Purpose · Craft and Structure · Medium

B *It describes how the team responded to a problem with the first version of the plastic.*

The sentence before the underlined portion identifies a problem: the first plastic "dissolved within days, far too quickly." The underlined sentence tells how the team fixed it, adjusting the recipe so the plastic lasted about two years. It shows the team's response to that shortcoming.

A The goal, a plastic that breaks down safely in seawater, is stated in the first sentence; the underlined portion describes a later step toward it.

C The safety test for fish appears in the final sentence, after the underlined portion, which concerns how long the plastic lasts.

D The text never explains why nets need a particular lifespan; it reports only that the adjusted plastic lasted about two years.

Review: Chapter 6, Text Structure and Purpose (page 82)

8. Correct answer: C RW-D08

Cross-Text Connections · Craft and Structure · Harder

C *By acknowledging that oral testimony contains errors but arguing that some errors are informative in their own right*

Text 2 concedes the factual point in Text 1 ("Interviewees certainly make factual errors") but argues that an exaggeration like "all winter" reveals how the strike felt and that such distortions "are themselves evidence." The author would accept that errors occur while rejecting the idea that they limit testimony to a fallback role.

A Text 2 does not deny the errors; it opens by agreeing that "interviewees certainly make factual errors," just as Text 1 claims.

B This is Text 1's position. Text 2 values testimony for insights that "documents rarely provide," not only as a substitute for missing records.

D Text 2 relies on newspapers to establish the true length of the strike, so it does not treat documents as less accurate than interviews.

Review: Chapter 7, Cross-Text Connections (page 98)

9. Correct answer: A RW-D09

Central Ideas and Details · Information and Ideas · Easier

A *Tomás comes to value his great-aunt's stories and begins working to preserve them.*

The text traces a change. At first Tomás "only half listened," but after realizing that "no one else alive remembered" the bakery, he begins "writing down her stories" and checking every detail. The main idea is his new appreciation of the stories and his effort to preserve them.

B The great-aunt tells stories every Sunday, and the text supplies no sign that she is unwilling to share details.

C The bakery is the detail that changes Tomás's attitude, but nothing suggests that he wants to learn its recipes.

D This describes Tomás only "at first"; the text's main point is how his attitude changes after that.

Review: Chapter 8, Central Ideas and Details (page 112)

10. Correct answer: D RW-D10

Central Ideas and Details · Information and Ideas · Medium

D *A study suggests that hearing adult song helps young sparrows develop typical songs.*

The text compares chicks that heard adult song with chicks that did not. Only the first group developed songs like the recordings, supporting the conclusion that exposure to adult song helped these birds develop typical songs under the study conditions. Choice D summarizes that finding.

A The study included no wild sparrows. It compared two indoor groups, so it offers no basis for a comparison with wild birds.

B The study compared early exposure with no exposure; it did not test learning at different ages, so it cannot support this claim.

C No group heard live adult birds, so the text gives no basis for comparing recordings with live teachers.

Review: Chapter 8, Central Ideas and Details (page 112)

11. Correct answer: B RW-D11

Command of Evidence: Quantitative · Information and Ideas · Easier

B *55 seconds in 60°C water but took 210 seconds in 10°C water.*

The claim is that warmer water dissolves sugar faster, so the evidence must pair a warm and a cold temperature with the correct times. The table shows 55 seconds at 60°C and 210 seconds at 10°C: the warmest water took the least time and the coldest water took the most.

A This reverses the table: 55 seconds goes with 60°C and 210 seconds with 10°C, so as written the choice contradicts the claim.

C The table lists 95 seconds, not 150 seconds, for 40°C, so this choice misreads the data and shows no difference between temperatures.

D This is accurate, but it says nothing about how dissolving time changes with temperature, so it does not support the claim.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: C RW-D12

Command of Evidence: Textual · Information and Ideas · Medium

C *"At 6:14 each morning, the minute the old express had once passed his signal box, Hollis set down his coffee and checked his watch, and only then did his shoulders relax."*

The portrayal has two parts: a routine from his working years and the comfort Hollis takes in it. Choice C shows both. He still marks the minute "the old express had once passed his signal box," and "only then did his shoulders relax," which shows relief.

A The photographs link Hollis to his railway past, but rarely opening a box is not a daily routine, and it shows no comfort.

B A spur-of-the-moment trip does not show Hollis finding comfort in a daily routine retained from his railway work.

D This gives a fact about the signal box, not about Hollis's own habits or how he feels about keeping them.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: C RW-D13

Command of Evidence: Quantitative · Information and Ideas · Harder

C *In 1945, streetcars and buses each carried more than 25 million riders, but by 1955 streetcars carried fewer than 5 million.*

The argument has two parts: a period when both systems carried many riders, then a later collapse in streetcar ridership. Choice C covers both. In 1945 streetcars (30 million) and buses (27 million) each carried more than 25 million riders, and by 1955 streetcars had fallen to 4 million.

A This is accurate, but it describes only bus growth. It shows neither the period of shared ridership nor the later streetcar collapse.

B This is accurate but incomplete. It never mentions the 1955 drop, so it cannot show that the collapse came only later.

D This misreads the graph. In 1955 buses carried 41 million riders while streetcars carried only 4 million.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

14. Correct answer: A RW-D14

Inferences · Information and Ideas · Medium

A *could not show whether the coating slows rusting, since the uncoated nails in that chamber did not rust either.*

In the dry chamber, neither coated nor uncoated nails rusted, so there was no observed difference to attribute to the coating. The humid chamber supplies the comparison in which the two groups differed. The dry results alone therefore could not show whether the coating slows rusting.

B The dry chamber does not isolate an effect of the coating, and "every kind of environment" goes far beyond two chambers tested for one month.

C No nails rusted in the dry chamber, so its results cannot show coated nails rusting faster than uncoated ones.

D This conflicts with the humid-chamber result: coated nails showed no rust there while uncoated nails did. It does not support a dry-air-only effect.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: D RW-D15

Inferences · Information and Ideas · Harder

D *written records alone might be unable to show which explanation is correct.*

Deciding between the two explanations would require knowing the tale's early history. But a story that "circulated orally for generations before anyone wrote it down" left no written trace of its early versions. Written records alone therefore might not be able to show whether the tale spread or arose independently.

A Nothing in the text supports proof of either explanation, and oral circulation makes the early history harder, not easier, to document.

B Because the tale was told aloud long before it was written, the place of the first written version need not be where the tale began.

C Oral circulation does not rule out spread by trade and migration; people can carry a story by word of mouth.

Review: Chapter 10, Inferences (page 150)

16. Correct answer: B RW-D16

Boundaries · Standard English Conventions · Easier

B *2024. Ten years earlier, the*

The text contains two complete sentences: volunteers counted 214 nests in 2024, and ten years earlier the same stretch had held only 61. Among these choices, B correctly separates them with a period and places "ten years earlier" with the count of 61 nests.

A This joins two complete sentences with only a comma, creating a comma splice; the boundary between the sentences is missing.

C This attaches "ten years earlier" to the first sentence, saying the 2024 count happened ten years earlier, which makes no sense.

D This also attaches "ten years earlier" to the 2024 count, which makes the first sentence illogical and leaves the second without its time phrase.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: B RW-D17

Form, Structure, and Sense · Standard English Conventions · Easier

B *gives*

The subject of the verb is the singular noun "collection"; "of old documents" is a phrase that comes between the subject and the verb. A singular subject needs a singular verb, so "gives" is correct.

A "Give" does not agree with the singular third-person subject "collection." The nearby plural noun "documents" is inside a prepositional phrase, not the subject.

C "Have given" uses the plural helping verb "have," so it does not agree with the singular subject "collection."

D "Are giving" uses the plural helping verb "are," which does not agree with the singular subject "collection."

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: D RW-D18

Boundaries · Standard English Conventions · Medium

D *spider—teaches*

The phrase "a trickster who is usually depicted as a spider" is a supplement that renames Ananse, and it opens with a dash. A supplement must close with the same mark that opened it, so a second dash is needed before the verb "teaches."

A A comma cannot close a supplement that opens with a dash; the two marks around an interruption must match.

B Leaving out the closing dash fails to separate the supplement from the main verb, so the interruption never ends.

C A semicolon separates two independent clauses, but "teaches moral and social lessons" has no subject of its own.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: A RW-D19

Form, Structure, and Sense · Standard English Conventions · Medium

A *their*

The pronoun refers to "the warblers," a plural noun, and it must show that the tags belonged to the birds. "Their" is the plural possessive pronoun, so it agrees with its antecedent in number and correctly shows possession.

B "Its" is a singular possessive pronoun, so it does not agree in number with the plural antecedent "the warblers."

C "It's" is a contraction of "it is," which makes no sense before "tags," and it is singular rather than plural.

D "They're" is a contraction of "they are"; "they are tags" makes no sense here, where a possessive pronoun is needed.

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: C RW-D20

Boundaries · Standard English Conventions · Medium

C *storytellers, but*

Choice C joins two independent clauses with a comma and the coordinating conjunction "but." The conjunction directly introduces the subject "the printed versions," without extra punctuation between them.

A A comma alone cannot join these two independent clauses. This choice omits the coordinating conjunction and creates a comma splice.

B The comma immediately after "but" separates the conjunction from its clause without an intervening parenthetical expression. That extra comma makes this choice incorrect.

D The comma before "but" is correct, but the comma after it wrongly separates the conjunction from the clause it introduces.

Review: Chapter 11, Boundaries (page 164)

21. Correct answer: D RW-D21

Form, Structure, and Sense · Standard English Conventions · Harder

D *the Marisol had its steam engines replaced twice*

The opening modifier, "Built in 1910 to carry passengers and mail across the sound," describes the ferry, so the subject of the main clause must be the ferry itself. Only choice D makes "the Marisol" the subject, which also gives "being taken out of service" a logical subject.

A The subject here is "steam engines," so the sentence says the engines were built in 1910 to carry passengers, a misplaced modifier.

B This makes "engineers" the subject, illogically saying that the engineers were built in 1910 to carry passengers and mail.

C The subject is "the steam engines," so the opening modifier again describes the engines rather than the ferry.

Review: Chapter 12, Form, Structure, and Sense (page 180)

22. Correct answer: A RW-D22

Boundaries · Standard English Conventions · Harder

A *years;*

The sentence lists three people, and each item contains a comma ("Rosa Delgado, a ticket agent"). When list items contain internal commas, semicolons separate the items, as the semicolon before "and Lena Park" shows. A semicolon is needed after "years" as well.

B A comma would blur the boundaries between list items that already contain commas and would not match the semicolon before the final item.

C A colon introduces a list or an explanation; it cannot separate one item in a list from the next.

D Without punctuation, the first item runs into the second, so the sentence seems to say Rosa Delgado worked for thirty years Samuel Obi.

Review: Chapter 11, Boundaries (page 164)

23. Correct answer: B RW-D23

Transitions · Expression of Ideas · Easier

B *For example,*

The first sentence makes a general claim, that storytellers use fixed phrases to begin tales, and the second gives one specific case, the English opening "Once upon a time." "For example" signals that the second sentence illustrates the first.

A "However" signals a contrast, but the second sentence supports the first sentence's claim rather than contradicting it.

C "As a result" signals an effect, but the English phrase is an instance of the general practice, not a consequence of it.

D "In contrast" signals a difference, yet "Once upon a time" is one of the fixed opening phrases the first sentence describes.

Review: Chapter 13, Transitions (page 195)

24. Correct answer: C RW-D24

Transitions · Expression of Ideas · Harder

C *Even so,*

The previous sentence reports a striking difference in beaks, which might seem to show inherited differences. The next sentence warns against drawing that conclusion. "Even so" signals a concession: despite the clear result, the researchers were cautious about what it meant.

A "Indeed" adds emphasis to the previous point, but the next sentence limits that point instead of strengthening it.

B "Thus" would present the caution as a result of the beak difference. Instead, the next sentence limits an apparent inheritance conclusion by introducing another possible influence, diet.

D "Specifically" introduces more detail about the previous claim, but the sentence raises a caution instead of describing the beaks further.

Review: Chapter 13, Transitions (page 195)

25. Correct answer: A RW-D25

Rhetorical Synthesis · Expression of Ideas · Easier

A *Horsecars were pulled by horses, while cable cars moved by gripping a steel cable that ran beneath the street.*

The goal is a difference in how the two kinds of cars were powered. Choice A names both power sources, horses for horsecars and a moving cable for cable cars, and uses "while" to set them side by side as a contrast.

B This gives the dates for the two systems, which shows a difference in timing, not in how the cars were powered.

C This states a similarity between the two systems, so it does not emphasize any difference between them.

D This describes only how cable cars were powered. Without the horsecars' power source, there is no difference to emphasize.

Review: Chapter 14, Rhetorical Synthesis (page 208)

26. Correct answer: D RW-D26

Rhetorical Synthesis · Expression of Ideas · Medium

D *Mules and Men (1935), by the American folklorist and writer Zora Neale Hurston, is a study of the folkways of African Americans in Florida and Louisiana.*

An unfamiliar audience needs to know both who Hurston was and what the book is. Choice D identifies Hurston as "the American folklorist and writer" and explains that *Mules and Men* "is a study of the folkways of African Americans in Florida and Louisiana."

A This gives Hurston's training and the publication date but does not explain the book's subject, which would help introduce it to unfamiliar readers.

B This places the book in time relative to another work, but it explains neither who Hurston was nor what the book contains.

C This gives background on Hurston's hometown but never mentions *Mules and Men*, the work the student wants to introduce.

Review: Chapter 14, Rhetorical Synthesis (page 208)

27. Correct answer: B RW-D27

Rhetorical Synthesis · Expression of Ideas · Harder

B *Although its thin, cloudy appearance caused concern, Kwolek's solution surprisingly yielded very strong fibers, leading to Kevlar, which is used in protective vests, boats and ropes.*

The goal has two parts: the surprise and the practical importance. Choice B explains that the unusual solution caused concern yet yielded surprisingly strong fibers, then identifies useful products made with Kevlar. Each other choice leaves out at least one part of the goal.

A This identifies Kwolek and an honor she received but says nothing surprising about the discovery or how the fiber is used.

C This describes the search and its result but does not emphasize the researchers' surprise, and it omits Kevlar's practical uses.

D This conveys Kevlar's practical importance but never mentions Kwolek's discovery or that the result was unexpected.

Review: Chapter 14, Rhetorical Synthesis (page 208)

Diagnostic Explanations: Math

1. Correct answer: C M-D01

Linear equations in one variable · Algebra · Easier

C *11*

Distribute, collect the x-terms on one side, and divide. The equation 4(x − 3) = 2x + 10 becomes 4x − 12 = 2x + 10, so 2x = 22 and x = 11. Check: 4(11 − 3) = 32 and 2(11) + 10 = 32.

1. Distribute 4: 4x − 12 = 2x + 10.
2. Subtract 2x from both sides: 2x − 12 = 10.
3. Add 12 to both sides: 2x = 22.
4. Divide both sides by 2: x = 11.

A This comes from moving −12 across the equals sign without changing its sign, which gives 2x = 10 − 12 = −2 and x = −1.

B This comes from multiplying only x by 4, writing 4x − 3 = 2x + 10, which gives 2x = 13 and x = 6.5.

D This is the value of 2x, not x. After reaching 2x = 22, divide both sides by 2 to get x = 11.

Calculator option: Graph y = 4(x − 3) and y = 2x + 10; the lines intersect at the point where x = 11.

Review: Chapter 16, Linear Equations in One Variable (page 239)

2. Correct answer: A M-D02

Percentages · Problem-Solving and Data Analysis · Easier

A *48*

To find a percent of a number, multiply by the percent written as a decimal: 15% of 320 is 0.15 × 320 = 48 volunteers. A quick mental check: 10% of 320 is 32 and 5% is 16, and 32 + 16 = 48.

1. Let n be the number of volunteers who worked at the booth.
2. Write 15% as the decimal 0.15.
3. Multiply: n = 0.15 × 320 = 48.

B This is the number of volunteers who did not work at the booth: 320 − 48 = 272, or 85% of 320.

C This adds 15 to 320, treating the percent as a number of students instead of finding 15% of 320.

D This increases 320 by 15% (1.15 × 320 = 368) instead of finding 15% of 320.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

3. Correct answer: A M-D03

Lines, angles, and triangles · Geometry and Trigonometry · Easier

A *63*

An exterior angle of a triangle equals the sum of the two interior angles that are not next to it. Angle ACD is the exterior angle at C, so 115 = 52 + x and x = 63. Check: angle ACB is 180 − 115 = 65 degrees, and 52 + 63 + 65 = 180.

1. Angle ACD is an exterior angle of triangle ABC; the remote interior angles are angles BAC and ABC.
2. Exterior angle theorem: 115 = 52 + x.
3. Subtract 52 from both sides: x = 63.

B This is the measure of angle ACB, the interior angle next to the 115° exterior angle, not the measure of angle ABC.

C This subtracts only 52 from 180, ignoring the third angle of the triangle, angle ACB, which also uses part of the 180°.

D This adds 52 and 115. The exterior angle equals the sum of the two remote interior angles, so subtract 52 from 115 instead.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: D M-D04

Linear functions · Algebra · Easier

D *f(x) = 3x − 4*

Each time x increases by 1, f(x) increases by 3 (from −4 to −1 to 2 to 5), so the slope is 3. When x = 0, f(x) = −4, so the y-intercept is −4. Therefore f(x) = 3x − 4. Check: f(3) = 9 − 4 = 5.

1. Find the slope: each time x increases by 1, f(x) increases by 3, so m = 3.
2. Find the y-intercept: f(0) = −4, so b = −4.
3. Write f(x) = mx + b: f(x) = 3x − 4.
4. Check another row: f(2) = 6 − 4 = 2, which matches the table.

A This swaps the slope and the y-intercept. It gives f(0) = 3, but the table shows f(0) = −4.

B This has the correct y-intercept but a slope of 1. It gives f(1) = −3, while the table shows f(1) = −1.

C This has the correct slope but the wrong sign on the intercept. It gives f(0) = 4 instead of −4.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

5. Correct answer: 1/2 or .5 or 0.5 M-D05

Linear equations in one variable · Algebra · Easier

Distribute and collect like terms: 6x + 3 = 2x + 5, so 4x = 2 and x = 2/4 = 1/2. Enter 1/2, .5, or 0.5. Dividing the wrong way (4 ÷ 2) gives the wrong answer 2, and distributing the 3 only to 2x (6x + 1) leads to the wrong answer 1.

1. Distribute 3: 6x + 3 = 2x + 5.
2. Subtract 2x and 3 from both sides: 4x = 2.
3. Divide both sides by 4: x = 2/4 = 1/2.

Calculator option: Graph y = 3(2x + 1) and y = 2x + 5; the lines intersect where x = 0.5.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: B M-D06

Equivalent expressions · Advanced Math · Easier

B $2x^2 - 9x + 8$

Subtracting a polynomial means subtracting every one of its terms, so distribute the negative sign: $3x^2 - 5x + 2 - x^2 - 4x + 6$. Combining like terms gives $(3 - 1)x^2 + (-5 - 4)x + (2 + 6) = 2x^2 - 9x + 8$.

1. Distribute the negative sign: $3x^2 - 5x + 2 - x^2 - 4x + 6$.
2. Combine the x^2 terms: $3x^2 - x^2 = 2x^2$.
3. Combine the x-terms: $-5x - 4x = -9x$.
4. Combine the constants: $2 + 6 = 8$, so the result is $2x^2 - 9x + 8$.

A This subtracts only the x^2 term and adds the others, giving $-5x + 4x = -x$ and $2 - 6 = -4$.

C This distributes the negative sign to x^2 and 4x but not to −6. Subtracting −6 means adding 6, so the constant is 8.

D This adds the two polynomials instead of subtracting them, combining $3x^2 + x^2$, $-5x + 4x$, and $2 + (-6)$.

Calculator option: Graph the original expression and each choice to rule out visible mismatches. Confirm the matching expression by distributing the minus sign and combining like terms; a screen match alone is not a proof.

Review: Chapter 20, Equivalent Expressions (page 289)

7. Correct answer: C M-D07

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

C *6*

The median of 15 values is the 8th value when the values are listed in order. Counting dots from the left, the 3 dots at 4 are values 1 to 3, the 4 dots at 5 are values 4 to 7, and the 2 dots at 6 are values 8 and 9. The 8th value is 6.

1. Count the values: $3 + 4 + 2 + 2 + 3 + 1 = 15$, so the median is the 8th value in order.
2. Values 1 to 3 are 4, and values 4 to 7 are 5.
3. Values 8 and 9 are 6, so the 8th value, the median, is 6.

A This is the median of the dot counts (1, 2, 2, 3, 3, 4), not the median of the 15 homerooms' results.

B This is the mode, the value with the most dots. The median is the middle value of the ordered list, which is 6.

D This is the midpoint of the smallest and largest values, $(4 + 9)/2$, not the middle of all 15 values.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

8. Correct answer: D M-D08

Linear equations in two variables · Algebra · Medium

D $y = -(4/3)x + 9$

Rewrite line k in slope-intercept form, $y = (3/4)x - 3$, so its slope is 3/4. A perpendicular line has the negative reciprocal slope, −4/3. Substituting (6, 1) into $y = -(4/3)x + b$ gives $1 = -8 + b$, so $b = 9$ and line j is $y = -(4/3)x + 9$.

1. Solve $3x - 4y = 12$ for y: $y = (3/4)x - 3$, so line k has slope 3/4.
2. Line j has the negative reciprocal slope, −4/3.
3. Substitute (6, 1) into $y = -(4/3)x + b$: $1 = -8 + b$, so $b = 9$.
4. Line j: $y = -(4/3)x + 9$.

A This line has slope 3/4, the same as line k, so it is parallel to line k, not perpendicular to it.

B This has the correct slope but uses the y-coordinate of (6, 1) as the y-intercept; it passes through (0, 1), not (6, 1).

C This uses the reciprocal 4/3 without changing the sign. Perpendicular slopes multiply to −1, but $(3/4)(4/3) = 1$.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: 12.5 or 25/2 M-D09

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

Convert kilometers to meters and hours to seconds: 45 km/h × 1,000 m per km ÷ 3,600 seconds per hour = 45,000 ÷ 3,600 = 12.5 meters per second. Enter 12.5 or 25/2. Dividing by 60 only once gives the wrong answer 750, and skipping the kilometer-to-meter step gives 0.0125.

1. One hour has $60 \times 60 = 3{,}600$ seconds.
2. In one hour the train travels $45 \times 1{,}000 = 45{,}000$ meters.
3. Meters per second: $45{,}000 \div 3{,}600 = 12.5$.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

10. Correct answer: B M-D10

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

B *5/2*

Factor: $2x^2 + 3x - 20 = (2x - 5)(x + 4)$. Setting each factor equal to 0 gives $x = 5/2$ or $x = -4$, so the positive solution is 5/2. Check: $2(25/4) + 3(5/2) - 20 = 12.5 + 7.5 - 20 = 0$.

1. Find two numbers whose product is $2 \times (-20) = -40$ and whose sum is 3: 8 and −5.
2. Factor by grouping: $2x^2 + 8x - 5x - 20 = 2x(x + 4) - 5(x + 4) = (2x - 5)(x + 4)$.
3. Solve: $2x - 5 = 0$ gives $x = 5/2$, and $x + 4 = 0$ gives $x = -4$.
4. The positive solution is 5/2.

A This is the reciprocal of the solution, which can come from solving $5x - 2 = 0$ instead of $2x - 5 = 0$.

C This takes the number in the factor $x + 4$ without changing its sign; $x + 4 = 0$ gives $x = -4$, which is negative.

D This sets $2x - 5 = 0$ but forgets to divide by 2; $2x = 5$ gives $x = 5/2$, not 5.

Calculator option: Graph $y = 2x^2 + 3x - 20$; it crosses the x-axis at $x = -4$ and $x = 2.5$, so the positive solution is 2.5, or 5/2.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

11. Correct answer: A M-D11

Linear functions · Algebra · Medium

A *After 8 bus rides, the balance on the card is $18.*

In B(n), the input n is the number of rides and the output B(n) is the balance in dollars. So $B(8) = 18$ means that after 8 rides the balance is $18. Check: $40 - 2.75(8) = 40 - 22 = 18$.

1. B(n) is the balance in dollars after n rides.
2. $B(8) = 18$ pairs the input $n = 8$ with the output 18.
3. Check: $40 - 2.75(8) = 40 - 22 = 18$, so after 8 rides the balance is $18.

B This reverses the input and the output. In $B(8) = 18$, the 8 is the number of rides and the 18 is the balance in dollars.

C The cost per ride, $2.75, and the starting balance, $40, come from the function's slope and intercept, not from the 8 and the 18.

D Over 8 rides the balance falls by $8 \times 2.75 = 22$ dollars, from $40 to $18; the 18 is the remaining balance, not the decrease.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

12. Correct answer: D M-D12

Linear inequalities in one or two variables · Algebra · Medium

D *(2, 3)*

Test each point in both inequalities. For (2, 3), the statement $3 > 2(2) - 3 = 1$ is true, and $2 + 3 = 5 < 6$ is true. Because (2, 3) satisfies both inequalities, it is a solution to the system; each other point fails at least one inequality.

1. Substitute each point into $y > 2x - 3$ and into $x + y < 6$.
2. (2, 3): $3 > 1$ is true, and $5 < 6$ is true.
3. Each other choice fails at least one inequality, so (2, 3) is the solution.

A For (0, −4), the first inequality fails: $-4 > 2(0) - 3 = -3$ is false, even though $x + y = -4 < 6$ is true.

B For (1, 5), $x + y = 6$, and $6 < 6$ is false: the point lies on the boundary line, which a strict inequality excludes.

C For (3, 4), the first inequality holds because $4 > 3$, but $x + y = 7$, and $7 < 6$ is false.

Calculator option: Graph $y > 2x - 3$ and $x + y < 6$; the solutions lie where the two shaded regions overlap. Points on a dashed boundary line, such as (1, 5), are not included.

Review: Chapter 19, Linear Inequalities (page 278)

13. Correct answer: 8 M-D13

Systems of two linear equations in two variables · Algebra · Medium

Add the two equations: $8x + 8y = 64$. Dividing both sides by 8 gives $x + y = 8$ without solving for x and y separately. (The solution is $x = 3.5$ and $y = 4.5$.) Entering 3.5 or 4.5 gives only one variable, and entering 64 forgets to divide by 8.

1. Add the equations: $(5x + 3x) + (3y + 5y) = 31 + 33$, so $8x + 8y = 64$.
2. Divide both sides by 8: $x + y = 8$.

Calculator option: Graph both lines; they intersect at (3.5, 4.5), and $3.5 + 4.5 = 8$.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

14. Correct answer: C M-D14

Area and volume · Geometry and Trigonometry · Medium

C *54*

The new banner is $1.5 \times 4 = 6$ feet tall and $1.5 \times 6 = 9$ feet wide, so its area is $6 \times 9 = 54$ square feet. Equivalently, scaling both dimensions by 1.5 multiplies the area by $1.5^2 = 2.25$, and $24 \times 2.25 = 54$.

1. New height: $1.5 \times 4 = 6$ feet.
2. New width: $1.5 \times 6 = 9$ feet.
3. New area: $6 \times 9 = 54$ square feet.

A This multiplies the original area, 24 square feet, by 1.5. When both dimensions are scaled by 1.5, the area is multiplied by 1.5^2.

B This adds 1.5 feet to each dimension (5.5×7.5) instead of multiplying each dimension by 1.5.

D This multiplies the area by $1.5^3 = 3.375$, the factor for volume; area scales by the square of the factor.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

15. Correct answer: D M-D15

Nonlinear functions · Advanced Math · Medium

D $f(x) = (x - 3)^2 - 4$

The graph opens upward and has its vertex at (3, −4). In vertex form, $f(x) = a(x - h)^2 + k$, the vertex is (h, k), and $a > 0$ makes the parabola open upward, so $f(x) = (x - 3)^2 - 4$ fits. Check: $f(1) = 4 - 4 = 0$, matching an x-intercept.

1. Read the vertex from the graph: (3, −4).
2. The parabola opens upward, so $a > 0$.
3. Vertex form with $h = 3$ and $k = -4$: $f(x) = a(x - 3)^2 - 4$; only choice D matches.
4. Check the x-intercept (1, 0): $(1 - 3)^2 - 4 = 0$.

A This parabola has its vertex at (−3, −4). A vertex at x = 3 requires $(x - 3)^2$, not $(x + 3)^2$.

B This parabola has its vertex at (3, 4), above the x-axis, but the graph's vertex is at (3, −4), below the x-axis.

C The negative leading coefficient makes this parabola open downward, but the graph opens upward.

Calculator option: Graph each choice; only $y = (x - 3)^2 - 4$ matches the vertex at (3, −4) and the x-intercepts at 1 and 5.

Review: Chapter 22, Nonlinear Functions (page 313)

16. Correct answer: A M-D16

Probability and conditional probability · Problem-Solving and Data Analysis · Harder

A *3/19*

Selecting "one of the late trips" limits the possibilities to the 114 late trips, so 114 is the denominator. Of those late trips, 18 were express trips. The probability is 18/114, which reduces to 3/19.

1. The condition "one of the late trips" makes the denominator the total number of late trips, 114.
2. The numerator is the number of late express trips, 18.
3. Probability = 18/114 = 3/19.

B This is 18/96, which compares late express trips with late local trips; the denominator must include all 114 late trips.

C This is 114/450, the probability that a randomly chosen trip was late; it does not answer the question about express trips.

D This is 150/450, the probability that a randomly chosen trip was an express trip; it ignores the condition that the trip was late.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

17. Correct answer: -33 M-D17

Equivalent expressions · Advanced Math · Harder

Expand the left side: $2x^2 - 6x + px - 3p = 2x^2 + (p - 6)x - 3p$. Matching the x-coefficients gives $p - 6 = 5$, so $p = 11$. Matching the constants gives $q = -3p = -33$. Entering 33 drops the negative sign, and entering 11 gives p instead of q.

1. Expand: $(2x + p)(x - 3) = 2x^2 - 6x + px - 3p$.
2. Group like terms: $2x^2 + (p - 6)x - 3p$.
3. Match the x-coefficients: $p - 6 = 5$, so $p = 11$.
4. Match the constants: $q = -3p = -3(11) = -33$.

Review: Chapter 20, Equivalent Expressions (page 289)

18. Correct answer: B M-D18

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

B *3*

Set the expressions equal: $x^2 - 4x + 12 = 2x + k$, so $x^2 - 6x + (12 - k) = 0$. Exactly one intersection point means exactly one solution, so the discriminant is 0: $(-6)^2 - 4(1)(12 - k) = 0$. Then $36 - 48 + 4k = 0$, so $k = 3$.

1. Set $x^2 - 4x + 12 = 2x + k$ and rearrange: $x^2 - 6x + (12 - k) = 0$.
2. One intersection point means one solution, so the discriminant $b^2 - 4ac$ equals 0.
3. Here $a = 1$, $b = -6$, and $c = 12 - k$: $36 - 4(12 - k) = 0$.
4. Solve: $36 - 48 + 4k = 0$, so $4k = 12$ and $k = 3$.

A This comes from a sign error, writing the constant term as 12 + k; then $36 - 4(12 + k) = 0$ gives $k = -3$.

C This forces the line through the parabola's vertex, (2, 8). A line with slope 2 through the vertex crosses the parabola twice, not once.

D This forgets to subtract 2x when combining the equations; $x^2 - 4x + (12 - k) = 0$ with a discriminant of 0 gives $k = 8$.

Calculator option: Graph the parabola and $y = 2x + k$ with a slider for k; the line touches the parabola at exactly one point when $k = 3$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

19. Correct answer: C M-D19

Systems of two linear equations in two variables · Algebra · Harder

C *50*

Let s and j be the current numbers of members in the senior and junior choirs. Then $s + j = 84$ and $j + 6 = 2(s - 6)$, so $j = 2s - 18$. Substituting gives $s + 2s - 18 = 84$, so $s = 34$ and $j = 50$. Check: after the move, $56 = 2 \times 28$.

1. Let s = current senior choir members and j = current junior choir members.
2. Total: $s + j = 84$.
3. After the move: $j + 6 = 2(s - 6)$, so $j = 2s - 18$.
4. Substitute: $s + (2s - 18) = 84$, so $3s = 102$ and $s = 34$.
5. Then $j = 84 - 34 = 50$.

A This is the number of members the senior choir would have after the move (34 − 6), not the junior choir's current size.

B This is the current number of members in the senior choir, but the question asks about the junior choir.

D This is the number the junior choir would have after 6 members join it (50 + 6), not the number it has now.

Calculator option: Graph $x + y = 84$ and $y + 6 = 2(x - 6)$, letting x stand for the senior choir and y for the junior choir; the lines intersect at (34, 50).

Review: Chapter 18, Systems of Two Linear Equations (page 264)

20. Correct answer: B M-D20

Right triangles and trigonometry · Geometry and Trigonometry · Harder

B *24*

tan P = opposite/adjacent = QR/PR = 12/35, so the legs are in the ratio 12 : 35, and by the Pythagorean theorem the hypotenuse corresponds to $\sqrt{(12^2 + 35^2)} = 37$. Since PQ = 74 = 2 × 37, the scale factor is 2, and QR = 2 × 12 = 24.

1. tan P = QR/PR = 12/35, so QR = 12t and PR = 35t for some positive number t.
2. Pythagorean theorem: PQ = $\sqrt{((12t)^2 + (35t)^2)} = 37t$.
3. Set 37t = 74, so t = 2.
4. QR = 12t = 24.

A This uses the 12 from the ratio as the side length without scaling. The hypotenuse is 74, not 37, so every side is twice as long.

C This is the hypotenuse of the 12 : 35 : 37 reference triangle, not a leg. The actual hypotenuse is 74, so QR = 2 × 12.

D This is PR, the side adjacent to angle P (2 × 35 = 70), not QR, the side opposite angle P.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

21. Correct answer: 11 M-D21

Nonlinear functions · Advanced Math · Harder

Use the two given values to find a and b. Since f(0) = a + b = 7 and f(3) = 8a + b = 35, subtracting gives 7a = 28, so a = 4 and b = 3. Then f(1) = 4(2) + 3 = 11. Doubling f(0) to get 14 wrongly ignores the constant b.

1. $f(0) = a(2)^0 + b = a + b = 7$.
2. $f(3) = a(2)^3 + b = 8a + b = 35$.
3. Subtract the first equation from the second: 7a = 28, so a = 4 and b = 7 − 4 = 3.
4. f(1) = 4(2) + 3 = 11.

Review: Chapter 22, Nonlinear Functions (page 313)

22. Correct answer: D M-D22

Nonlinear functions · Advanced Math · Harder

D *8*

Substitute both points. From (1, 0): $a(1 - 3)^2 + k = 4a + k = 0$. From (0, −10): $a(0 - 3)^2 + k = 9a + k = -10$. Subtracting gives 5a = −10, so a = −2, and then k = −4a = 8. The vertex of the graph is (3, 8).

1. Substitute (1, 0): 4a + k = 0.
2. Substitute (0, −10): 9a + k = −10.
3. Subtract the first equation from the second: 5a = −10, so a = −2.
4. Substitute back: 4(−2) + k = 0, so k = 8.

A This treats f(0) = −10 as k. In this form, k is the y-coordinate of the vertex, (3, k), not the y-intercept.

B This has the right size but the wrong sign. From 4a + k = 0 with a = −2, k = −4(−2) = 8.

C This is the value of a, the coefficient that sets the parabola's direction and width, not the constant k.

Calculator option: After solving, graph $y = -2(x - 3)^2 + 8$ and confirm that it passes through (1, 0) and (0, −10).

Review: Chapter 22, Nonlinear Functions (page 313)

PART II

Reading and Writing

How the Reading and Writing Section Works

Words in Context

Text Structure and Purpose

Cross-Text Connections

Central Ideas and Details

Command of Evidence: Textual and Quantitative

Inferences

Boundaries

Form, Structure, and Sense

Transitions

Rhetorical Synthesis

CHAPTER 4

How the Reading and Writing Section Works

The Reading and Writing section looks different from most reading tests you have taken. There are no long passages with ten questions each. Instead, every question brings its own short text, and the questions arrive in a predictable order. This chapter shows you how the section is built, gives you one dependable method and previews how Chapters 5 through 14 teach each skill.

One short passage per question

Each Reading and Writing question has its own passage, or its own pair of passages. A passage runs from 25 to 150 words, and a passage pair counts as one total within that range. Every question is four-option multiple choice with a single best answer. There is no calculator in this section, and none is needed.

Passages come from four subject areas: literature, history/social studies, humanities and science. On the test, literary passages are excerpts from published works, and the other passages are written for the test from factual information. In this book, literary passages are either original or verbatim public-domain excerpts identified by author, title and year. Passages about real topics were fact-checked; the rest are labeled as hypothetical or fictional.

Short passages change how you should read. Every sentence can matter, so do not skim. Because you never have to hold more than about 150 words in mind, you can afford to read closely and reread the key sentence.

The four domains and eleven skills

The section tests 11 skills in four domains. The table shows where each skill is taught in this book and the kind of question that tests it.

Chapter	Skill	Domain	Typical question
5	Words in Context	Craft and Structure	Which choice completes the text with the most logical and precise word or phrase?
6	Text Structure and Purpose	Craft and Structure	Which choice best states the main purpose of the text?
7	Cross-Text Connections	Craft and Structure	Based on the texts, how would the author of Text 2 most likely respond to the claim in Text 1?
8	Central Ideas and Details	Information and Ideas	Which choice best states the main idea of the text?
9	Command of Evidence: Textual	Information and Ideas	Which finding, if true, would most directly support the researchers' hypothesis?

Chapter	Skill	Domain	Typical question
9	Command of Evidence: Quantitative	Information and Ideas	Which choice most effectively uses data from the table to complete the statement?
10	Inferences	Information and Ideas	Which choice most logically completes the text?
11	Boundaries	Standard English Conventions	Which choice completes the text so that it conforms to the conventions of Standard English?
12	Form, Structure, and Sense	Standard English Conventions	Which choice completes the text so that it conforms to the conventions of Standard English?
13	Transitions	Expression of Ideas	Which choice completes the text with the most logical transition?
14	Rhetorical Synthesis	Expression of Ideas	The student wants to emphasize a difference between two studies. Which choice most effectively uses relevant information from the notes to accomplish this goal?

Notice that the two Standard English Conventions skills share the same question. You tell them apart by the choices: Boundaries choices test sentence and clause connections, often through punctuation or connecting words. Form, Structure, and Sense choices test usage such as verb forms, agreement or possessives. Identify the rule being tested rather than relying only on how the choices look.

The order of questions in a module

Both Reading and Writing modules follow the same order, so you can predict what kind of question is coming and how hard it is likely to be.

Order	Domain	Skills, in their usual order	How difficulty runs
First	Craft and Structure	Words in Context, Text Structure and Purpose, Cross-Text Connections	Grouped by skill; each group runs from easiest to hardest
Second	Information and Ideas	Central Ideas and Details, Command of Evidence, Inferences	Grouped by skill; each group runs from easiest to hardest
Third	Standard English Conventions	Boundaries and Form, Structure, and Sense, mixed together	One combined run from easiest to hardest
Fourth	Expression of Ideas	Transitions, then Rhetorical Synthesis	Grouped by skill; each group runs from easiest to hardest

The domain order is fixed, and questions on the same skill are grouped together. The order of skills inside each domain is the usual pattern, not a guarantee. The Command of Evidence questions, both textual and quantitative, come together in one group. Each module's 2 pretest questions sit among the questions on the same skill, so you will not notice them.

Because each skill group restarts at an easier level, difficulty in this section rises and falls several times in a module. Chapter 3 explains how to use that pattern for pacing.

Text complexity

Passages are written at three levels of complexity, roughly matching grades 6–8, grades 9–11 and grades 12–14 (senior year of high school through the first two years of college). Harder questions often use more complex text, but not always: a short, plain passage can carry a hard question, and a dense passage can carry an easy one.

When a passage feels dense, slow down and break it apart.

- **Find the main clause.** Ask "Who or what is doing what?" and ignore the descriptive phrases for a moment.
- **Watch the signal words.** Words such as *however*, *because*, *although* and *therefore* tell you how the ideas connect.
- **Notice the qualifiers.** Words such as *some*, *most*, *may*, *often* and *primarily* limit a claim. Harder questions often turn on them.
- **Restate in plain words.** If you can say the passage's point in one short sentence of your own, you understand it well enough to answer.

Graphics: tables, bar graphs and line graphs

Command of Evidence: Quantitative questions include a graphic, and the only kinds used are tables, bar graphs and line graphs. No calculation is needed: you compare values, spot the larger or smaller one, or notice a trend.

Read a graphic in this order.

1. Read the title, so you know what was measured.
2. Read the axis labels or column headings, including units.
3. Read the key, if there is one, so you know which bar or line is which group.
4. Read the sentence the question asks you to complete, and note exactly which groups, years or measures it names.
5. Find only those values in the graphic.

TRAP A choice can describe the graphic accurately and still be wrong. If it reports a true value for the wrong group, the wrong year or the wrong measure, it does not complete the claim. Chapter 9 teaches you to match a claim's exact variables.

The general method

Each skill chapter teaches its own method, but all of them share a four-step core that works on any Reading and Writing question.

1. **Read the question first.** It tells you your task: choosing a word, a purpose, a piece of evidence, a punctuation mark or a transition. Knowing the task tells you what to look for as you read.
2. **Read the passage with that task in mind.** For reading questions, find the main claim and the sentence that matters most. For conventions questions, read the whole sentence around the blank, not just the words beside it.

3. **Predict.** Before you look at the choices, put the answer in your own words: a rough word, a short statement or the punctuation rule you expect. A prediction protects you from choices that sound good but do not fit.
4. **Eliminate.** Compare each choice with your prediction. Cross out each choice you can reject and name the reason it fails. Choose the one that survives, then reread the sentence with it in place.

Naming the reason a choice fails makes elimination reliable. These reasons come up most often.

Reason a choice fails	What it looks like
Too broad	It claims more than the passage covers.
Too narrow	It describes one detail instead of the whole point.
Opposite	It reverses the passage's claim or direction.
Unsupported	It sounds reasonable, but the passage never says or implies it.
True but not responsive	It is accurate, but it does not answer the question asked.
Wrong relationship	It connects ideas with the wrong logic, such as cause instead of contrast.
Breaks a rule	It creates a grammar or punctuation error.

TIP Use Bluebook's Option Eliminator as you work. Crossing out a choice only when you can name its flaw keeps you honest and makes your second pass faster.

How to read a passage pair

Cross-Text Connections questions give you two short texts, labeled Text 1 and Text 2, that discuss the same topic. Together they still total no more than 150 words. The question usually asks how one author would respond to the other, or what both authors would agree on.

Map the pair before you look at the choices.

1. **Text 1's claim:** state it in a few words.
2. **Text 2's claim:** state it in a few words.
3. **The relationship:** does Text 2 agree, qualify (agree with limits), reject or offer a different explanation?
4. **The direction:** check which author is responding to which. "How would the author of Text 2 respond to Text 1" is a different question from the reverse.

TRAP A choice that matches only one text. On a "both authors would agree" question, a statement that fits Text 1 perfectly is still wrong if Text 2 would dispute it.

How Chapters 5 through 14 are organized

Each skill chapter follows the same order.

1. **What the question looks like:** wording and format.
2. **How to recognize it:** the clues that identify the skill.

3. **The method:** a numbered routine.
4. **Worked examples:** at least five, including a harder-module example, each with the thinking, the answer and why the other choices fail.
5. **Common traps:** a table of traps and how to spot them.
6. **Harder-module variations:** how the skill gets harder at the top of the scale.
7. **Check yourself:** quick recall prompts with brief answers.

A Practice Set follows each chapter, arranged from easier to harder, and then explanations for every answer choice. Each explanation ends by naming the chapter to review.

Two worked examples

The two examples below apply the general method to a reading question and a conventions question; the skill chapters go much deeper.

Worked example 1: a reading question

In a hypothetical study, librarians at a small public library tracked how often patrons borrowed books displayed face-out, with the cover facing the room, compared with books shelved spine-out. Over six months, the face-out books were borrowed about three times as often. However, staff had chosen the face-out books because they were new releases, a group of books that this library's patrons already borrowed often before the display began. The study's results therefore _____

Which choice most logically completes the text?

- **A** prove that displaying a book face-out causes patrons to borrow it.
- **B** cannot show whether the display or the books' newness explains the higher borrowing rate.
- **C** suggest that the library's patrons prefer older books to new releases.
- **D** indicate that the library should display every book face-out.

Read the question first. "Most logically completes the text" is an Inferences question (Chapter 10). Your task is to finish the passage's line of reasoning.

Read the passage with the task in mind. The first two sentences give a result: face-out books were borrowed more often. Then *however* signals a problem. The face-out books were also new releases, which were already popular. Two things were different about the face-out books, so either one could explain the result.

Predict. Something like: "...cannot tell us that face-out display caused the extra borrowing."

Eliminate. A is *too broad*: "prove" claims far more than the study can show, given the problem that *however* introduced. C is the *opposite* of the passage, which says new releases were already borrowed often. D is *unsupported*: a recommendation goes beyond what a flawed comparison can show. B matches the prediction.

Answer: B.

Worked example 2: a conventions question

A hand-drawn trail map hangs near the entrance of the town hall in the fictional mountain town of Pine Hollow. Its creator—a former surveyor who had walked nearly every path in the _____ drew the entire map from memory.

Which choice completes the text so that it conforms to the conventions of Standard English?

- **A** valley,
- **B** valley;
- **C** valley
- **D** valley—

Read the question first. This is the standard conventions question, and the choices differ only in punctuation, so this is a Boundaries question (Chapter 11).

Read the whole sentence. The core sentence is "Its creator ... drew the entire map from memory." The dash after *creator* opens a *supplement*: extra information that could be removed without breaking the sentence. Here the supplement is an *appositive*, a noun phrase that renames the noun next to it ("a former surveyor who had walked nearly every path in the valley" renames "its creator").

Predict. A supplement opened with a dash must be closed with a dash. The blank needs a dash.

Eliminate. A creates a mismatched pair: a dash cannot be closed with a comma. B is wrong because a semicolon must separate two complete sentences, and "Its creator—a former surveyor who had walked nearly every path in the valley" is not one. C leaves the supplement open and fails to mark the return to the main clause. D closes the supplement correctly.

Answer: D.

CHECK In both examples, the decisive clue was one small signal: *however* in the first, the opening dash in the second. Train yourself to look for the signal before you look at the choices.

Before you start Chapter 5

- Read the question first, then the passage, then predict, then eliminate.
- Know the order: Craft and Structure, Information and Ideas, Standard English Conventions, Expression of Ideas.
- Difficulty restarts with each skill group, while the conventions questions form one combined run.
- Graphics are only tables, bar graphs and line graphs, and they never require calculation.
- For a passage pair, map both claims and their relationship before reading the choices.

ONLINE (OPTIONAL) The website has a page for each of the 30 skills: what it asks, how to spot it, its traps and two video lessons. Its video library also covers the test format, the testing app and the graphing calculator.

All 30 skills
skillbyskillprep.com/skills

Video lessons
skillbyskillprep.com/videos

CHAPTER 5

Words in Context

Words in Context questions ask you to choose the word or phrase that best fits a short passage, or to decide what a word means in the sentence where it appears. They look like vocabulary questions, but they are really reading questions. The passage always contains the clues you need, and the right answer is the choice those clues support most exactly.

What the question looks like

Words in Context belongs to the Craft and Structure domain, which opens each Reading and Writing module. Questions on the same skill are grouped together and run from easier to harder, and Words in Context questions usually come first.

You will see two versions.

- **Blank completion.** A passage of 25 to 150 words contains a blank (_____). The question asks, "Which choice completes the text with the most logical and precise word or phrase?"
- **Meaning in context.** A word from the passage appears in quotation marks in the question, and sometimes it is also underlined in the passage. The question asks, "As used in the text, what does the word '...' most nearly mean?"

In both versions, the four choices are usually the same part of speech, so grammar rarely eliminates anything. Two words in the blank-completion question matter. *Logical* means the choice fits what the passage is saying. *Precise* means it says exactly that, with the right strength and the right tone, rather than something close by.

How to recognize it

Read the question first and notice the form of the choices (single words, transition words or full clauses) without weighing them yet. The table separates Words in Context from three question types that also use a blank.

What you see	Question type	Where it is taught
"most logical and precise word or phrase"; the choices are single words or short phrases	Words in Context	This chapter
"most nearly mean"	Words in Context	This chapter
Choices such as "However," "Therefore," and "For example,"	Transitions	Chapter 13
"most logically completes the text"; the blank ends the passage and the choices are full clauses	Inferences	Chapter 10
"conforms to the conventions of Standard English"; the choices differ in punctuation or verb form	Boundaries; Form, Structure, and Sense	Chapters 11 and 12

The method

Use the same five steps every time. With practice, they take about a minute.

1. **Cover the choices.** Read the whole passage and treat the blank as a gap. If you read the choices first, a familiar word can pull you in before you know what the passage needs.
2. **Find the clue.** Locate the words that control the meaning of the blank, and name the type of clue using the table below.
3. **Predict.** Say a simple word or phrase that would fit, such as "small," "not proven," or "changing." A rough prediction is fine. Even "something negative" helps.
4. **Match.** Compare each choice with your prediction, not with the passage in general. Cross out choices that point the wrong way.
5. **Test precision and tone.** Read the sentence again with your answer in place. Ask whether it has the right strength (mild or strong?) and the right feeling (warm or cold?) for this passage.

For a "most nearly mean" question, cover the quoted word in the passage, predict a replacement from the clues, and then match.

Four kinds of context clues

Clue type	Signal words	What it tells you	Mini example
Contrast	but, yet, although, however, while, unlike, by contrast	The blank means roughly the opposite of a nearby idea.	"Although the first draft rambled, the final essay was _____." Predict: short and focused.
Restatement	a colon, a dash, "that is," "in other words"	The blank means the same as a nearby phrase.	"The room was _____: every surface held a stack of papers." Predict: cluttered.
Cause and effect	because, since, so, therefore, as a result	The blank names a cause or result that fits the logic.	"Because the trail had washed out, the park _____ it." Predict: closed.
Example	for example, such as, including, or a list	The blank names what the examples have in common.	"Her meals were _____: rice, beans and little else." Predict: plain.

TIP Before you predict a word, decide whether the blank is positive, negative or neutral, and whether it is strong or mild. Those two labels often eliminate two choices, and they are your fallback when you cannot predict a word at all.

Worked examples

Try each example yourself before you read the explanation.

Example 1: A contrast clue

Early maps of the fictional island of Tessaly were _____, showing only the coastline and a single harbor. By contrast, the map that a survey team completed decades later recorded every stream, ridge and footpath on the island.

Which choice completes the text with the most logical and precise word or phrase?

A) detailed; B) sparse; C) colorful; D) accurate

Think it through. "By contrast" tells you that the early maps were the opposite of the later map, which "recorded every stream, ridge and footpath." The phrase "showing only the coastline and a single harbor" restates the same idea. Predict: "bare" or "having little detail."

Answer: B. *Sparse* means thinly filled, which matches the prediction exactly.

Why the others fail. *Detailed* describes the later map, so it sits on the wrong side of the contrast. *Colorful* has no clue in the passage. *Accurate* concerns correctness, but the passage contrasts the amount of detail, not whether either map is correct. A map can be accurate and still show very little.

Example 2: A restatement clue

The treaty's wording proved _____: each clause could be read in two different ways, and both nations later claimed that the text supported their position.

A) concise; B) hostile; C) ambiguous; D) temporary

Think it through. The colon signals a restatement, so the words after it explain the blank: each clause "could be read in two different ways." Predict: "unclear" or "open to more than one reading."

Answer: C. *Ambiguous* means open to more than one interpretation.

Why the others fail. *Concise* means brief, but length is never discussed. *Hostile* adds a feeling that the passage does not show. *Temporary* concerns time, and nothing indicates that the treaty expired.

Example 3: A cause clue and a sensible-sounding trap

Because the old market square flooded after nearly every heavy storm, the city's new design for it is _____: the paving stones rest on a bed of gravel that lets rainwater soak into the ground instead of collecting on the surface.

A) permeable; B) decorative; C) temporary; D) sloped

Think it through. "Because" names the problem, flooding, and the colon explains the solution: rainwater can "soak into the ground." Predict: "lets water pass through."

Answer: A. *Permeable* means allowing liquid to pass through, which is exactly what the explanation describes.

Why the others fail. *Sloped* is the tempting choice, because a sloped surface could also carry water away. But the passage explains a different solution, so *sloped* is sensible in general and unsupported here. *Decorative* and *temporary* have no clue at all.

TRAP A choice can be a reasonable idea about the topic and still be wrong. The answer must match the passage's own explanation, not a solution you might have proposed.

Example 4: "Most nearly mean" and a second meaning

Critics admired the realism of the novelist's early settings but found her dialogue wooden. For her third book, she spent months listening to conversations on buses and in diners, studying the rhythm of everyday speech. Reviewers who had once complained about her characters now praised their voices as remarkably natural.

As used in the text, what does the word "wooden" most nearly mean?

A) sturdy; B) stiff; C) carved; D) old-fashioned

Think it through. Cover "wooden" and look for clues. The dialogue was criticized, and the cure was learning "the rhythm of everyday speech," after which the voices sounded "natural." Predict: "unnatural" or "awkward."

Answer: B. *Stiff* describes speech that sounds forced rather than natural.

Why the others fail. *Sturdy* and *carved* use the physical meaning of wood, which makes no sense for dialogue. *Old-fashioned* is a criticism a reader might make of dialogue, but the passage contrasts unnatural speech with natural speech, not old speech with new.

NOTE In "most nearly mean" questions, the word's familiar everyday meaning is often the trap. If that meaning seems to fit perfectly, test it against the clues again.

Example 5: Tone decides

When Amara's younger brother finally finished his birdhouse, it leaned to one side and the roof did not quite meet the walls. Still, he carried it to her in both hands, beaming. Amara, who had watched him work on it for three afternoons, held it _____ and told him it was the finest birdhouse on the street.

A) critically; B) tenderly; C) indifferently; D) suspiciously

Think it through. Every choice describes a way of holding something, so meaning alone will not decide. Tone will. The brother is "beaming," Amara has watched him work for three afternoons, and she praises the result. Predict: "kindly" or "lovingly."

Answer: B. *Tenderly* matches the warm, protective feeling of the scene.

Why the others fail. *Critically* picks up the details about the leaning birdhouse, but Amara's words show affection, not judgment. *Indifferently* contradicts her three afternoons of attention. *Suspiciously* has no support anywhere in the scene.

Example 6: Harder-module example with two clues

The architect's plan for the riverside district has drawn wide praise for its parks and footpaths, and few critics dispute that the design is handsome. Its housing proposal has proved more _____. Some residents welcome the taller apartment buildings it would allow, while others warn that the towers would block river views and raise rents, and months of public meetings have produced no agreement.

A) popular; B) contentious; C) expensive; D) obscure

Think it through. This passage has two clues. "More" compares the housing proposal with the widely praised parks, so the blank must move away from praise. The next sentence then restates the idea: residents are split, and the meetings "produced no agreement." Predict: "disputed."

Answer: B. *Contentious* means causing disagreement, which both clues support.

Why the others fail. *Expensive* grabs the word "rents," but rising rents are one argument inside the disagreement, not what the blank describes. *Popular* is the wrong side of the comparison. *Obscure*, meaning little known, contradicts months of public meetings.

Common traps

Trap	How to spot it
Wrong side of the contrast	The choice describes the idea that "but," "unlike" or "by contrast" pushes against.
Familiar meaning	In a "most nearly mean" question, the choice gives the word's everyday sense, but the sentence uses a different one.
On topic but unsupported	The choice suits the subject (an art word in a museum passage), yet no clue points to it.
Wrong strength	The direction is right but the force is not: "furious" when the passage shows mild annoyance, or "proved" when it shows "suggested."
Wrong tone	The choice is cold in a warm scene, or casual in a formal one.
Detail grab	The choice echoes a single nearby word ("rents" leading to "expensive") instead of the idea the blank needs.

Harder-module variations

Harder questions often use more complex text: longer sentences, more academic vocabulary and more literary language. The method stays the same, but these variations need extra care.

- **Negatives around the blank.** Phrases such as "far from _____," "hardly _____" or "not entirely _____" reverse the meaning. Rewrite the sentence in positive terms before you predict. "The results were far from conclusive" means "the results left the question open."
- **Phrase-length choices.** When the options are phrases such as "call into question" or "rule out," compare their verbs and their strength. Evidence that only suggests something cannot rule out every alternative.
- **Second meanings of everyday words.** Words such as *sound* (reliable), *novel* (new), *temper* (soften), *qualify* (limit) and *betray* (reveal) often appear in their less common senses.
- **Two clues in tension.** A passage may praise one feature and question another. Find the clue that controls the sentence containing the blank.
- **All four choices on topic.** Every choice may suit the subject, so your prediction, not the topic, has to do the eliminating.

Check yourself

1. What are the five steps of the Words in Context method?
2. Which signal words point to a contrast clue, and what does a contrast clue tell you about the blank?
3. What does a colon right after the blank usually signal?
4. In a "most nearly mean" question, why should you be cautious when a word's most familiar meaning seems to fit?

5. How should you handle a blank inside a phrase such as "far from _____"?
6. **Answers:** (1) Cover the choices, find the clue, predict, match, and test precision and tone. (2) Words such as but, although, however and by contrast; the blank means roughly the opposite of the nearby idea. (3) A restatement: the words after the colon explain the blank. (4) These questions often use a less common meaning, so the familiar one may be the trap. (5) Rewrite the sentence in positive terms first, then predict.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/words-in-context

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C0501

> In a hypothetical greenhouse trial, a botanist grew two groups of basil seedlings under identical lamps. One group was watered every morning, and the other was watered only once a week. The seedlings that received daily water grew tall and leafy. In contrast, the seedlings watered once a week remained _____, with thin stems and only a few small leaves at the end of the trial.

Which choice completes the text with the most logical and precise word or phrase?

A fragrant **B** stunted **C** colorful **D** sturdy

---

2 RW-C0502

> Marisol had promised her grandmother that she would keep the window boxes blooming all summer, but by July the petunias drooped no matter how often she watered them. Every evening she knelt beside the boxes, pinching off dead blossoms and testing the soil with her fingertip. Her brother teased her for fussing over "a few flowers," yet Marisol remained _____: she refused to let a single box fail.

Which choice completes the text with the most logical and precise word or phrase?

A indifferent **B** forgetful **C** amused **D** determined

---

3 RW-C0503

> Planners in the fictional city of Larkport noticed that a narrow downtown street was often jammed with delivery trucks during the morning rush. Because the trucks were blocking buses and cyclists, the city decided to _____ deliveries on that street to the hours before 7:00 a.m., when traffic is light. After the change, buses on the street ran much closer to their schedules.

Which choice completes the text with the most logical and precise word or phrase?

A restrict **B** celebrate **C** duplicate **D** disguise

---

4 RW-C0504

> At a small museum in the fictional town of Bellgrove, one gallery features a local painter who worked with _____ materials. One canvas includes strips of newspaper glued beneath the paint. Another work is painted on an old wooden door, and a third mixes oil paint with sand that the artist scooped from a nearby beach.

Which choice completes the text with the most logical and precise word or phrase?

A costly **B** fragile **C** unconventional **D** identical

---

5 RW-C0505

> Many plants replace their leaves again and again over the course of their lives. Welwitschia mirabilis, a plant of the Namib Desert in southwestern Africa, follows a different pattern. It produces only two leaves, which are leathery and shaped like long straps. These leaves are remarkably _____: rather than being shed and replaced, they keep growing from their bases for the plant's entire life, even as their far ends slowly wear away.

Which choice completes the text with the most logical and precise word or phrase?

A seasonal **B** fragile **C** persistent **D** decorative

6 RW-C0506

> For weeks, Ines had sketched the central figure of the mural on the library wall, and she was eager to begin painting it. But when the lead artist halted work for the third time that morning to remeasure the wall, Ines felt her patience thinning. She wanted to paint, not wait. Still, she checked her frustration, set down her brush and listened carefully as the lead artist explained the new measurements.

As used in the text, what does the word "checked" most nearly mean?

A restrained **B** examined **C** marked **D** verified

7 RW-C0507

> The Dutch painter Rachel Ruysch (1664–1750) specialized in still lifes, many featuring flowers, fruit and insects, and wealthy buyers paid high prices for her work. Her father, a professor of anatomy and botany, probably introduced her to the study of exotic flowers. Ruysch's paintings are admired for being _____: her petals, leaves and insects are rendered with such accuracy that they reveal a deep knowledge of real plant and animal specimens.

Which choice completes the text with the most logical and precise word or phrase?

A improvised **B** abstract **C** symbolic **D** meticulous

8 RW-C0508

> In a book published in 1898, the English reformer Ebenezer Howard proposed building "garden cities" of about 30,000 people each. Howard wanted each garden city to be _____, providing the jobs and amenities of a large city so that it would not be merely a suburb whose residents depended on somewhere else for work. A belt of farmland closed to builders would surround each one. Two such towns were founded in England during his lifetime: Letchworth in 1903 and Welwyn Garden City in 1920.

Which choice completes the text with the most logical and precise word or phrase?

A temporary **B** self-sufficient **C** crowded **D** ornamental

9 RW-C0509

> Seeds of some alpine wildflowers will not sprout in the same autumn that they fall, even when conditions seem favorable. In a hypothetical laboratory study, a team of botanists found that chemicals in the seed coat of one such species arrest germination until the seeds have spent several weeks in cold, damp conditions, much like a winter beneath snow. Once that period ends, the chemicals break down, and the seeds sprout readily when warmed.

As used in the text, what does the word "arrest" most nearly mean?

A halt **B** capture **C** attract **D** punish

---

10 RW-C0510

> The following poem describes a gardener at the end of autumn.
>
> She kneels where the tomato vines have failed,
> their stems gone brittle, leaves curled into fists,
> the stakes all leaning like tired sentries.
> Yet she does not mourn the _____ rows.
> She splits the last soft fruit with her thumb,
> lifts out the seeds and dries them on the sill,
> then folds them into paper marked for spring.

Which choice completes the text with the most logical and precise word or phrase?

A crowded **B** fertile **C** barren **D** distant

---

11 RW-C0511

> Conservators at a hypothetical regional museum were asked to clean a landscape painting whose varnish had yellowed with age, dulling the blues of its sky. Removing old varnish can be risky, because a solvent strong enough to dissolve it may also loosen the paint underneath. Rather than removing the varnish all at once, the conservators worked _____, first testing a solvent on a tiny corner and then cleaning the canvas a few square centimeters at a time over several months.

Which choice completes the text with the most logical and precise word or phrase?

A thoroughly **B** incrementally **C** collaboratively **D** hurriedly

---

12 RW-C0512

> In a hypothetical case, early field notes on a rare mountain orchid recorded only a single species of night-flying moth carrying its pollen, and for years the orchid was described as depending on that moth alone. A follow-up study complicates the picture without overturning it. Over two seasons, motion-triggered cameras recorded the moth carrying pollen between flowers far more often than any other visitor, but two species of bee, active during the day, also carried pollen on several occasions. The moth's role, in other words, appears to be _____ rather than exclusive.

Which choice completes the text with the most logical and precise word or phrase?

A negligible **B** essential **C** unverified **D** predominant

---

13 RW-C0513

> When the letter from the county office arrived, Grandfather read it twice at the kitchen window and then folded it into his shirt pocket. Lena waited for him to shout, or to sigh, or to lower himself heavily into his chair, the way he had the spring a late frost blackened half the orchard's blossoms. He did none of these things. His face remained composed as he laced his boots, and he walked out to the rows of young pear trees as though it were any other morning, pausing only to straighten a stake the wind had loosened.

As used in the text, what does the word "composed" most nearly mean?

A calm **B** written **C** constructed **D** satisfied

14 RW-C0514

> Critics of a new zoning code in the fictional city of Port Ellery warned that allowing small shops on residential streets would bring heavy traffic and noise to quiet blocks. A two-year review by the city's planning office offers a mixed verdict. Measured noise levels barely changed, and most residents surveyed reported walking to nearby shops more often than before the code took effect. Still, the critics' warnings were not entirely _____: on several blocks, delivery vans now idle along the curb each morning, just as opponents of the code had predicted.

Which choice completes the text with the most logical and precise word or phrase?

A justified **B** unanimous **C** unfounded **D** anticipated

15 RW-C0515

> The leaves of many seaside shrubs are coated in wax, and one proposed explanation is that the coating shields leaves from damage caused by salt spray. In a hypothetical greenhouse experiment, a team of plant biologists gently wiped the wax from the leaves of some shrubs, left others untreated and then misted all of them with salt water for six weeks. The wiped shrubs lost more leaves. Because wiping may also have injured the leaf surface in other ways, however, the team cautioned that its results _____ the proposed explanation but do not settle the question.

Which choice completes the text with the most logical and precise word or phrase?

A definitively establish **B** lend support to **C** directly contradict **D** have no bearing on

Practice Set Explanations

1. Correct answer: B RW-C0501

Words in Context · Craft and Structure · Easier

B *stunted*

The signal "In contrast" sets the weekly-watered seedlings against those that "grew tall and leafy," and the phrase after the blank restates their condition: "thin stems and only a few small leaves." The blank needs a word meaning small and underdeveloped. "Stunted," which means held back in growth, fits both clues exactly.

A The passage compares how much the seedlings grew, not how they smelled, so nothing supports describing them as fragrant.

C Color is never mentioned; the clue after the blank describes thin stems and few leaves, which concern growth rather than appearance.

D "Sturdy" means strong and solid, the opposite of seedlings with thin stems and only a few small leaves.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: D RW-C0502

Words in Context · Craft and Structure · Easier

D *determined*

The colon introduces a restatement of the blank: Marisol "refused to let a single box fail." Her nightly care for the drooping flowers and her steadiness despite her brother's teasing point the same way. "Determined," meaning firmly committed to reaching a goal, matches this evidence precisely.

A "Indifferent" means not caring, which contradicts Marisol's nightly work and her refusal to let any box fail.

B Marisol checks the boxes every evening, so the text shows the opposite of forgetfulness.

C Her brother is the one who is teasing; the clue after the colon describes Marisol's persistence, not amusement.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: A RW-C0503

Words in Context · Craft and Structure · Easier

A *restrict*

The cause clue "Because the trucks were blocking buses and cyclists" shows that the city needed to limit the problem, and the phrase "to the hours before 7:00 a.m." describes confining deliveries to certain times. "Restrict," meaning to keep within limits, fits both clues, and the improved bus schedules confirm that the limit worked.

B The trucks were causing a problem, so the city had no reason to honor the deliveries; the clue calls for limiting them.

C Doubling the deliveries would make the morning jams worse, which conflicts with the improved bus schedules described at the end.

D The change concerns when deliveries happen, not how the trucks look; hiding the trucks would not clear the street.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: C RW-C0504

Words in Context · Craft and Structure · Easier

C *unconventional*

The sentences after the blank are examples: newspaper strips, an old door and beach sand. What these materials share is that none is a standard art supply. "Unconventional," meaning not following usual practice, names the quality that all three examples have in common.

A Newspaper, an old door and beach sand are cheap or free, so the examples point away from costly materials.

B Nothing in the examples suggests that the works break easily; they show unusual choices, not delicate ones.

D Each work uses a different material, so the examples contradict the idea that the materials are identical.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: C RW-C0505

Words in Context · Craft and Structure · Medium

C *persistent*

The colon introduces an explanation of the blank: the leaves are never "shed and replaced" but "keep growing from their bases for the plant's entire life." The contrast with plants that replace their leaves "again and again" points the same way. "Persistent," meaning lasting for a long time, precisely describes leaves that remain for the plant's whole life.

A Seasonal leaves are shed and replaced, which is the opposite of what the colon says about this plant's two leaves.

B Although the leaf tips wear away, the explanation stresses that the leaves keep growing for the plant's entire life, which signals endurance rather than fragility.

D The passage describes how long the leaves last, not how they look, so "decorative" has no support.

Review: Chapter 5, Words in Context (page 69)

6. Correct answer: A RW-C0506

Words in Context · Craft and Structure · Medium

A *restrained*

Ines is impatient, yet "Still" signals that she does not act on that feeling: she sets down her brush and listens carefully. In this context, "checked" describes holding an

emotion back. "Restrained," meaning kept under control, captures that meaning.

B "Examined" is a common meaning of "checked," but Ines is not inspecting her frustration; her actions show that she holds it back.

C "Marked" refers to making a check mark, and nothing is written or labeled in the scene.

D "Verified" means confirmed as accurate, which does not fit a feeling; the context concerns self-control, not confirmation.

Review: Chapter 5, Words in Context (page 69)

7. Correct answer: D RW-C0507

Words in Context · Craft and Structure · Medium

D *meticulous*

The colon introduces an explanation of the blank: Ruysch's petals, leaves and insects are "rendered with such accuracy" that they show "a deep knowledge of real plant and animal specimens." The blank must describe extremely careful, exact work. "Meticulous," meaning showing great attention to detail, fits that explanation precisely.

A "Improvised" means done without prior planning, but the passage gives no information about planning. Its clues concern accuracy and attention to detail.

B Abstract art does not aim to depict real objects exactly, but the text stresses accurate depictions of real specimens.

C Still lifes can carry symbolic meanings, but the explanation after the colon concerns accuracy, not hidden meaning, so "symbolic" is unsupported here.

Review: Chapter 5, Words in Context (page 69)

8. Correct answer: B RW-C0508

Words in Context · Craft and Structure · Medium

B *self-sufficient*

The rest of the sentence explains the blank: each garden city would provide "the jobs and amenities of a large city" so that it would not be "merely a suburb" whose residents "depended on somewhere else for work." A town that meets its own needs is "self-sufficient," the choice that precisely captures Howard's goal.

A Nothing suggests the towns were meant to be short-lived; two were founded as lasting communities, so "temporary" has no support.

C Howard set a population of about 30,000 and surrounded each town with farmland, details that point away from crowding.

D The word "garden" might suggest decoration, but the explanation concerns jobs and amenities, not ornament.

Review: Chapter 5, Words in Context (page 69)

9. Correct answer: A RW-C0509

Words in Context · Craft and Structure · Medium

A *halt*

The chemicals keep the seeds from sprouting "until" a cold period has passed; once the chemicals break down, the seeds "sprout readily." The chemicals therefore stop germination for a time. "Halt," meaning to bring to a stop, matches this use of "arrest."

B "Capture" reflects the police meaning of "arrest," but chemicals cannot seize a process; the context describes stopping germination.

C This choice draws on "arresting" in the sense of eye-catching, but the chemicals prevent sprouting rather than drawing attention to it.

D Punishment implies wrongdoing, which does not apply to a seed; the context describes pausing growth until conditions change.

Review: Chapter 5, Words in Context (page 69)

10. Correct answer: C RW-C0510

Words in Context · Craft and Structure · Medium

C *barren*

The opening lines show the garden at its end: the vines "have failed," their stems are "brittle," and the leaves are curled. "Yet she does not mourn" signals that the rows are something a gardener might grieve over. "Barren," meaning no longer productive, describes the failed rows precisely; the gardener saves seeds from the last fruit rather than expecting more.

A The poem describes dying vines, not plants packed closely together, so nothing supports "crowded."

B "Fertile" means productive, the opposite of rows whose vines have failed and gone brittle.

D The gardener kneels right beside the rows, so describing them as distant contradicts the scene.

Review: Chapter 5, Words in Context (page 69)

11. Correct answer: B RW-C0511

Words in Context · Craft and Structure · Medium

B *incrementally*

"Rather than removing the varnish all at once" sets up a contrast with an approach done in small stages, and the rest of the sentence confirms it: a test on "a tiny corner," then "a few square centimeters at a time over several months." "Incrementally," meaning in small steps, fits this description most precisely.

A The conservators may have been thorough, but the contrast with "all at once" calls for a word about working in small stages, which "thoroughly" does not convey.

C Nothing in the passage describes teamwork; the details concern the pace and scale of each step of the cleaning.

D Months of work, a few square centimeters at a time, is the opposite of hurried work.

Review: Chapter 5, Words in Context (page 69)

12. Correct answer: D RW-C0512

Words in Context · Craft and Structure · Harder

D *predominant*

The follow-up study "complicates the picture without overturning it": the moth carried pollen "far more often than any other visitor," but bees did so too. The blank must describe a role that is the largest but not the only one, as "rather than exclusive" confirms. "Predominant," meaning greatest in frequency or importance, states exactly that.

A "Negligible" means too small to matter, but the moth carried pollen far more often than any other visitor.

B "Essential" claims the orchid cannot reproduce without the moth, yet bees also carried pollen, so the evidence shows frequency, not necessity.

C The cameras directly recorded the moth carrying pollen, so its role has been confirmed rather than left unverified.

Review: Chapter 5, Words in Context (page 69)

13. Correct answer: A RW-C0513

Words in Context · Craft and Structure · Harder

A *calm*

Lena expects an emotional reaction (shouting, sighing, sinking into a chair), but Grandfather does "none of these things" and goes about his morning "as though it were any other." His face therefore shows no visible distress. "Calm," meaning free from outward agitation, is the meaning of "composed" here.

B A letter can be composed, or written, but here the word describes Grandfather's face, which reveals no emotion.

C "Constructed" means built from parts, which does not describe a facial expression in this scene.

D Grandfather hides any reaction, but nothing shows that he is pleased by the letter; "satisfied" goes beyond the evidence of self-control.

Review: Chapter 5, Words in Context (page 69)

14. Correct answer: C RW-C0514

Words in Context · Craft and Structure · Harder

C *unfounded*

"Still" turns from the positive findings back toward the critics, and the colon supplies evidence that one warning came true: delivery vans now idle on several blocks, "just as opponents of the code had predicted." The sentence must say the warnings had some basis. Because "unfounded" means lacking a basis, "not entirely unfounded" says exactly that.

A "Not entirely justified" would mean the warnings were partly wrong, but the evidence after the colon shows a warning that came true.

B Whether all the critics agreed is never discussed; the sentence concerns whether their warnings had any basis.

D "Not entirely anticipated" makes little sense here, because the colon notes that the problem appeared "just as opponents of the code had predicted."

Review: Chapter 5, Words in Context (page 69)

15. Correct answer: B RW-C0515

Words in Context · Craft and Structure · Harder

B *lend support to*

Shrubs without wax lost more leaves, which is consistent with the idea that wax protects against salt. But the team notes a second possible cause, injury from wiping, and says the results "do not settle the question." The blank needs a phrase that is positive but limited. "Lend support to" fits both the direction of the result and the caution.

A "Definitively establish" is too strong, because the team explicitly says that the results "do not settle the question."

C The wiped shrubs fared worse, which fits the idea that wax protects leaves, so the results do not contradict the explanation.

D The experiment tested the wax explanation directly, so saying the results have no bearing on it ignores what the team found.

Review: Chapter 5, Words in Context (page 69)

CHAPTER 6

Text Structure and Purpose

Text Structure and Purpose questions ask what a text is doing, not only what it says. Every author has a job in mind: to describe something, to argue a point, to challenge an idea or to capture a moment in a story. These questions ask you to name that job for the whole text, for one underlined sentence, or for the order of the parts. Once you can label what each sentence does, these questions become predictable.

What the question looks like

Text Structure and Purpose is part of the Craft and Structure domain, so these questions appear early in each Reading and Writing module, usually right after the Words in Context questions. Each one has a short passage, sometimes a poem, and one of three questions.

Question form	Question	What you are naming
Main purpose	Which choice best states the main purpose of the text?	Why the whole text was written
Underlined function	Which choice best describes the function of the underlined portion in the text as a whole?	The job that one sentence or phrase does for the text
Overall structure	Which choice best describes the overall structure of the text?	The moves the text makes, in order

The choices usually begin with a verb: "To argue that...," "To describe...," "It introduces...," "It qualifies...." Check that verb first.

How to recognize it

You are in this skill when the question uses the word *purpose*, *function* or *structure*. Do not confuse it with Central Ideas and Details (Chapter 8), which asks for the main *idea*: what the text says. Main *purpose* questions ask what the text *does*. Compare three labels for a short passage about a town that tested a four-day school week:

Label	Answer
Topic	A four-day school week
Main idea	The town's trial found that attendance rose but some families struggled to find childcare.
Main purpose	To report the mixed results of a trial

A choice that only names the topic is never enough for a purpose question.

The method

1. **Label as you read.** After each sentence, give it a two- or three-word job: "common belief," "study," "result," "objection," "example," "author's view," "shift in mood." These labels are your map of the text.

2. **Decide the author's stance.** Ask whether the author is presenting information or taking a position. Evaluative words such as *should*, *flawed*, *persuasive* and *overlooks* signal a position.
3. **Predict before you read the choices.** For main purpose, use a verb plus a short object, such as "to describe a test of a method." For underlined function, ask, "What would the text lose without this sentence?" For overall structure, list your labels in order, usually two to four of them.
4. **Check the verb first.** Eliminate any choice whose verb does not fit the text's stance, such as "argue" for a neutral description or "criticize" for an approving one.
5. **Then check the rest of the choice.** It must match the whole text (for purpose) or the specific sentence (for function), with nothing extra and nothing missing.

Purpose verbs and what they signal

Verb	What the text or sentence does	Common signals
describe, explain	Presents information without taking a side	how, why, process and sequence words
argue, claim	Takes a position that someone could dispute	should, must, flawed, better
challenge, question	Pushes back against an idea	however, but, "commonly assumed," "overlooks"
qualify	Limits a claim without rejecting it	though, only, in some cases, may
illustrate	Gives a specific example of a general point	for example, consider, in one case
contrast, compare	Sets two things side by side	unlike, whereas, similarly
concede	Admits a point that cuts against the author's own view	admittedly, granted, to be sure

TIP "Qualify" does not mean "agree" or "reject." A sentence that qualifies a claim accepts part of it and draws a boundary around the rest.

Worked examples

Example 1: Main purpose of an informational text

Some farmers plant clover between rows of corn to protect the soil. In a hypothetical field trial, a team of agronomists compared plots of corn grown with clover between the rows and plots grown on bare soil. After two seasons, the soil in the clover plots held more moisture and lost less topsoil to heavy rains, and corn yields in the two kinds of plots were nearly the same.

Which choice best states the main purpose of the text?

- A) To argue that every farm should plant clover between its rows of corn
- B) To describe a comparison of two growing methods and its results
- C) To explain how clover seeds are chosen and planted
- D) To criticize farmers for ignoring the problem of soil loss

Think it through. Labels: practice (clover), study (plots compared), results (better soil, similar yields). The author reports and never recommends anything. Predict: "to describe a trial comparing two methods."

Answer: B.

Why the others fail. A has the wrong verb, because the text never says what farmers should do. C names a topic the text does not discuss. D invents a criticism; the farmers in the first sentence are already trying to protect their soil.

Example 2: Main purpose of a literary passage

Every Saturday for a year, Wen had walked past the corner bakery without going in, telling himself the bread cost too much. This Saturday the baker was outside sweeping, and she nodded to him as if he were a regular. Wen stopped. He looked at the round loaves in the window, dusted with flour, and then at his own reflection standing there. He pushed the door open before he could change his mind.

- A) To explain why the bakery's bread is so expensive
- B) To describe the bakery's loaves in detail
- C) To show a small gesture prompting a character to break a long habit
- D) To suggest that Wen regrets going into the bakery

Think it through. Labels: habit (a year of walking past), trigger (the baker's nod), pause, decision (he opens the door). Predict: "to show Wen finally going in after a small encouragement."

Answer: C.

Why the others fail. A repeats Wen's excuse, but the text never explains the price. B is too narrow, because the loaves are a single detail. D goes beyond the text, which ends at the moment of decision.

Example 3: An underlined sentence that illustrates

Small changes to a building can change how people move through it. <u>When a hypothetical office building painted its stairwell bright yellow and posted a sign beside the elevators pointing to it, the number of employees taking the stairs doubled within a month.</u> Building managers value changes like these because they cost little and can be tested quickly before an owner commits to a costly renovation.

Which choice best describes the function of the underlined portion in the text as a whole?

- A) It introduces the main claim of the text.
- B) It offers a specific example supporting the claim in the first sentence.
- C) It raises an objection to the claim in the first sentence.
- D) It explains why building managers prefer large renovation projects.

Think it through. The first sentence is a general claim. The underlined sentence names one building and one result, which is evidence for that claim. Predict: "an example that supports the opening claim."

Answer: B.

Why the others fail. A fails because the claim comes first and the underlined sentence supports it. C fails because the result agrees with the claim. D reverses the last sentence, which says managers value small, testable changes.

Example 4: An underlined sentence that marks a change

For decades, visitors to a hypothetical history museum entered through a grand hall lined with portraits of the museum's founders. Last year, the staff moved those portraits to a side gallery and filled the hall with objects donated by local families. The director says the change was meant to show that the museum belongs to the whole community.

- A) It describes a change that the rest of the text explains.
- B) It provides evidence that visitors disliked the original hall.
- C) It summarizes the history of the museum's founders.
- D) It presents the director's argument against the change.

Think it through. Labels: old arrangement, the change (underlined), the reason. The underlined sentence is the hinge between the first and last sentences. Predict: "it reports the change that the director then explains."

Answer: A.

Why the others fail. B invents visitor reactions that the text never mentions. C fails because the founders appear only as subjects of portraits. D reverses the director's view, since the director supports the change.

Example 5: Overall structure of a poem

Poems are printed with their line breaks. Here, a slash (/) marks each break.

Each morning I count the cracks along the curb, / the same gray square, the same bent sign, the same / brown hedge that no one trims. I keep my eyes down. / Today a single poppy, red as a shout, / has pushed up through the gravel by the stop, / and I stand there, late, and look at everything.

Which choice best describes the overall structure of the text?

- A) The speaker describes a routine, notices something unexpected, and responds with new attention.
- B) The speaker lists complaints about a neighborhood and then proposes solutions.
- C) The speaker recalls a childhood walk and compares it with the present.
- D) The speaker describes a flower and then explains how it came to grow there.

Think it through. Look for the turn. Poems often pivot on a word such as "but," "yet" or "today," or on a shift in tense or mood. Here the first three lines are routine and downcast ("the same," "I keep my eyes down"). "Today" introduces the poppy, and the speaker ends by looking "at everything." Labels: routine, surprise, response.

Answer: A.

Why the others fail. B treats the details as complaints, but the speaker proposes nothing. C invents a childhood memory. D gets the order wrong, and the poem never explains how the poppy grew.

Example 6: Harder-module example: challenge or qualify?

Critics have long explained the loose brushwork and blurred edges in the late landscapes of a fictional painter, Ilse Varga, as the result of failing eyesight. Her letters from those years do mention trouble reading small print. Yet the same letters describe her scraping and repainting single passages dozens of times to soften them, and several late works keep the crisp detail of her early style in their foreground figures. The blur, it seems, was at least partly a choice.

Which choice best describes the function of the underlined portion in the text as a whole?

- A) It presents evidence that the text goes on to reject entirely.
- B) It concedes a point in favor of the critics' view before the text argues that their explanation is incomplete.
- C) It states the main claim that the rest of the text supports.
- D) It explains why the painter stopped painting landscapes.

Think it through. Labels: common view (eyesight), support for that view (underlined), counterevidence ("Yet"), conclusion. The word "do" in "do mention" grants the critics a real point, and "at least partly" in the last sentence shows that the author is limiting the critics' explanation, not throwing it out. The underlined sentence is a concession.

Answer: B.

Why the others fail. A is too strong: "entirely" contradicts "at least partly." C mislocates the main claim, which is the final sentence. D invents an event; nothing says she stopped painting landscapes.

TRAP In harder questions, two choices often differ only in strength, such as "refutes" versus "qualifies." Find the qualifier in the text (partly, some, may, often) and let it decide.

Common traps

Trap	How to spot it
Topic, not purpose	The choice names what the text is about but not what it does.
Wrong verb	"Argue" for a neutral report, or "criticize" when the author only describes.
Too narrow	For main purpose, the choice fits one sentence or detail, not the whole text.
Wrong neighbor	For underlined function, the choice describes what a different sentence does.
Half right	For overall structure, the first part matches but a later part does not, or the order is wrong.
Too strong	The choice says "proves," "rejects" or "always" when the text only suggests or qualifies.

Harder-module variations

Harder questions often use denser text, but the labels still work.

- **Late-arriving purpose.** The first sentence may state a view that the author goes on to question, often after a concession ("Admittedly," "To be sure," "does have"). Do not label the purpose until you have read the last sentence.

- **Partial underlines.** When only a clause or phrase is underlined, ask what that piece adds to its own sentence as well as to the whole text.
- **Literary purposes.** For fiction and poems, choices often use verbs such as "convey," "portray" or "depict," as in "to portray a change." Find the turn, then name what changes and how.

Check yourself

1. What is the difference between a text's main idea and its main purpose?
2. What question should you ask about an underlined sentence?
3. Why should you check the verb in each choice before anything else?
4. What does it mean for a sentence to qualify a claim?
5. In a poem, what kinds of words or shifts often mark the turn?
6. **Answers:** (1) The main idea is what the text says; the main purpose is what the text does, such as describe, argue or challenge. (2) "What would the text lose without this sentence?" (3) A wrong verb, such as "argue" for a neutral text, eliminates a choice no matter how accurate the rest of it is. (4) It accepts part of the claim and limits the rest. (5) Words such as but, yet, still or today, and shifts in tense, mood or time.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/
text-structure-and-purpose

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C0601

> Gardeners who grow tomatoes indoors sometimes find that healthy, flowering plants produce little fruit. Outdoors, wind and visiting bees shake tomato flowers, helping pollen fall where it is needed. Indoors, neither is present. For a hypothetical class project, a student gently tapped the flowering stems of half of her indoor tomato plants each morning and left the other half alone. By the end of the summer, the tapped plants had produced about twice as many tomatoes.

Which choice best states the main purpose of the text?

A To argue that home gardeners should stop growing tomato plants indoors

B To explain why bees are especially drawn to the flowers of tomato plants

C To describe a test of a simple way to increase indoor tomato harvests

D To compare the flavor of tomatoes grown indoors with those grown outdoors

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2 RW-C0602

> Theo had always thought his aunt's garden was a mess: beans climbing the fence, squash vines spilling across the path, marigolds tucked between rows of lettuce. On his first morning helping her, he offered to pull out the marigolds so that the rows would look neater. His aunt laughed and explained that every plant in the garden had been placed exactly where it was for a reason. Theo looked again at the crowded beds and began to wonder what else he had misjudged.

Which choice best describes the function of the underlined portion in the text as a whole?

A It reveals that the garden's disorder is intentional, prompting Theo to rethink his view.

B It shows that Theo's aunt is annoyed by his offer to help her clear the flower beds.

C It lists the vegetables and flowers that grow in different parts of the aunt's garden.

D It explains why Theo decides to stop helping his aunt with the work in her garden.

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3 RW-C0603

> Many crosswalks are painted with plain white stripes. In the fictional town of Alder Falls, a council member proposed repainting the downtown crosswalks with bright designs created by students at the local high school. Supporters said that colorful crosswalks would be easier for drivers to notice. Before making a decision, the council agreed to repaint two crosswalks as a trial and to count how often drivers stopped for pedestrians at each one over three months.

Which choice best states the main purpose of the text?

A To criticize drivers in Alder Falls for ignoring pedestrians in crosswalks

B To explain how high school students create designs for public art projects

C To argue that colorful crosswalks are safer than crosswalks with plain stripes

D To describe a proposal and the town's plan to test it before deciding

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4 RW-C0604

> The following poem describes a city park in winter.
>
> The fountain sleeps beneath a lid of ice;
> the benches hold a single row of snow.
> No children climb the oak, no vendors call,
> and yet the sparrows know the park is theirs:
> they hop along the railings, bright and loud,
> as busy as a summer Sunday crowd.

Which choice best states the main purpose of the text?

A To explain how city workers protect fountains from freezing weather

B To portray a winter park that remains lively because of the sparrows in it

C To recall the games that children play in the park during the summer

D To argue that parks should have more vendors during the winter months

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5 RW-C0605

> Dodder, a group of leafless, parasitic, twining plants, lives in an unusual way. A dodder seedling sprouts with a small root that anchors it in the soil, then sends up a slender stem that grows in a spiral until it touches another plant. The stem wraps around this host and pushes rootlike organs called haustoria into it, drawing out water and nutrients. After the stem establishes a connection to the host, the dodder's own root rots away. From then on, the dodder depends on the host plant for its water and mineral nutrients.

Which choice best describes the function of the underlined portion in the text as a whole?

A It explains how a dodder seedling finds a host plant to attach itself to.

B It describes a change that leaves the dodder dependent on its host for water and mineral nutrients.

C It identifies a weakness that allows farmers to control dodder easily.

D It contrasts the roots of dodder with the roots of other parasitic plants.

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6 RW-C0606

> Jacob Lawrence's *Migration Series* is a set of sixty small paintings, each paired with a caption, that tells the story of African Americans who left the rural South for Northern cities during and after World War I. Lawrence, himself the son of migrants, researched the subject and wrote the narrative before he began painting, and he then worked on all of the panels at once in tempera. After the series was shown at a New York gallery in 1941, the Phillips Collection and the Museum of Modern Art bought it jointly and divided the panels between them. Lawrence thus became the first African American artist represented in the Museum of Modern Art's collection.

Which choice best describes the overall structure of the text?

A It presents a claim about Lawrence's style, then supports that claim with evidence drawn from several individual panels.

B It compares how two museums have exhibited Lawrence's paintings, then judges which museum's approach was more successful.

C It explains why a historical migration took place, then describes how several different artists responded to it.

D It introduces a series of paintings, describes how the artist made it, and then tells how two museums acquired it.

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7 RW-C0607

> Curitiba, a city in southern Brazil, is known as a model of environmentally minded urban planning. Its population had been growing rapidly for decades when, in the early 1970s, the city began an extensive renewal. Curitiba created new lakes and flood-control systems, built large parks, introduced recycling programs and zoning rules, and set up specialized bus services. Much of this work was directed by Jaime Lerner, an architect and engineer who served three terms as the city's mayor.

Which choice best states the main purpose of the text?

A To explain the measures behind a city's reputation for environmentally minded planning

B To argue that other cities should copy the specialized bus services that Curitiba created

C To describe the career of an architect who later became the mayor of a city

D To explain why Curitiba's population grew so rapidly for several decades

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8 RW-C0608

> According to a hypothetical survey at a city art museum, most visitors spend less than a minute in front of any single painting. Hoping to change that habit, the museum's education staff placed a bench in front of one large landscape and posted a card inviting visitors to sit for five minutes and list everything they noticed. Over the following month, visitors who sat on the bench stayed in front of the landscape for an average of seven minutes. The staff now plans to add benches to three more galleries.

Which choice best describes the function of the underlined portion in the text as a whole?

A It describes the subject of the landscape painting that the staff chose for the experiment.

B It explains why most museum visitors spend so little time looking at each painting.

C It reports the result of the experiment, which helps explain the plan in the final sentence.

D It suggests that the survey described in the first sentence produced inaccurate results.

---

9 RW-C0609

> For her first week as a night guard at the museum, Odile walked her rounds quickly, eyes on the floor, counting doors. The galleries felt enormous in the dark, and the paintings were only shapes she had to pass. On the eighth night she paused, without quite meaning to, before a small painting of a woman reading beside a window. Her flashlight caught the yellow of the curtain, then the open page, then the woman's half-smile. Odile stood there until her radio crackled. When she moved on, she found herself looking up.

Which choice best states the main purpose of the text?

A To explain the routine duties of a guard who works the night shift at a museum

B To describe in careful detail a painting of a woman reading beside a window

C To suggest that Odile dislikes her new job and intends to leave it soon

D To depict a moment when a character begins to see her surroundings differently

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10 RW-C0610

> Many gardeners assume that sunflower heads turn to follow the sun throughout their lives. For a hypothetical science project, a student tracked twenty sunflowers in a school garden from early growth through full bloom. While the plants were young, their heads did swing from east to west each day and back again overnight. Once the flowers fully opened, however, most of the heads stopped moving and stayed facing east for the rest of the season.

Which choice best describes the overall structure of the text?

A It presents a common belief, describes a test of it, and reports results that only partly confirm it.

B It describes a disagreement between two groups of gardeners, then explains why one group's view proved correct.

C It explains how sunflowers move toward light, then gives instructions for planting them in a school garden.

D It reports a surprising discovery about sunflowers, then traces the history of how scientists first made it.

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11 RW-C0611

> When the fictional city of Marrow Bay converted a downtown parking lot into a small plaza with trees, benches and a weekend market, several business owners objected. Losing the parking spaces, they warned, would drive away customers who arrived by car. <u>During the plaza's first year, sales at shops facing it rose by an average of 12 percent, according to a study commissioned by the city.</u> Planners from neighboring towns have since visited the plaza to consider similar projects.

Which choice best describes the function of the underlined portion in the text as a whole?

A It explains why some business owners opposed converting the parking lot.

B It describes the trees, benches and market that the city included in the new plaza.

C It offers evidence that runs counter to a concern described earlier in the text.

D It indicates that neighboring towns have already built plazas of their own.

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12 RW-C0612

> Indoor farms grow leafy greens under lamps whose color can be finely adjusted, and growers have long debated the best mix of red and blue light. In a hypothetical trial, a team of horticulturists grew lettuce under five lamp settings ranging from 5 percent to 30 percent blue light. Plants grown with 20 percent blue light were the most compact and had the thickest leaves, traits that indoor farms value because such plants pack and ship well. The team cautions, though, that it tested a single lettuce variety over one growing cycle, and that other crops may respond differently to the same mix. Even so, the team plans to recommend the 20 percent setting to the lettuce growers who funded the trial.

Which choice best describes the function of the underlined portion in the text as a whole?

A It explains why the team chose to study lettuce rather than another crop.

B It acknowledges limits on how broadly the trial's results can be applied.

C It contradicts the result reported in the sentence that comes before it.

D It suggests that the growers who funded the trial will reject the team's advice.

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13 RW-C0613

> A curator at a hypothetical museum argues that unfinished paintings are too often treated as curiosities, prized mainly for what they reveal about an artist's working methods. In her view, the bare patches of canvas and loose underdrawings in such works can be as expressive as any finished passage. In several portraits she has studied, an abrupt shift from a carefully modeled face to a sketchy, barely begun background seems to direct a viewer's attention as deliberately as a spotlight would. She does not claim that every unfinished painting was left that way on purpose, only that the effects of incompleteness deserve to be judged on their own terms.

Which choice best states the main purpose of the text?

A To present a curator's view that unfinished paintings have expressive value beyond what they reveal about method

B To argue that artists often left paintings unfinished on purpose in order to direct viewers' attention

C To explain the techniques that painters used to create underdrawings and the sketched backgrounds of portraits

D To criticize museums for choosing to display unfinished paintings in the same galleries as finished ones

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14 RW-C0614

> The following poem describes a person repainting an old kitchen.
>
> I scraped the yellow off the kitchen wall
> to make the room look new, the way the listing
> had promised: bright and modern, clean and plain.
> Beneath the yellow, green; beneath the green,
> a column drawn in pencil, rows of dates,
> and children's names, each mark a little higher,
> nineteen fifty-one, fifty-three, fifty-four.
> Someone had stood here, heels against the wall,
> while someone else leaned in to draw the line.
> I set the scraper down. I bought a frame.

Which choice best describes the overall structure of the text?

A The speaker describes a room's appearance, then compares it with the kitchens in other houses.

B The speaker lists reasons for a decision, then questions whether the decision was wise.

C The speaker recalls events from childhood, then explains how they shaped an adult career.

D The speaker begins a task with one goal, finds something unexpected, and changes course.

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15 RW-C0615

> Many museums hang paintings in chronological order, guiding visitors from older works to newer ones as though art history were a single road. A hypothetical museum recently abandoned that arrangement in its main galleries, grouping works by theme instead, so that a seventeenth-century harbor scene might hang beside a contemporary painting of a container port. Chronological hanging does have real strengths: it makes the influence of one generation of artists on the next easy to trace. But the museum's director argues that thematic grouping reveals conversations across centuries that a timeline keeps hidden, and visitor surveys suggest that people linger longer in the reorganized galleries.

Which choice best describes the function of the underlined portion in the text as a whole?

A It explains why the museum decided to abandon chronological hanging in its galleries.

B It states the central claim that the rest of the text is meant to support with evidence.

C It concedes a merit of the older arrangement before the text turns to the case for the new one.

D It suggests that the museum's director now regrets the change to thematic grouping of works.

Practice Set Explanations

1. Correct answer: C RW-C0601

Text Structure and Purpose · Craft and Structure · Easier

C *To describe a test of a simple way to increase indoor tomato harvests*

The text identifies a problem (indoor tomato plants that flower but produce little fruit), offers a likely cause (no wind or bees to shake the flowers) and then describes the student's test: tapping half the plants each morning, which doubled the harvest. Its main purpose is to describe that test and its result.

A The text never tells gardeners to stop growing tomatoes indoors; it describes a way to make indoor plants more productive.

B Bees are mentioned only as one source of shaking outdoors; the text never explains what draws them to the flowers.

D Flavor is never discussed; the only outcome measured is the number of tomatoes the plants produced.

Review: Chapter 6, Text Structure and Purpose (page 82)

2. Correct answer: A RW-C0602

Text Structure and Purpose · Craft and Structure · Easier

A *It reveals that the garden's disorder is intentional, prompting Theo to rethink his view.*

Before the underlined sentence, Theo sees the garden as "a mess" and wants to tidy it. His aunt's explanation that every plant was placed "for a reason" shows that the disorder is intentional, and the next sentence shows Theo reconsidering: he begins "to wonder what else he had misjudged." The sentence prompts his change in view.

B The aunt laughs and explains her reasoning calmly; nothing in her response suggests that she is annoyed.

C The plants are listed in the first sentence; the underlined sentence explains why they are arranged as they are.

D Theo does not stop helping; the final sentence shows him rethinking his opinion of the garden instead.

Review: Chapter 6, Text Structure and Purpose (page 82)

3. Correct answer: D RW-C0603

Text Structure and Purpose · Craft and Structure · Easier

D *To describe a proposal and the town's plan to test it before deciding*

The text presents a proposal (colorful crosswalks), the supporters' reasoning (they would be easier to notice) and the council's decision to run a three-month trial "before making a decision." It reports these steps without judging them, so its main purpose is to describe the proposal and the plan to test it.

A The text never says that drivers in Alder Falls ignore pedestrians; the trial is meant to measure drivers' behavior, not condemn it.

B Students are mentioned as the designers, but the text never explains how they create their designs.

C The safety claim belongs to supporters, and the council is still testing it, so the text does not argue that it is true.

Review: Chapter 6, Text Structure and Purpose (page 82)

4. Correct answer: B RW-C0604

Text Structure and Purpose · Craft and Structure · Easier

B *To portray a winter park that remains lively because of the sparrows in it*

The first lines list what the park lacks in winter: running water, people on the benches, children and vendors. The turn "and yet" introduces the sparrows, which "hop along the railings, bright and loud," as busy as a summer crowd. The poem's purpose is to show that the emptied park is still full of life.

A The ice on the fountain is an image of winter stillness; the poem never describes how workers protect fountains.

C Children appear only as absent ("No children climb the oak"); the poem never describes their summer games.

D Vendors are mentioned only to show how quiet the park is; the poem makes no argument about what parks should have.

Review: Chapter 6, Text Structure and Purpose (page 82)

5. Correct answer: B RW-C0605

Text Structure and Purpose · Craft and Structure · Medium

B *It describes a change that leaves the dodder dependent on its host for water and mineral nutrients.*

Before the underlined sentence, the dodder has both its own anchoring root and the haustoria that tap the host. The underlined sentence describes the loss of that root, and the final sentence states the result: the dodder depends on the host for water and mineral nutrients. The sentence marks the loss of its direct connection to the soil.

A How the seedling finds a host is described earlier, in the sentence about the stem growing in a spiral until it touches another plant.

C Farming and control methods are never mentioned, so the text gives no basis for calling the root's loss a weakness farmers can use.

D No other parasitic plants are discussed, so the sentence cannot be contrasting dodder with them.

Review: Chapter 6, Text Structure and Purpose (page 82)

6. Correct answer: D RW-C0606

Text Structure and Purpose · Craft and Structure · Medium

D *It introduces a series of paintings, describes how the artist made it, and then tells how two museums acquired it.*

The first sentence identifies the series and its subject. The second describes Lawrence's process: researching, writing the narrative and painting all the panels at once. The last two sentences explain what happened after the 1941 exhibition, when two museums bought the series together and divided it. Choice D names these three parts in order.

A The text mentions tempera but makes no claim about Lawrence's style, and it never discusses individual panels as evidence.

B The museums bought and divided the series; the text never describes how either museum displayed it or judges their approaches.

C The text names the migration as the series' subject but never explains its causes, and Lawrence is the only artist discussed.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: A RW-C0607

Text Structure and Purpose · Craft and Structure · Medium

A *To explain the measures behind a city's reputation for environmentally minded planning*

The first sentence names Curitiba's reputation as a model of environmentally minded planning, and the rest of the text explains what earned it: the lakes, flood-control systems, parks, recycling programs, zoning rules and bus services of the city's renewal. Jaime Lerner is mentioned as the person who directed that work. The main purpose is to explain the measures behind the reputation.

B Bus services are one item in a list of changes, and the text never recommends that other cities copy them.

C Lerner appears only in the final sentence, as the director of the renewal; his career is not the focus of the text.

D Rapid growth is mentioned only as background to the renewal; the text never explains what caused it.

Review: Chapter 6, Text Structure and Purpose (page 82)

8. Correct answer: C RW-C0608

Text Structure and Purpose · Craft and Structure · Medium

C *It reports the result of the experiment, which helps explain the plan in the final sentence.*

The text moves from a problem (brief viewing times) to an experiment (the bench and the card) to a result and a decision. The underlined sentence gives the result: visitors who sat stayed about seven minutes. That success explains why "the staff now plans to add benches to three more galleries."

A The text never describes what the landscape shows; the underlined sentence reports how long visitors stayed in front of it.

B The text gives no reason for the short viewing times, and the underlined sentence concerns the experiment's result instead.

D The underlined sentence describes visitors who changed their behavior, which does not call the earlier survey's findings into question.

Review: Chapter 6, Text Structure and Purpose (page 82)

9. Correct answer: D RW-C0609

Text Structure and Purpose · Craft and Structure · Medium

D *To depict a moment when a character begins to see her surroundings differently*

At first Odile hurries through the galleries with her "eyes on the floor," treating the paintings as "only shapes she had to pass." One painting stops her, and afterward "she found herself looking up." The text's purpose is to depict this shift from ignoring the art around her to noticing it.

A Her rounds are background; the text focuses on the night she stops to look, not on the duties of her job.

B The painting's details matter because of Odile's reaction to them, so describing the painting is too narrow to be the main purpose.

C Nothing suggests Odile plans to leave; the ending shows her becoming more engaged with her surroundings, not less.

Review: Chapter 6, Text Structure and Purpose (page 82)

10. Correct answer: A RW-C0610

Text Structure and Purpose · Craft and Structure · Medium

A *It presents a common belief, describes a test of it, and reports results that only partly confirm it.*

The text opens with a belief (sunflower heads follow the sun throughout their lives), then describes the student's project tracking twenty plants. The results confirm the belief for young plants but not for mature ones, most of which "stayed facing east." Choice A captures all three parts, including the partly confirming result.

B Only one belief is presented, and no second group of gardeners appears, so the text does not describe a disagreement.

C The text never gives planting instructions; the school garden is simply where the observations took place.

D The text says nothing about scientists or the history of a discovery; it reports one student's observations.

Review: Chapter 6, Text Structure and Purpose (page 82)

11. Correct answer: C RW-C0611

Text Structure and Purpose · Craft and Structure · Medium

C *It offers evidence that runs counter to a concern described earlier in the text.*

The business owners warned that losing parking "would drive away customers." The underlined sentence reports that sales at nearby shops instead rose by an average of 12 percent in the plaza's first year. It therefore supplies evidence against the owners' concern, which also helps explain why other towns are now interested.

A The owners' reason is given in the sentence before the underlined portion; the underlined sentence reports later sales results.

B The plaza's trees, benches and market are described in the first sentence, not in the underlined one.

D The final sentence says planners visited to consider similar projects, not that their towns have built plazas, and the underlined sentence does not mention other towns.

Review: Chapter 6, Text Structure and Purpose (page 82)

12. Correct answer: B RW-C0612

Text Structure and Purpose · Craft and Structure · Harder

B *It acknowledges limits on how broadly the trial's results can be applied.*

The underlined sentence follows the trial's main result and notes that only "a single lettuce variety" was tested "over one growing cycle," so "other crops may respond differently." It does not dispute the result; it limits how far the result can be extended. The final sentence fits this reading: the team still recommends the setting, but only to lettuce growers.

A The sentence notes that only lettuce was tested but gives no reason for that choice; it addresses what the results can and cannot show.

C The sentence accepts the 20 percent result and restricts its scope; a contradiction would claim that the result was wrong.

D Nothing suggests the funders will reject the advice; the final sentence says the team plans to recommend the setting to them.

Review: Chapter 6, Text Structure and Purpose (page 82)

13. Correct answer: A RW-C0613

Text Structure and Purpose · Craft and Structure · Harder

A *To present a curator's view that unfinished paintings have expressive value beyond what they reveal about method*

The text presents the curator's argument that unfinished paintings are undervalued when seen only as evidence of method, and it supports her point with portraits whose sketchy backgrounds direct attention. The final sentence limits her claim to judging "the effects of incompleteness." Choice A states this purpose without overstating it.

B The curator discusses the expressive effects of unfinished paintings, not how often artists deliberately left them unfinished. This choice adds an unsupported claim about artistic intention.

C Underdrawings are mentioned as features worth appreciating, but the text never explains how painters made them.

D The curator objects to how unfinished paintings are valued, not to their being displayed, and she says nothing about which galleries they should hang in.

Review: Chapter 6, Text Structure and Purpose (page 82)

14. Correct answer: D RW-C0614

Text Structure and Purpose · Craft and Structure · Harder

D *The speaker begins a task with one goal, finds something unexpected, and changes course.*

The speaker starts scraping to make the kitchen "look new." Under the layers of paint, the speaker finds a pencil column of dates and children's names, imagines the people who made it, and then sets "the scraper down" and buys "a frame," choosing to preserve the marks. Choice D follows this sequence: goal, discovery, change of course.

A Only one kitchen appears in the poem; it never compares the room with kitchens in other houses.

B The speaker does not list reasons or doubt the final choice; the poem ends with two short, decisive actions.

C The names and dates belong to children from the 1950s, not to the speaker's own childhood, and no career is mentioned.

Review: Chapter 6, Text Structure and Purpose (page 82)

15. Correct answer: C RW-C0615

Text Structure and Purpose · Craft and Structure · Harder

C *It concedes a merit of the older arrangement before the text turns to the case for the new one.*

The underlined sentence praises the arrangement the museum gave up: chronological hanging "does have real strengths." The next sentence begins with "But" and presents the director's argument and the survey evidence for thematic grouping. The underlined sentence therefore grants a point to the other side before the text makes its case.

A The sentence names an advantage of chronological hanging, which is a reason to keep it, not a reason the museum abandoned it.

B The text's emphasis falls on the case for thematic grouping that follows "But"; the underlined sentence is a concession, not the central claim.

D The director argues for the thematic galleries, and the surveys support the change, so nothing suggests regret.

Review: Chapter 6, Text Structure and Purpose (page 82)

CHAPTER 7

Cross-Text Connections

Cross-Text Connections questions give you two short texts on the same subject and ask how their authors relate. One author may agree with the other, push back, accept part of the other's claim, or explain the same evidence in a different way. You never need outside knowledge. You need a clear, one-line summary of each author's position before you look at the choices, because the wrong answers are built to sound right about one text at a time.

What the question looks like

Cross-Text Connections belongs to the Craft and Structure domain, and its questions usually follow the Text Structure and Purpose questions near the start of each module. Together, the two texts fall within the usual 25 to 150 words, so each is only a few sentences long.

The question takes one of three forms:

- "Based on the texts, how would the author of Text 2 most likely respond to the ... in Text 1?"
- "Based on the texts, both authors would most likely agree with which statement?"
- "Based on the texts, how would the author of Text 1 most likely respond to ... in Text 2?"

In "respond" questions, the choices usually begin "By arguing that...," "By agreeing that...," "By pointing out that..." or similar. In "agree" questions, the choices are statements.

How to recognize it

Look for two passages labeled Text 1 and Text 2 and a question that begins "Based on the texts." A single passage that presents two views ("Some critics argue... Others reply...") is not a cross-text question. It is usually a Text Structure and Purpose question (Chapter 6) or a Central Ideas and Details question (Chapter 8).

The method: map, predict, match

1. **Read the question first.** Note who is responding and to what. In "How would the author of Text 2 respond to the conclusion in Text 1?", Text 2's author is speaking, and the target is Text 1's conclusion, not Text 1 in general.
2. **Map Text 1 in one line.** Write or think a short summary: the claim plus its basis. Example: "Survey shows X, so the authors conclude Y."
3. **Map Text 2 the same way.** Then add its stance toward the target: agree, qualify, reject or offer an alternative explanation.
4. **Predict the response in your own words.** Example: "Text 2 would say the result might come from Z, not Y."
5. **Match against both maps.** The right choice has the right speaker, the right target, and the speaker's true stance at the right strength.

Four ways one author can respond to another

Response type	What the responding author does	Signals in the text	How the correct choice often sounds
Agree or extend	Accepts the claim and may add support	similarly, likewise, confirms, also	"By agreeing that…"
Qualify	Accepts part of the claim but limits it	may, only, in some cases, but, though	"By noting that the claim holds only when…"
Reject	Denies the claim, often with new evidence	however, fails to, no evidence, did not	"By pointing to evidence that…"
Alternative explanation	Accepts the observation but offers a different cause	another possibility, may simply, could reflect	"By suggesting that X, rather than Y, explains…"

TRAP The most common wrong answer is true of one text only. A choice can summarize Text 2 perfectly and still be wrong if it does not respond to the target in Text 1, or if Text 1's author would reject it in an "agree" question.

For "both authors would agree" questions, test every choice twice: would the author of Text 1 accept it, and would the author of Text 2? Keep it only if both answers are yes. For "Text 1 responds to Text 2" questions, remember that Text 1 was not written as a reply. Apply Text 1's reasoning to Text 2's claim, as if the author of Text 1 had just read it.

Worked examples

Example 1: Text 2 rejects with new evidence

Text 1: *Some city officials argue that brighter streetlights make residents feel safer and therefore lead more people to walk at night. In the fictional city of Orland, the council recently approved brighter lights for every residential street on that basis.*

Text 2: *A hypothetical study in a different city counted evening pedestrians on streets that received brighter lights and on similar streets that did not. A year later, the number of walkers had risen by the same amount on both kinds of streets. In that city, at least, the new lights did not appear to explain the increase.*

Based on the texts, how would the author of Text 2 most likely respond to the officials' argument in Text 1?

- A) By agreeing that brighter lights lead more people to walk at night
- B) By pointing to evidence that brighter lights may not increase evening walking
- C) By arguing that streetlights should be removed from residential streets
- D) By claiming that residents of Orland feel unsafe walking at night

Map. Text 1: brighter lights cause more night walking. Text 2: in one city, walking rose equally with and without new lights. Stance: reject, carefully ("In that city, at least").

Answer: B. The word "may" matches Text 2's cautious wording.

Why the others fail. A reverses Text 2's finding. C recommends something neither text proposes. D is off target, because Text 2 never discusses Orland's residents.

Example 2: Text 2 offers an alternative explanation

Text 1: *After a hypothetical city painted large murals on a dozen downtown buildings, shops on those blocks reported more customers than the year before. Local officials credit the murals with drawing new visitors downtown.*

Text 2: *The mural project began the same spring that the city reopened a downtown train station that had been closed for two years of repairs. Riders leaving the station pass the mural blocks first. Any increase in foot traffic on those blocks could owe as much to the trains as to the art.*

Based on the texts, how would the author of Text 2 most likely respond to the officials' view in Text 1?

- A) By suggesting that the reopened station may have contributed to the increase
- B) By agreeing that the murals drew many new visitors downtown
- C) By arguing that the city should paint murals near every train station
- D) By claiming that shops on the mural blocks did not gain customers

Map. Text 1: more customers, caused by murals. Text 2: more customers, with the reopened station as another possible cause. Text 2 accepts the observation and questions how much of the increase should be credited to the murals.

Answer: A.

Why the others fail. B ignores Text 2's doubt. C adds a recommendation that neither text makes. D is the classic over-reach: Text 2 accepts the increase and questions only its cause.

Example 3: What both authors would agree with

Text 1: *Allowing visitors to photograph paintings has real benefits. A photo lets a visitor study a detail again at home, and it lets people share art with friends and family who may never travel to the museum.*

Text 2: *Photography in galleries has costs. A flash may fade some fragile pigments over time, and crowds holding up phones can block other visitors' views. Even so, an outright ban would give up benefits that many visitors value. Rules against flash and limits on crowding can address the problems while keeping those benefits.*

Based on the texts, both authors would most likely agree with which statement?

- A) Photography in museums offers benefits that many visitors value.
- B) Flash photography can gradually fade some pigments.
- C) Museums should ban cameras from their galleries.
- D) A photograph can fully replace a visit to a museum.

Test each choice twice. A: Text 1 lists benefits, and Text 2 refers to "benefits that many visitors value." Both say yes. B: Text 2 says yes, but Text 1 never mentions flash. C: Text 2 rejects a ban, and Text 1 favors allowing photos. D: neither text says photos replace visits.

Answer: A. Choice B is the one-text trap: it is accurate about Text 2 but has no support in Text 1.

Example 4: Text 1 responds to Text 2

Text 1: *Libraries that drop late fees often see former borrowers return. In a hypothetical survey of one city's residents, many people who had stopped using the library said they stayed away because they owed small fines and felt embarrassed about them.*

Text 2: *The fictional town of Brisco plans to raise its library's late fees. Town officials argue that higher fines will push borrowers to return books on time and will raise money for new purchases.*

Based on the texts, how would the author of Text 1 most likely respond to the plan in Text 2?

- A) By warning that higher fines could lead some people to stop using the library
- B) By agreeing that higher fines will raise money for new books
- C) By arguing that the library should buy fewer new books
- D) By claiming that most borrowers already return books early

Map. Text 1: fines drive some people away, so dropping them brings borrowers back. Text 2: raise fines. Applied to Text 2's plan, Text 1's reasoning predicts lost borrowers.

Answer: A.

Why the others fail. B echoes Text 2's own argument, which Text 1 does not support. C and D bring in claims that appear in neither text.

Example 5: Harder-module example: qualify, not reject

Text 1: *A team of researchers studying a hypothetical city found that residents of neighborhoods with more parks reported exercising more often. The team concluded that building parks increases how much people exercise.*

Text 2: *The link between parks and exercise is real, and a nearby park plainly makes a walk or a game easier. But people who already enjoy exercise may choose to live near parks in the first place. Until studies follow the same residents before and after a new park opens, the size of the effect remains uncertain, even if its direction is not in doubt.*

Based on the texts, how would the author of Text 2 most likely respond to the team's conclusion in Text 1?

- A) By rejecting the idea that parks have any effect on how much residents exercise
- B) By accepting that parks likely encourage exercise while questioning how large the effect is
- C) By agreeing that the study shows exactly how much new parks increase exercise
- D) By arguing that cities should stop building parks until more studies are done

Map. Text 1: parks cause more exercise. Text 2: yes in direction ("the direction is not in doubt"), but the size is uncertain because active people may choose park-rich neighborhoods. Stance: qualify.

Answer: B.

Why the others fail. A is the strongest trap. It picks up Text 2's doubt but pushes it too far, contradicting "the direction is not in doubt." C ignores the doubt entirely. D invents a recommendation.

> **TIP** In harder pairs, look for the sentence where the responding author grants something ("is real," "does show," "plainly"). That sentence rules out every choice that says the author rejects the claim outright.

Common traps

Trap	How to spot it
One-text choice	It is true of one text, but the other author would not say it, or it ignores the target in the question.
Wrong direction	It describes Text 1 responding when the question asks about Text 2, or the reverse.
Too strong	It says "rejects entirely" or "proves" when the author only qualifies or suggests.
Denying the facts	It says the responding author disputes the evidence when the author accepts the evidence and questions its cause.
Off target	It responds to a different part of the other text than the one the question names.
Outside both texts	It adds a recommendation or claim that neither text makes.

Harder-module variations

- **Partial agreement.** The responding author accepts the evidence or the direction of an effect but limits the conclusion, so choices differ mainly in strength. Match the author's exact words (may, likely, uncertain, only).
- **No direct mention.** Text 2 may never name Text 1's study or claim. You must see that its general point applies.
- **Shared assumptions.** In "agree" questions, the correct choice may be something both authors take for granted rather than something either states outright.

Check yourself

1. What two things should you note in the question before reading the texts?
2. Name the four main ways one author can respond to another.
3. How do you test a choice in a "both authors would agree" question?
4. What is the difference between rejecting evidence and offering an alternative explanation?
5. Why is a choice that accurately summarizes Text 2 sometimes wrong?
6. **Answers:** (1) Who is responding, and the exact target of the response. (2) Agree or extend, qualify, reject, and offer an alternative explanation. (3) Ask whether each author would accept it; keep it only if both would. (4) Rejecting evidence denies the facts; an alternative explanation accepts the facts and offers a different cause. (5) It may not respond to the target in the other text, or the other author may not accept it.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/cross-text-connections

Practice Set

12 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C0701

> **Text 1**
> Many home gardeners believe that talking to houseplants helps them grow. In a hypothetical survey of 300 gardeners, those who said they talked to their plants every day were far more likely to describe their plants as thriving. Some even play recordings of conversation for their plants while they are away. For these gardeners, the survey confirms that plants respond to the sound of a human voice.
>
> **Text 2**
> A gardener who talks to houseplants every day is, by definition, spending time with them every day. That attention probably means noticing dry soil, yellowing leaves and insect pests sooner than other gardeners do. A survey that simply compares talkers with non-talkers cannot separate the effect of speaking from the effect of this extra care.

Based on the texts, how would the author of Text 2 most likely respond to the gardeners' conclusion in Text 1?

A By agreeing that the survey proves that plants respond to the sound of a human voice

B By suggesting that the gardeners' closer attention, rather than their talking, may explain the results

C By arguing that the gardeners in the survey were not honest about how often they spoke to their plants

D By claiming that houseplants grow best when gardeners give them as little attention as possible

---

2 RW-C0702

> **Text 1**
> Planners in the fictional city of Corvale added protected bike lanes to Harbor Street, separating cyclists from car traffic with low concrete curbs. Within a year, the number of people biking on the street had doubled. The planners credit the curbs: in public meetings, many residents had said they wanted to bike but felt unsafe riding beside moving cars.
>
> **Text 2**
> When residents of a hypothetical mid-sized city were asked why they rarely rode the bicycles they owned, the most common answer was fear of traffic. A painted line on the pavement does little to calm that fear. A lane physically separated from cars addresses it directly, which is why such lanes can attract people who would never ride in a painted one.

Based on the texts, both authors would most likely agree with which statement?

A Painted bike lanes are more effective than lanes separated from traffic by curbs.

B Most residents of cities have no real interest in riding bicycles.

C Bike lanes should be built only on streets that carry little car traffic.

D Separating bike lanes from cars can increase cycling by easing safety fears.

---

3 RW-C0703

> **Text 1**
> Some museum educators argue that audio guides distract visitors from the art. With a recorded voice in their ears, these educators say, visitors spend more time looking at labels and keypads than at the paintings themselves, and they move through the galleries at the pace of the recording rather than their own.
>
> **Text 2**
> At a hypothetical art museum, staff recorded how long visitors spent looking at paintings, comparing visitors who carried audio guides with those who did not. Guide users looked at each painting with a recorded stop for about twice as long as other visitors did. Many said the recordings pointed out details, such as a faint signature or a reflection in a window, that drew them to look more closely.

Based on the texts, how would the author of Text 2 most likely respond to the educators' argument in Text 1?

A By noting that, in one museum, audio guides were linked to visitors looking longer at paintings

B By agreeing that audio guides cause many visitors to hurry through the museum's galleries

C By arguing that museums should replace their wall labels with recorded descriptions

D By claiming that most museum visitors would rather look at screens than at paintings

---

4 RW-C0704

> **Text 1**
> Trees along city streets do more than make neighborhoods look pleasant. Their shade cools sidewalks and buildings on hot days, and their roots absorb rainwater that would otherwise rush into storm drains. In a hypothetical study of one neighborhood, blocks with full tree cover were several degrees cooler at midday in summer than nearby blocks without trees.
>
> **Text 2**
> The fictional town of Pell Ridge plans to remove the aging maples that line its main street so that the road can be widened by one lane. Town officials describe the trees as mainly decorative. They say that a row of new flower planters will keep the street just as attractive at a fraction of the cost of caring for the old trees.

Based on the texts, how would the author of Text 1 most likely respond to the town officials' view in Text 2?

A By agreeing that the maples are mainly decorative and can be replaced by flower planters

B By arguing that the road should be widened by more than one lane to handle traffic

C By pointing out that the officials overlook practical benefits that street trees provide

D By suggesting that flower planters cool a street more effectively than trees can

---

5 RW-C0705

> **Text 1**
> Planting strips of wildflowers along the edges of farm fields is often promoted as a way to help pollinating insects. In a hypothetical two-year study, a team of ecologists found that farms with wildflower strips hosted nearly twice as many bees as similar farms without them over both summers. The team concluded that the strips increase local bee populations.
>
> **Text 2**
> Finding more bees on farms with flower strips does not show that more bees exist in the area. The flowers may simply draw bees away from nearby meadows and gardens, concentrating them where the counting takes place. To learn whether the strips truly increase populations, researchers would need to track bee numbers across the surrounding landscape as well as on the farms.

Based on the texts, how would the author of Text 2 most likely respond to the team's conclusion in Text 1?

A By arguing that the findings could reflect bees moving to the farms, not more bees overall

B By agreeing that the study shows that wildflower strips increase local populations of bees

C By claiming that wildflower strips harm bees by drawing them close to farm machinery

D By suggesting that the team counted only one species of bee and overlooked the others

---

6 RW-C0706

> **Text 1**
> Before modern conservation science, restorers sometimes repainted damaged areas of old pictures freely, even adding details they imagined the artist had intended. Today many conservators follow a wiser principle of restraint: new paint should cover only areas where the original paint is missing, and it should be removable, so that future experts with better tools and knowledge can undo it.
>
> **Text 2**
> A conservator at a hypothetical regional museum faced a difficult choice when a large patch of sky flaked off a nineteenth-century landscape. She filled the gap with a pale blue that blends in from a few steps away but can be spotted up close. Just as sensibly, she used a paint that dissolves in a mild solvent, so the repair can be taken off later without affecting the original layers beneath it.

Based on the texts, both authors would most likely agree with which statement?

A Restorers should add the details that the original artist probably intended to include.

B Repairs to a painting are best when they can be removed without harming the original.

C Damaged paintings should be displayed without any repairs, so viewers see their true state.

D Modern conservators rarely face difficult choices when they treat damaged paintings.

---

7 RW-C0707

> **Text 1**
> Some planners argue that cities can reduce car traffic by eliminating minimum parking requirements, the rules that force new buildings to include a set number of parking spaces. Without those rules, they say, developers will build fewer spaces, parking will become scarcer and costlier, and more residents will choose to walk, bike or ride transit.
>
> **Text 2**
> Dropping parking minimums may work well in neighborhoods with frequent buses and trains. In a hypothetical study of one region, though, new apartments built without required parking in areas with little transit still ended up with nearly as many spaces as before, because developers expected renters to want them. Where residents have few alternatives to driving, the policy may change less than its supporters expect.

Based on the texts, how would the author of Text 2 most likely respond to the planners' argument in Text 1?

A By rejecting the idea that parking rules affect how many spaces developers build in any neighborhood

B By arguing that cities should require more parking spaces in new apartment buildings than they do now

C By agreeing that eliminating parking minimums will reduce car traffic in every kind of neighborhood

D By suggesting that the effect the planners predict may depend on whether residents can avoid driving

---

8 RW-C0708

> **Text 1**
> Museum wall labels should be brief, however much a curator has to say. In a hypothetical study at one gallery, visitors read nearly every word of labels shorter than 50 words but skipped most of the text in labels longer than 150 words. A long label, in other words, is often an unread label, and the information in it never reaches most visitors.
>
> **Text 2**
> A new exhibition at a fictional museum, which opens next month, pairs each of its forty paintings with a 300-word wall label covering the work's history, the artist's technique and the painting's later owners. The curator, who spent two years researching the collection, believes that visitors deserve the full story behind each work and that shorter labels oversimplify complicated paintings.

Based on the texts, how would the author of Text 1 most likely respond to the curator's approach in Text 2?

A By praising the curator for giving visitors much more information about each painting

B By arguing that the exhibition should include fewer than forty paintings to avoid crowding

C By cautioning that many visitors will likely skip labels that long, leaving the story unread

D By claiming that most visitors already know the history of the paintings on display

---

9 RW-C0709

> **Text 1**
> In a hypothetical experiment, pea seedlings were grown in pots of dry soil beside a small speaker that played a recording of running water. Over several days, most of the seedlings' roots bent toward the speaker, even though the soil there was just as dry as the soil elsewhere. The experimenters suggested that pea roots can sense the vibrations of flowing water and use them to locate moisture.
>
> **Text 2**
> Sound alone may not explain such a result. If the speaker's casing was slightly warmer than the surrounding soil, or if the soil nearest the speaker was packed more loosely, roots might have grown in that direction for reasons that have nothing to do with vibration. Repeating the test with a silent speaker in the same position would help rule out these possibilities.

Based on the texts, how would the author of Text 2 most likely respond to the experimenters' suggestion in Text 1?

A By agreeing that the experiment shows that pea roots can detect the sound of flowing water

B By noting that other factors could explain the roots' growth and proposing a way to test them

C By claiming that pea seedlings are unable to grow their roots in soil that is completely dry

D By arguing that the experiment should have been done with a plant other than the pea

---

10 RW-C0710

> **Text 1**
> Conservators at a hypothetical museum examined a portrait long attributed to the workshop of a celebrated painter. Infrared images revealed a confident, heavily revised underdrawing beneath the paint: the sitter's hands had been redrawn three times. Because workshop assistants usually copied a master's finished designs without alteration, the team argues, these revisions show that the master personally designed the portrait.
>
> **Text 2**
> Revisions in an underdrawing indicate that whoever made it was working out a composition rather than copying one, and that is strong evidence against treating a portrait as a routine workshop copy. Some workshops, however, trained senior assistants to invent new compositions in the master's manner. A revised underdrawing therefore narrows the list of possible designers without singling out the master.

Based on the texts, how would the author of Text 2 most likely respond to the team's conclusion in Text 1?

A By accepting that the revisions argue against a routine copy but denying that they identify the master

B By arguing that a senior assistant, rather than the master, most likely designed the portrait in question

C By agreeing that the revisions in the underdrawing prove that the master personally designed the portrait

D By rejecting the claim that the underdrawing beneath the portrait shows any real signs of revision

---

11 RW-C0711

> **Text 1**
> Seed banks store frozen seeds of thousands of crop varieties so that plants lost to disease, drought or disaster can be grown again. Supporters regard these vaults as an essential safeguard, yet even they acknowledge a limitation: a seed kept frozen for decades does not evolve alongside the pests and climate conditions that it may eventually face.
>
> **Text 2**
> In some farming communities, traditional crop varieties are preserved by planting them every season and saving seed from the strongest plants. Because these varieties keep growing in the field, they keep adapting to local conditions. Such living collections are vulnerable in ways a vault is not, however: a single flood or failed harvest can erase a variety grown in only a few small fields.

Based on the texts, both authors would most likely agree with which statement?

A Frozen seed banks are the most dependable of all the ways to preserve crop varieties.

B Traditional varieties grown in fields should replace the varieties stored in seed banks.

C Seed banks protect crop varieties against floods better than any other method does.

D Varieties kept only in frozen storage lack an advantage of varieties grown each season.

---

12 RW-C0712

> **Text 1**
> When the fictional city of Danbury Flats replaced a four-lane downtown road with two lanes, wider sidewalks and a center row of trees, traffic on the road fell by about a third within a year. City officials concluded that the redesign had persuaded many drivers to leave their cars at home and walk, bike or take the bus instead.
>
> **Text 2**
> Before crediting a road redesign with changing how people travel, planners should ask a simple question: where did the missing cars go? Traffic counts taken on a redesigned road reveal nothing about parallel streets a few blocks away. If drivers simply shifted to those routes, the total number of car trips may have barely changed, even as the redesigned road itself grew quieter.

Based on the texts, how would the author of Text 2 most likely respond to the officials' conclusion in Text 1?

A By concluding that the redesign had no effect at all on how residents of Danbury Flats travel

B By agreeing that the drop in traffic shows that residents chose walking and buses over driving

C By suggesting that the conclusion may be premature because drivers could have switched streets

D By claiming that traffic on the redesigned road did not really decline after the change

---

Practice Set Explanations

1. Correct answer: B RW-C0701

Cross-Text Connections · Craft and Structure · Easier

B *By suggesting that the gardeners' closer attention, rather than their talking, may explain the results*

Text 2 accepts that daily talkers report healthier plants but points to another difference between the groups: daily talkers also spend more time with their plants and likely notice problems sooner. Because the survey "cannot separate the effect of speaking from the effect of this extra care," the author would say that attention, not talking, may explain the results.

A Text 2 argues that the survey cannot isolate the effect of talking, so its author would not accept the gardeners' conclusion.

C Text 2 never questions whether the gardeners reported their habits truthfully; it questions what caused the difference.

D Text 2 suggests that extra attention helps plants, which is the opposite of claiming that plants thrive with little attention.

Review: Chapter 7, Cross-Text Connections (page 98)

2. Correct answer: D RW-C0702

Cross-Text Connections · Craft and Structure · Easier

D *Separating bike lanes from cars can increase cycling by easing safety fears.*

Text 1 credits Corvale's curbs for doubling cycling on Harbor Street, noting that residents had felt "unsafe riding beside moving cars." Text 2 says that separated lanes address the "fear of traffic" that keeps people off their bicycles. Both texts therefore support the idea that physical separation can increase cycling by reducing safety worries.

A Text 2 says painted lines do little to calm fear, and Text 1 credits curbs, so neither author would rank painted lanes higher.

B Both texts describe people who want to bike but feel unsafe, which contradicts the claim that most residents have no interest.

C Neither text proposes limiting bike lanes to quiet streets; both focus on separating cyclists from car traffic, not avoiding it.

Review: Chapter 7, Cross-Text Connections (page 98)

3. Correct answer: A RW-C0703

Cross-Text Connections · Craft and Structure · Easier

A *By noting that, in one museum, audio guides were linked to visitors looking longer at paintings*

The educators in Text 1 claim that audio guides pull attention away from paintings. Text 2 reports that, at one museum, guide users looked at paintings with recorded stops about twice as long as other visitors, and many said the recordings helped them notice details. The author of Text 2 would point to this evidence as a challenge to the educators' claim.

B Text 2's findings show guide users lingering longer, not hurrying, so its author would dispute this point rather than agree with it.

C Text 2 never discusses wall labels or proposes replacing them; it reports how long visitors looked at paintings.

D Text 2 suggests that the recordings drew visitors' eyes to details in the paintings, so this claim has no support.

Review: Chapter 7, Cross-Text Connections (page 98)

4. Correct answer: C RW-C0704

Cross-Text Connections · Craft and Structure · Easier

C *By pointing out that the officials overlook practical benefits that street trees provide*

The officials in Text 2 treat the maples as "mainly decorative" and replaceable by planters. Text 1 argues that street trees "do more than make neighborhoods look pleasant": their shade cools streets and their roots absorb rainwater. The author of Text 1 would therefore say that the officials overlook the trees' practical benefits.

A Text 1 opens by rejecting the idea that street trees are only decorative, so its author would not accept the officials' view.

B Text 1 says nothing about traffic or road width; its focus is on what trees contribute to a street.

D Text 1 credits trees with cooling streets and never discusses planters, so this claim has no support in Text 1.

Review: Chapter 7, Cross-Text Connections (page 98)

5. Correct answer: A RW-C0705

Cross-Text Connections · Craft and Structure · Medium

A *By arguing that the findings could reflect bees moving to the farms, not more bees overall*

Text 2 accepts that farms with strips had more bees but says the flowers "may simply draw bees away from nearby meadows and gardens." If so, the higher counts show bees relocating, not increasing. The author of Text 2 would therefore say that the team's conclusion goes beyond its evidence and that wider tracking is needed.

B Text 2 states that more bees on the farms "does not show that more bees exist in the area," so its author would reject this conclusion.

C Text 2 never suggests that the strips harm bees; it questions only whether they increase bee numbers.

D Text 2 does not mention bee species at all; its concern is where the counting took place.

Review: Chapter 7, Cross-Text Connections (page 98)

6. Correct answer: B RW-C0706

Cross-Text Connections · Craft and Structure · Medium

B *Repairs to a painting are best when they can be removed without harming the original.*

Text 1 endorses the "wiser principle" that new paint "should be removable" so that future experts can undo it. Text 2 presents a removable repair approvingly ("Just as sensibly"), noting that it can be "taken off later without affecting the original layers." Both authors therefore value repairs that can be reversed without harming the original painting.

A Text 1 describes inventing details as an older practice that modern restraint rejects, and Text 2's conservator only fills the area of loss.

C Both texts describe repairs being made; neither argues that damaged paintings should be left untreated.

D Text 2 describes a conservator who "faced a difficult choice," and neither text suggests that such choices are rare.

Review: Chapter 7, Cross-Text Connections (page 98)

7. Correct answer: D RW-C0707

Cross-Text Connections · Craft and Structure · Medium

D *By suggesting that the effect the planners predict may depend on whether residents can avoid driving*

Text 2 grants that dropping minimums "may work well" where transit is frequent, but reports that in low-transit areas developers built nearly as many spaces anyway. Its conclusion, that "where residents have few alternatives to driving, the policy may change less," limits the planners' argument to certain conditions rather than rejecting it.

A Text 2 accepts that the policy may work in neighborhoods with frequent transit, so its author does not reject the argument everywhere.

B Text 2 questions how much the policy changes, but it never recommends requiring more parking in new buildings.

C Text 2's study of low-transit areas directly challenges the idea that the policy works in every neighborhood.

Review: Chapter 7, Cross-Text Connections (page 98)

8. Correct answer: C RW-C0708

Cross-Text Connections · Craft and Structure · Medium

C *By cautioning that many visitors will likely skip labels that long, leaving the story unread*

Text 1 reports that visitors skipped most of the text in labels longer than 150 words and concludes that "a long label, in other words, is often an unread label." The curator in Text 2 uses 300-word labels. The author of Text 1 would therefore warn that the full story the curator wants to tell may not reach most visitors.

A Text 1 argues that long labels often go unread, so its author would doubt that 300-word labels actually inform visitors.

B Text 1 addresses the length of labels, not the number of paintings, so this response has no basis in Text 1.

D Text 1 says nothing about what visitors already know; its point concerns whether visitors read long labels.

Review: Chapter 7, Cross-Text Connections (page 98)

9. Correct answer: B RW-C0709

Cross-Text Connections · Craft and Structure · Medium

B *By noting that other factors could explain the roots' growth and proposing a way to test them*

Text 2 offers other possible causes for the roots' growth, such as a warmer speaker casing or looser soil near the speaker, and proposes repeating the test with "a silent speaker in the same position." Its author would say the vibration explanation is not yet established and that a further test is needed to rule out the alternatives.

A Text 2 opens by saying "Sound alone may not explain such a result," so its author would not accept the experiment as proof.

C Text 1 reports that the roots did grow in dry soil, and Text 2 never disputes that the roots grew.

D Text 2 never questions the choice of peas; its concern is whether conditions near the speaker explain the roots' growth.

Review: Chapter 7, Cross-Text Connections (page 98)

10. Correct answer: A RW-C0710

Cross-Text Connections · Craft and Structure · Harder

A *By accepting that the revisions argue against a routine copy but denying that they identify the master*

Text 2 agrees that revisions are "strong evidence against" a routine copy, but notes that some senior assistants also invented compositions. A revised underdrawing therefore "narrows the list of possible designers without singling out the master." Its author would accept part of the team's reasoning while denying that it proves the master's authorship.

B Text 2 says the evidence does not single out the master, but it never claims an assistant was the more likely designer; this overstates its position.

C Text 2 explicitly says that the revisions do not single out the master, so its author would not accept them as proof.

D Text 2 takes the revisions as given and reasons about what they mean; it never disputes that they exist.

Review: Chapter 7, Cross-Text Connections (page 98)

11. Correct answer: D RW-C0711

Cross-Text Connections · Craft and Structure · Harder

D *Varieties kept only in frozen storage lack an advantage of varieties grown each season.*

Text 1 concedes that frozen seeds do not evolve alongside changing pests and climate. Text 2 says that varieties grown each season "keep adapting to local conditions." Together, the texts indicate that stored seeds miss an advantage that field-grown seeds have, so both authors would accept the statement in choice D.

A Text 1 notes a limitation of seed banks, and Text 2 does not rank methods, so neither author calls vaults the most dependable.

B Text 2 points out that living collections can be wiped out by a single flood, so neither text argues that they should replace seed banks.

C Text 2 says only that vaults avoid some risks that field plantings face; neither text compares seed banks with every other preservation method.

Review: Chapter 7, Cross-Text Connections (page 98)

12. Correct answer: C RW-C0712

Cross-Text Connections · Craft and Structure · Harder

C *By suggesting that the conclusion may be premature because drivers could have switched streets*

Text 2 does not dispute that the redesigned road grew quieter, but it warns that counts on one road "reveal nothing about parallel streets." If drivers shifted routes, car trips overall "may have barely changed." The author would therefore call the officials' conclusion premature, since another explanation has not been ruled out.

A Text 2 raises a possibility ("may have barely changed") and does not conclude that the redesign had no effect; this choice is too strong.

B Text 2 argues that the traffic drop alone cannot show a change in travel habits, so its author would not agree.

D Text 2 accepts that the redesigned road "grew quieter"; it questions the cause of the decline, not whether it happened.

Review: Chapter 7, Cross-Text Connections (page 98)

CHAPTER 8

Central Ideas and Details

Information and Ideas is the second content domain in each Reading and Writing module, after Craft and Structure. Its questions ask what a text says, what evidence supports a claim about it, and what follows logically from it. Central Ideas and Details questions usually open the group, and they are the most direct reading task on the test.

The correct choice is nearly always the passage restated in different words. The wrong choices borrow its vocabulary, stretch its claims or cover only one corner of it.

What the question looks like

Each question gives you one passage of 25 to 150 words, literary (prose or poetry) or informational (science, history, social studies or the humanities), and asks one of three things.

Stem wording	What it asks for
Which choice best states the main idea of the text?	The point the whole passage makes
According to the text, why (or what, or how)…?	One detail the passage states directly
Based on the text, what is true about [a person or thing]?	A detail or trait the passage clearly shows, common with literary texts

Main idea questions want the whole passage in one sentence. Detail questions want one fact, usually tied to a word in the stem: a group, a cause, a result, a surprise. Either way, you can point to the words that prove the answer. Outside knowledge never rescues a choice.

How to recognize it

Sort short-passage questions by what the stem asks you to do.

- If the stem asks what the text *says*, it is Central Ideas and Details.
- If it asks what the text *does* (the function of a sentence, the purpose or structure of the whole), it is Text Structure and Purpose (Chapter 6).
- If it asks which quotation, finding or data would support or weaken a claim, it is Command of Evidence (Chapter 9).
- If the passage ends with a blank and the stem asks which choice most logically completes the text, it is Inferences (Chapter 10).

The standard of proof differs. For Central Ideas, the answer is in the passage, reworded; for Inferences, it follows from the passage but is not written there. If you are reasoning several steps beyond the text on a main idea question, you have probably drifted into a trap.

The method

1. **Read the stem first.** Decide whether it wants the main idea or a detail. For a detail, use Highlights & Notes to mark the subject the stem names.
2. **Read the whole passage once,** asking what it is about and what the author says about it.
3. **Find the turn.** Many passages set up a common view or background fact, then pivot with *but*, *however*, *yet* or *in fact*. The main idea nearly always sits after the turn.
4. **Predict in your own words** before you look at the choices. Ten words is plenty.
5. **Compare each choice with your prediction,** not with the other choices. Use the Option Eliminator to strike any choice that is too narrow, too broad or not in the passage.
6. **Run the pointing test** on the survivor: put your finger on the sentence or sentences that support it. If you cannot point, it is not the answer.

TIP A prediction protects you from attractive wording. Read the choices cold and all four can sound plausible; read them with a prediction in mind and the misfits stand out.

Scope: the main idea check

Every main idea choice faces two tests.

- **Does it cover enough?** A choice that restates one sentence, often the opening background, is too narrow. Ask whether the rest of the passage is about that sentence or merely starts there.
- **Does it go too far?** A choice that claims more than the passage does is too broad. Watch for *all*, *every*, *always*, *only* and *proves*, and for a subject larger than the passage's own, such as all ocean life when the passage studied one reef. When a passage reports one study, the main idea is about what that study found or suggested.

Reading literary passages

Literary passages open with an introduction such as "The following text is from Thomas Hardy's 1874 novel *Far from the Madding Crowd*." Read it: it gives the period and sometimes a sentence about who is speaking or what has just happened.

Literary questions usually ask what is true about a character or scene. The answer comes from what the text shows: actions, words and the narrator's comments. The most common literary trap gives a character a feeling, motive or plan the text never shows.

Worked examples

Worked example 1: Ordinary cargoes (main idea)

Museum displays of shipwrecks tend to feature gold coins, cannons and other spectacular finds. In a hypothetical study, a team of maritime archaeologists argues that ordinary cargoes deserve more attention. The team cataloged the plain storage jars, roof tiles and sacks of grain carried by more than forty small merchant ships that sank in one sea over three centuries. By comparing where each kind of cargo was made with where each ship went down, the team traced which ports traded with one another and how those routes shifted over time.

Which choice best states the main idea of the text?

- A) Museum displays of shipwrecks focus too heavily on gold coins and cannons.
- B) Studying the ordinary cargoes of many wrecks can reveal how trade routes worked and changed.
- C) Storage jars and roof tiles were the goods most often carried by merchant ships.
- D) Maritime archaeology has shown how trade worked in every sea over three centuries.

Thinking. The first sentence is background; the turn comes in the second, where the team "argues that ordinary cargoes deserve more attention." The rest explains why. Prediction: plain cargoes from many wrecks reveal trade routes. **Answer: B.** You can point to the second and fourth sentences.

Why the traps fail. A restates only the opening background and adds a judgment ("too heavily") the passage does not make. C recycles passage words: the passage lists these goods but never ranks them. D is too broad, stretching one sea to every sea.

Worked example 2: The shipyard contracts (detail)

In a hypothetical eighteenth-century shipyard, boys who signed on as apprentices agreed to work for seven years without wages. In return, the master shipwright promised to teach them the trade and to provide food, lodging and clothing. Surviving contracts reveal one further promise that historians had not expected: at the end of the term, each apprentice was to receive a full set of the tools of the trade. Without such tools, a newly trained carpenter could not have found work in another yard.

According to the text, what did the surviving contracts reveal that historians had not expected?

- A) Apprentices worked for seven years without wages.
- B) The master shipwright provided food, lodging and clothing.
- C) Apprentices were promised a set of tools when their terms ended.
- D) Newly trained carpenters often moved to other shipyards.

Thinking. The qualifier "had not expected" points to one sentence, the one that begins "Surviving contracts reveal one further promise." **Answer: C.**

Why the traps fail. A and B are true according to the passage, but nothing marks them as unexpected. D goes beyond the text: carpenters needed tools to find work elsewhere, but the passage never says they often moved. "True but not what the stem asked" is the signature trap of detail questions.

Worked example 3: Far from the Madding Crowd (literary)

The following text is from Thomas Hardy's 1874 novel *Far from the Madding Crowd*. Gabriel Oak is a young farmer.

Mr. Oak carried about him, by way of watch, what may be called a small silver clock; in other words, it was a watch as to shape and intention, and a small clock as to size. This instrument being several years older than Oak's grandfather, had the peculiarity of going either too fast or not at all. The smaller of its hands, too, occasionally slipped round on the pivot, and thus, though the minutes were told with precision, nobody could be quite certain of the hour they belonged to. The stopping peculiarity of his watch Oak remedied by thumps and shakes, and he escaped any evil consequences from the other two defects by constant comparisons with and observations of the sun and stars, and by pressing his face close to the glass of his neighbours' windows, till he could discern the hour marked by the green-faced timekeepers within.

Based on the text, what is true about Oak?

- A) He trusts his watch because it tells the minutes with precision.
- B) He plans to replace his watch with a clock like his neighbours'.
- C) He is embarrassed that his watch is older than his grandfather.
- D) He makes up for his watch's faults by checking the time against other sources.

Thinking. Most of the passage describes the watch's three defects; the last sentence says what Oak does about them: he thumps and shakes it, and he checks the sun, the stars and his neighbours' clocks ("the green-faced timekeepers within"). *Remedied* means fixed; *discern* means make out. **Answer: D.**

Why the traps fail. A is half right: the minutes are precise, but the hour is uncertain, so Oak does not rely on the watch alone. B invents a plan. C invents a feeling; the age of the watch is stated, but nothing suggests embarrassment.

Worked example 4: The cleaner fish (main idea)

Cleaner wrasses are small reef fish that eat parasites off the skin of larger fish, which visit the cleaners' fixed "stations" to be groomed. To learn whether the cleaners matter beyond the fish they groom, a hypothetical team of marine biologists removed every cleaner wrasse from several small patch reefs and left nearby reefs untouched. After eighteen months, the reefs without cleaners were visited by about half as many fish species as the untouched reefs, and the fish that remained there were smaller on average.

Which choice best states the main idea of the text?

- A) In the hypothetical study, removing cleaner wrasses reduced the variety of fish using a reef, suggesting that cleaners affect a reef's wider fish community.
- B) Cleaner wrasses eat parasites off the skin of larger fish at fixed stations.
- C) Small fish species determine the health of every ecosystem in the ocean.
- D) Larger fish groom one another when cleaner wrasses are absent.

Thinking. The first sentence is background, the second poses the study's question and the third answers it. A good main idea joins question and result. **Answer: A.**

Why the traps fail. B is too narrow: it repeats the background and ignores the study. C is too broad twice over, from cleaner wrasses to "small fish species" and from a few patch reefs to "every ecosystem in the ocean." D is not in the passage at all.

Worked example 5: Factory clocks (harder: a qualified main idea)

A common account of early factory life holds that clocks were simply tools of employers, used to enforce long, rigid working days. A hypothetical study of letters and court records from mill towns of the 1840s complicates that picture without overturning it. Mill owners did use bells and clocks to impose discipline, and some were accused of setting the factory clock back to stretch the working day. But the same records show workers using the clock in turn: they carried their own watches to catch altered clocks, cited exact hours in their complaints and organized campaigns for a shorter working day around the clock's authority.

Which choice best states the main idea of the text?

- A) Clocks in early factories were used only by employers to lengthen the working day.
- B) Although employers used clocks to control workers, workers also used clock time to defend their own interests.
- C) Early factory workers rejected clock time and tracked their hours by the sun instead.
- D) Campaigns for a shorter working day succeeded because workers owned watches.

Thinking. The key phrase is "complicates that picture without overturning it," so the answer must keep both halves: employers did use clocks for control (not overturned), and workers used clocks too (complicated). **Answer: B.**

Why the traps fail. A restates the common account the passage qualifies; "only" is exactly what the study complicates. C overshoots the other way: the workers used clock time rather than rejecting it. D invents a result and a cause; the passage says workers organized campaigns, not that they succeeded.

Common traps

Trap	What it looks like	How to spot it
Too narrow	A true detail from one sentence, often the background	Ask whether the rest of the passage is about it
Too broad	A claim larger than the passage's subject or scope	Look for *all*, *every*, *always* or a bigger field
Half right	A correct first half joined to an unsupported second half	Read every word; one false clause kills a choice
Recycled words	Passage vocabulary rearranged into a new claim	Point to the sentence; matching words are not matching meaning
The common view	The opinion the passage sets up in order to challenge	Find the turn and read what follows it
Outside knowledge	True in the world but not stated in the passage	If you need anything beyond the text, reject it
Invented feeling	A character's emotion, motive or plan the text never shows	Find the words that show it; if none exist, eliminate

Harder-module variations

If your second module is the harder one, expect these changes.

- **Denser passages,** with more complex sentences and vocabulary.
- **Qualified main ideas,** as in Worked example 5. When you see *complicates*, *qualifies* or *only partly*, expect the answer to hold two ideas in balance.
- **Main ideas assembled from parts.** No single sentence states the point, so you must combine the study's question and its result.
- **Older literary prose and poetry.** Read for the gist of each sentence rather than stalling on one unfamiliar word.
- **Choices that differ by one word:** *most* versus *some*, *caused* versus *coincided with*, *suggests* versus *proves*.
- **Detail questions with a qualifier** such as "unlike earlier studies" or "unexpectedly," pointing to one sentence.

Same-skill questions are grouped from easier to harder, so the last Central Ideas question in a module is usually the hardest. Give it the full method, and use Mark for Review if you need to return.

Check yourself

1. What two tests does every main idea choice have to pass?
2. Name four words or phrases that often signal the turn in a passage.
3. On a detail question, what should you do before you read the choices?
4. A choice gives a character a feeling the passage never describes. Which trap is it?
5. What is the pointing test, and when do you use it?
6. **Answers:** (1) It must cover the whole passage, and it must not go beyond it. (2) *But*, *however*, *yet* and *in fact*. (3) Mark the subject the stem names and find the sentence it refers to. (4) Invented feeling. (5) Pointing to the sentence or sentences that support a choice; use it on every choice that survives elimination.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/central-ideas-and-details

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C0801

> Many coral reef fish spend their first weeks of life drifting in open water before settling on a reef. In a hypothetical study, a team of marine biologists placed underwater speakers beside patches of empty reef. Some speakers played recordings of a healthy reef, including the crackle of snapping shrimp and the scrape of grazing fish; others stayed silent. Over six weeks, about twice as many young fish settled on the patches with recordings as on the silent patches.

Which choice best states the main idea of the text?

A Snapping shrimp are the loudest animals that live on coral reefs.

B In a study, recordings of healthy-reef sounds appeared to draw young fish to settle nearby.

C Young reef fish spend their entire lives drifting in open water.

D Silent patches of reef are safer places for young fish to settle.

---

2 RW-C0802

> Before building a hypothetical observatory, a team of astronomers compared three mountain sites over two years. The tallest peak had the thinnest air above it, which usually helps telescopes see clearly, but clouds formed around its summit on most afternoons and often lingered after dark. The team chose a slightly lower peak nearby, where the skies were clear on far more nights.

According to the text, why did the team decide against building the observatory on the tallest peak?

A The air above it was thicker than the air above the other sites.

B It was too far from the other two mountain sites.

C It was too close to the bright lights of a town.

D Clouds often covered its summit after dark.

---

3 RW-C0803

> The following text is from Kate Chopin's 1899 novel *The Awakening*.
>
> Edna had attempted all summer to learn to swim. She had received instructions from both the men and women; in some instances from the children. Robert had pursued a system of lessons almost daily; and he was nearly at the point of discouragement in realizing the futility of his efforts. A certain ungovernable dread hung about her when in the water, unless there was a hand near by that might reach out and reassure her.

Based on the text, what is true about Edna?

A She feels afraid in the water unless someone is close enough to reach her.

B She has refused to take lessons from anyone except Robert.

C She learned to swim quickly because Robert's lessons were so frequent.

D She prefers to practice swimming alone, far from other people.

---

4 RW-C0804

> In many nineteenth-century textile mills, weavers were paid by the piece: they earned a set amount for each length of cloth they finished. A hypothetical study of one mill's account books found that when the mill installed faster looms, it also lowered the amount it paid for each piece. Weavers then produced much more cloth each week, yet their weekly earnings rose only slightly.

Which choice best states the main idea of the text?

A Piece rates were used in almost every industry of the nineteenth century.

B Faster looms caused weavers' weekly earnings to fall sharply.

C Faster looms raised weavers' output, but a lower piece rate kept their pay from rising much.

D Mill account books are the most reliable source on nineteenth-century wages.

---

5 RW-C0805

> Archaeologists often date a layer of soil by the objects found in it, but objects do not always stay where they were dropped. Burrowing animals, tree roots and later digging can push a coin or bead down into an older layer. For this reason, a team excavating a hypothetical village site dated each layer from groups of objects found together rather than from any single object. The team also checked whether charcoal from the same layer gave a similar radiocarbon date, and where the objects and the charcoal disagreed, it recorded the layer's age as uncertain.

Which choice best states the main idea of the text?

A Because objects can move between soil layers, the team relied on several kinds of evidence to date each layer.

B Burrowing animals have disturbed most archaeological sites so badly that they cannot be dated.

C Radiocarbon dates from charcoal are less reliable than dates based on coins.

D The village site contained more coins and beads than the team had expected.

---

6 RW-C0806

> Sea otters often float on their backs while eating, using their chests as tables. In a hypothetical year-long survey along a rocky coastline, marine biologists recorded what individual otters ate. Although the population as a whole ate dozens of kinds of prey, most individual otters relied heavily on just two or three kinds, such as sea urchins or crabs. Mothers and their pups often favored the same prey, which suggests that young otters may learn feeding habits from their mothers.

According to the text, what did the survey show about most individual sea otters?

A They regularly ate dozens of different kinds of prey.

B They depended largely on a small number of prey types.

C They learned to float on their backs by watching their mothers.

D They changed their favorite prey from one season to the next.

---

7 RW-C0807

> The following text is from Thomas Hardy's 1882 novel *Two on a Tower*. A visitor has asked a young astronomer how long he plans to keep using an old tower as his observatory.
>
> 'About a year longer—till I have obtained a practical familiarity with the heavens. Ah, if I only had a good equatorial!'
>
> 'What is that?'
>
> 'A proper instrument for my pursuit. But time is short, and science is infinite,—how infinite only those who study astronomy fully realize,—and perhaps I shall be worn out before I make my mark.'
>
> She seemed to be greatly struck by the odd mixture in him of scientific earnestness and melancholy mistrust of all things human. Perhaps it was owing to the nature of his studies.

Based on the text, what is true about the young astronomer?

A He already owns the instrument that he believes his work requires.

B He expects to understand astronomy completely within about a year.

C His visitor finds his attitude toward his studies entirely ordinary.

D He is devoted to astronomy but doubts that he will have time to achieve his goals.

---

8 RW-C0808

> When a broken ancient bowl is restored for display, conservators must decide how much of it to rebuild. Filling every gap with matching clay and paint lets visitors see the bowl's original shape, but it can also hide which parts are ancient and which are new. Many conservators therefore fill missing areas in a slightly different shade from the surrounding surface. From across a gallery, the bowl looks whole; up close, a careful viewer can tell exactly where the old surface ends and the repairs begin.

Which choice best states the main idea of the text?

A Ancient bowls should be displayed in pieces rather than repaired.

B Matching clay and paint is the most accurate way to restore an ancient bowl.

C Conservators often choose repairs that restore a bowl's shape while keeping the new parts distinguishable from the original.

D Visitors generally prefer restored bowls to broken ones.

---

9 RW-C0809

> Payroll records can tell historians what workers were paid, but not always how long they actually spent at work. A collection of diaries kept by cannery workers in a fictional coastal town around 1910 helps fill that gap. The cannery's payroll listed a ten-hour shift. The diaries, however, describe workers arriving at dawn and waiting, unpaid, until fishing boats returned with a catch, which sometimes added three or four hours to the workday.

According to the text, what do the diaries reveal that the payroll records do not?

A Workers often spent unpaid hours at the cannery waiting for work to begin.

B The cannery paid workers more than its payroll records showed.

C Fishing boats usually returned before dawn, so shifts began early.

D The cannery's official shifts lasted longer than ten hours.

---

10 RW-C0810

> Counting meteors by eye might seem too simple to be scientifically useful, but large numbers of counts can reveal patterns that no single observer could detect. In a hypothetical project, hundreds of volunteers recorded how many meteors they saw per hour during the same annual meteor shower, along with how dark their sky was. Individual counts varied widely, partly because observers under bright city skies missed fainter meteors. Once each count was adjusted for sky brightness, the combined data showed the shower's activity peaking sharply on a single night.

Which choice best states the main idea of the text?

A Observers under bright city skies saw more meteors than observers under dark skies.

B Meteor counts made by individual volunteers are too unreliable to be useful.

C Meteor showers are easiest to observe from cities.

D When adjusted for sky brightness, many volunteers' meteor counts together revealed a clear pattern in a shower's activity.

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11 RW-C0811

> Many people picture ancient builders moving giant stones only with enormous crews. A hypothetical experiment at a fictional archaeology field school tested another possibility. Students placed a two-ton concrete block on a wooden sledge and pulled it first across dry sand and then across sand that had been wetted. On wet sand, the sledge needed noticeably less pulling force, because the damp grains stuck together instead of piling up in front of the runners.

Which choice best states the main idea of the text?

A Ancient builders depended mainly on enormous crews to move heavy stones.

B An experiment suggests that wetting sand could have made heavy stones easier to drag on sledges.

C Concrete blocks are easier to move than stone blocks of the same weight.

D Dry sand is a better surface than wet sand for dragging a heavy sledge.

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12 RW-C0812

> On many reefs, corals of the same species release their eggs and sperm into the water on the same few nights each year, timing that improves the odds of fertilization. In a hypothetical laboratory study, marine biologists kept fragments of one coral species in two tanks. One tank was lit to mimic the monthly cycle of moonlight; the other was kept under constant dim light. Fragments in the moonlit tank spawned within a single night of one another. Fragments under constant light also spawned, but their spawning was spread across nearly two weeks. Water temperature was held steady in both tanks.

According to the text, what is true about the coral fragments kept under constant dim light?

A They did not spawn at any point during the study.

B They spawned within a single night of one another.

C They spawned, but over a much longer span of time than the fragments in the moonlit tank did.

D They spawned later than the other fragments because their water was warmer.

---

13 RW-C0813

> The following text is from Elizabeth Gaskell's 1855 novel *North and South*.
>
> For several miles before they reached Milton, they saw a deep lead-coloured cloud hanging over the horizon in the direction in which it lay. It was all the darker from contrast with the pale gray-blue of the wintry sky; for in Heston there had been the earliest signs of frost. Nearer to the town, the air had a faint taste and smell of smoke; perhaps, after all, more a loss of the fragrance of grass and herbage than any positive taste or smell. Quick they were whirled over long, straight, hopeless streets of regularly-built houses, all small and of brick. Here and there a great oblong many-windowed factory stood up, like a hen among her chickens, puffing out black "unparliamentary" smoke, and sufficiently accounting for the cloud which Margaret had taken to foretell rain.

Based on the text, what is true about the cloud that Margaret saw as she approached Milton?

A It confirmed Margaret's belief that rain was about to fall on Milton.

B It brought the first signs of frost to Heston.

C It carried the fragrance of grass and herbage toward the travelers.

D It came from the town's factories, though she had first taken it as a sign of coming rain.

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14 RW-C0814

> An astronomer who reports a distant star's present color is making a claim that goes beyond what the telescope shows. The starlight reaching the instrument may have left the star thousands of years ago, so it records the star's condition at that time; the star may since have changed. That light may also have crossed clouds of interstellar dust, which scatter blue light more than red and can make a star appear redder than it is. To describe the star's present color, then, the astronomer must rely on assumptions about how long the light traveled and what lay along its path.

Which choice best states the main idea of the text?

A A statement about a distant star's color depends on assumptions about the light's travel, not only on what is observed.

B Interstellar dust makes distant stars look redder than they really are.

C Telescopes cannot provide useful information about the colors of distant stars.

D Most stars that appear red have changed color since their light left them.

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15 RW-C0815

> When a whale dies and sinks to the deep seafloor, its body becomes a sudden feast in a place where food is usually scarce. Scavengers such as hagfish and sleeper sharks strip away the soft tissue within months. In a hypothetical long-term study of several such "whale falls," marine biologists found that once the soft tissue was gone, the skeleton supported a different set of animals: worms that bore into bone fed on the fats stored inside it. Later still, bacteria breaking down the remaining fats released sulfur compounds that supported clams and other animals that never fed on the whale directly.

Which choice best states the main idea of the text?

A Scavengers remove the soft tissue of a sunken whale within months of its arrival on the seafloor.

B Because food is scarce on the deep seafloor, most animals there depend on whale carcasses.

C A sunken whale carcass can support a series of different communities as it breaks down over time.

D Bone-boring worms release sulfur compounds that support clams near a whale carcass.

Practice Set Explanations

1. Correct answer: B RW-C0801

Central Ideas and Details · Information and Ideas · Easier

B *In a study, recordings of healthy-reef sounds appeared to draw young fish to settle nearby.*

The text describes a hypothetical study: speakers played healthy-reef sounds beside some empty reef patches, and about twice as many young fish settled on those patches as on silent ones. The best statement of the main idea names the study's central result, as this choice does, without adding claims the text never makes.

A The text mentions snapping shrimp only as one sound on the recordings. It never compares how loud reef animals are, so this claim is unsupported.

C This contradicts the first sentence, which says young fish drift only for their first weeks before settling on a reef.

D The text says nothing about safety. It reports only that fewer young fish settled on the silent patches, which does not show those patches were safer.

Review: Chapter 8, Central Ideas and Details (page 112)

2. Correct answer: D RW-C0802

Central Ideas and Details · Information and Ideas · Easier

D *Clouds often covered its summit after dark.*

The second sentence gives the reason: although the tallest peak's thin air "usually helps telescopes see clearly," clouds formed around its summit "on most afternoons and often lingered after dark." The team then chose a peak with clear skies on more nights, so frequent nighttime clouds were the deciding problem.

A The text says the tallest peak had the thinnest air, the opposite of this choice, so thick air was not the team's reason.

B The text never mentions the distances between the sites, so this reason has no support in the passage.

C Nothing in the text mentions towns or artificial light near the tallest peak. The problem the text names is cloud cover.

Review: Chapter 8, Central Ideas and Details (page 112)

3. Correct answer: A RW-C0803

Central Ideas and Details · Information and Ideas · Easier

A *She feels afraid in the water unless someone is close enough to reach her.*

The last sentence says that "a certain ungovernable dread hung about her when in the water, unless there was a hand near by that might reach out and reassure her." In plain terms, Edna is afraid in the water unless someone is close enough to help, which is exactly what this choice states.

B The text says she received instructions from men, women and even children, so she did not limit her lessons to Robert.

C She has "attempted all summer" without success, and Robert is nearly discouraged by "the futility of his efforts," so she has not learned quickly.

D This reverses the text. Her dread eases only when "a hand near by" could reassure her, so she wants people close, not far away.

Review: Chapter 8, Central Ideas and Details (page 112)

4. Correct answer: C RW-C0804

Central Ideas and Details · Information and Ideas · Easier

C *Faster looms raised weavers' output, but a lower piece rate kept their pay from rising much.*

The text explains piece-rate pay and then reports one hypothetical study: after faster looms arrived, the mill paid less per piece, so weavers made much more cloth but earned only slightly more. The main idea combines both effects, higher output and nearly flat pay, which only this choice does.

A The text discusses piece rates only in textile mills and in one mill's records. A claim about almost every industry is far too broad.

B The text says weekly earnings "rose only slightly," so a claim that they fell sharply contradicts the passage.

D The study uses account books, but the text never compares sources or calls account books the most reliable. That judgment is unsupported.

Review: Chapter 8, Central Ideas and Details (page 112)

5. Correct answer: A RW-C0805

Central Ideas and Details · Information and Ideas · Medium

A *Because objects can move between soil layers, the team relied on several kinds of evidence to date each layer.*

The first two sentences present a problem: objects can be pushed down into older layers. The rest of the text explains the team's response, dating layers from groups of objects and checking them against charcoal dates. A statement of the main idea must connect the problem and the response, as this choice does.

B The text says animals can move objects, not that most sites are ruined. The team still dated its layers, so this choice overstates the problem.

C The text uses charcoal as a check on the objects and never ranks the two methods, so this comparison is unsupported.

D Coins and beads appear only as examples of objects that can move. The text says nothing about how many the team found or expected.

Review: Chapter 8, Central Ideas and Details (page 112)

6. Correct answer: B RW-C0806

Central Ideas and Details · Information and Ideas · Medium

B *They depended largely on a small number of prey types.*

The third sentence answers the question: "most individual otters relied heavily on just two or three kinds" of prey, and "a small number of prey types" restates that detail. The sentence carefully separates individual otters from the population as a whole, which is exactly where the most tempting wrong answer goes astray.

A This describes the population as a whole. The text says most individual otters relied on just two or three kinds, so the choice confuses the group with the individual.

C The text links mothers and pups only through shared prey choices. It never says that floating is learned from mothers.

D The survey lasted a year, but the text never mentions seasonal changes in diet, so this detail is unsupported.

Review: Chapter 8, Central Ideas and Details (page 112)

7. Correct answer: D RW-C0807

Central Ideas and Details · Information and Ideas · Medium

D *He is devoted to astronomy but doubts that he will have time to achieve his goals.*

The astronomer speaks of "my pursuit" with clear commitment, yet he worries that "time is short" and that he may be "worn out before I make my mark." The narrator confirms this "odd mixture" of "scientific earnestness and melancholy mistrust." Only the choice that combines devotion with doubt captures what the text shows about him.

A He exclaims "if I only had a good equatorial!" and calls it "a proper instrument for my pursuit," so he lacks the instrument he wants.

B The year refers only to gaining "a practical familiarity with the heavens." He also calls science "infinite," so he does not expect complete mastery.

C The visitor is "greatly struck by the odd mixture" in him, which is the opposite of finding his attitude ordinary.

Review: Chapter 8, Central Ideas and Details (page 112)

8. Correct answer: C RW-C0808

Central Ideas and Details · Information and Ideas · Medium

C *Conservators often choose repairs that restore a bowl's shape while keeping the new parts distinguishable from the original.*

The text presents a problem with perfect matching, which hides which parts are new, and then a common solution: filling gaps in a slightly different shade so the bowl looks whole from a distance while the repairs show up close. The main idea is this balance between restoring the bowl's shape and keeping the repairs identifiable.

A The text describes how conservators repair bowls, not whether they should avoid repairs, so this recommendation is unsupported.

B The text says perfect matching can hide which parts are new, which is why many conservators avoid it. This choice reverses that point.

D The text never reports what visitors prefer. It focuses on conservators' choices about how visible repairs should be.

Review: Chapter 8, Central Ideas and Details (page 112)

9. Correct answer: A RW-C0809

Central Ideas and Details · Information and Ideas · Medium

A *Workers often spent unpaid hours at the cannery waiting for work to begin.*

The text sets payroll records, which listed a ten-hour shift, against diaries describing workers who arrived at dawn and waited "unpaid" for the boats, sometimes adding three or four hours to the day. That unpaid waiting time is the information the diaries supply and the payroll leaves out.

B The diaries concern time spent waiting, not the amount of pay. Nothing in the text suggests the payroll understated wages.

C The diaries say workers arrived at dawn and then waited for the boats to return, so the boats came back after dawn, not before.

D The payroll listed a ten-hour shift. The extra hours came from unpaid waiting outside that shift, so this choice misstates the records.

Review: Chapter 8, Central Ideas and Details (page 112)

10. Correct answer: D RW-C0810

Central Ideas and Details · Information and Ideas · Medium

D *When adjusted for sky brightness, many volunteers' meteor counts together revealed a clear pattern in a shower's activity.*

The first sentence sets up the idea that many simple counts can reveal patterns no single observer could see, and the rest of the text shows it happening: once the counts were adjusted for sky brightness, the combined data revealed a sharp peak. The correct choice states this central point about combined, adjusted data.

A The text says city observers "missed fainter meteors," which means they saw fewer meteors, not more. This choice reverses the detail.

B Individual counts varied, but the text's point is that the combined, adjusted counts were useful, so calling them useless contradicts the passage.

C The text indicates the opposite: bright city skies hide fainter meteors. Nothing suggests that cities are better places for viewing.

Review: Chapter 8, Central Ideas and Details (page 112)

11. Correct answer: B RW-C0811

Central Ideas and Details · Information and Ideas · Medium

B *An experiment suggests that wetting sand could have made heavy stones easier to drag on sledges.*

After noting a common assumption about ancient builders, the text describes a hypothetical experiment in which a heavy block on a sledge moved with less force over wet sand than over dry sand. The main idea is what the experiment suggests: wetting sand could have eased the work of dragging heavy loads.

A This is the common picture the text sets up before testing another possibility. The experiment points to a technique that eased dragging, not to reliance on huge crews.

C The students used concrete only as a stand-in for a heavy stone. The text never compares concrete with stone.

D The text reports the opposite result: the sledge needed "noticeably less pulling force" on wet sand.

Review: Chapter 8, Central Ideas and Details (page 112)

12. Correct answer: C RW-C0812

Central Ideas and Details · Information and Ideas · Harder

C *They spawned, but over a much longer span of time than the fragments in the moonlit tank did.*

The fifth sentence answers the question: fragments under constant light "also spawned, but their spawning was spread across nearly two weeks," while the moonlit fragments spawned within one night of one another. Both parts matter: these fragments did spawn, and their timing was far less synchronized.

A The text states that these fragments "also spawned," so the claim that they never spawned contradicts the passage.

B That timing describes the fragments in the moonlit tank. This choice gives one group's result to the other group.

D Water temperature "was held steady in both tanks," so the text gives no basis for a temperature difference between the two groups.

Review: Chapter 8, Central Ideas and Details (page 112)

13. Correct answer: D RW-C0813

Central Ideas and Details · Information and Ideas · Harder

D *It came from the town's factories, though she had first taken it as a sign of coming rain.*

The first sentence introduces "a deep lead-coloured cloud" over Milton, and the last sentence explains it: the factories' black smoke was "sufficiently accounting for the cloud which Margaret had taken to foretell rain." Connecting those two sentences shows that the cloud was smoke, not the rain cloud Margaret first assumed it to be.

A Margaret "had taken" the cloud "to foretell rain," but factory smoke explains it instead, so her belief was mistaken rather than confirmed.

B The text says Heston already had "the earliest signs of frost"; that detail explains the pale wintry sky, not anything the cloud caused.

C Near the town the air had lost "the fragrance of grass and herbage," so the cloud did not bring that fragrance. The text suggests the reverse.

Review: Chapter 8, Central Ideas and Details (page 112)

14. Correct answer: A RW-C0814

Central Ideas and Details · Information and Ideas · Harder

A *A statement about a distant star's color depends on assumptions about the light's travel, not only on what is observed.*

The text argues that reporting a star's present color "goes beyond what the telescope shows" and gives two reasons: the light is old, and dust may have altered it. Its conclusion is that the astronomer must rely on assumptions about the light's travel. The correct choice states that general point rather than a single supporting detail.

B This is one supporting detail, and the text says dust "can" redden a star, not that it always does. It also leaves out light-travel time.

C The text says a claim about color requires assumptions, not that telescopes are useless. This choice overstates the argument.

D The text says only that a star "may" have changed. It makes no claim about most red stars, so this choice is unsupported.

Review: Chapter 8, Central Ideas and Details (page 112)

15. Correct answer: C RW-C0815

Central Ideas and Details · Information and Ideas · Harder

C *A sunken whale carcass can support a series of different communities as it breaks down over time.*

The text follows a carcass through stages: scavengers remove the soft tissue, bone-boring worms feed next, and later bacteria support clams and other animals. Phrases such as "once the soft tissue was gone" and "later still" signal a sequence. The main idea is that one carcass hosts a succession of communities, which only this choice states.

A This is accurate but describes only the first stage. The text goes on to describe later communities, so this choice is too narrow to be the main idea.

B The text calls a whale fall a sudden feast, not the food source most deep-sea animals depend on. The claim about "most animals" goes beyond the passage.

D The text credits bacteria, not worms, with releasing the sulfur compounds, so this choice mixes up two different stages.

Review: Chapter 8, Central Ideas and Details (page 112)

CHAPTER 9

Command of Evidence: Textual and Quantitative

Command of Evidence questions hand you a claim and ask you to find the evidence that does a specific job: support it, weaken it or illustrate it. The College Board treats Command of Evidence as one skill with two kinds of evidence. Textual evidence is a quotation from a literary work or a research finding described in words. Quantitative evidence is data from a table, bar graph or line graph. Evidence questions come together in each module, within the Information and Ideas domain.

The skill is narrower than it looks. You never decide whether a claim is true in the real world, only whether a piece of evidence bears on it in the way the stem asks.

What the question looks like

Stem	Kind of evidence	Your job
Which finding, if true, would most directly support the hypothesis?	Textual: a finding	Choose the result the hypothesis predicts
Which finding, if true, would most directly weaken the claim?	Textual: a finding	Choose the result that makes the claim less likely
Which quotation from [the work] most effectively illustrates the claim?	Textual: a quotation	Choose the quotation that shows every part of the claim
Which choice most effectively uses data from the table to complete the statement?	Quantitative	Choose accurate data that finishes the sentence's point
Which choice best describes data from the graph that support the claim?	Quantitative	Choose accurate data that makes the claim's comparison

If true means exactly what it says. Treat every finding as established fact, and do not reject one because it is surprising or new to the passage; findings are supposed to be new information. Your only question is what each finding would do to the claim.

Data questions need no calculation beyond reading and comparing values, and there is no calculator in the Reading and Writing section.

How to recognize it

Look for three signals: a claim, hypothesis or conclusion in the passage, often credited to a researcher, historian or student; a stem that uses *support*, *weaken*, *illustrate* or *uses data*; and, for quantitative questions, a table or graph. Central Ideas questions (Chapter 8) ask what the passage says; evidence questions ask what would back up or undercut something it claims. Some quantitative questions end with a blank, as Inferences questions (Chapter 10) do, but here the choices are data from the graphic.

The method

Findings and quotations

1. **Isolate the claim.** Use Highlights & Notes to mark the sentence that states it, and break it into parts: who, what, under what conditions, compared with what.
2. **Predict the evidence.** Ask what you would expect to see if the claim were true. For a weaken question, ask what would make the claim less likely, such as contrary evidence or another explanation for the result.
3. **Test every part.** The right choice addresses the specific claim, not just its topic. A two-part claim needs evidence for both parts.
4. **Eliminate the neutral.** Many wrong choices are on topic but would not change your confidence in the claim at all.

Tables and graphs

1. **Read the frame first:** title, axis labels, units, legend and categories.
2. **Name what the claim compares:** two groups, before and after, or a trend over time.
3. **Check accuracy.** Strike any choice that misreads a value, a year, a series or a unit.
4. **Check relevance.** Among the accurate choices, keep the one that makes the claim's comparison. Accuracy is necessary, but it is not enough.

TIP On data questions, the passage tells you what to look for and the graphic tells you whether it is there. Read the claim, then the graphic, then the choices.

Supporting and weakening

Suppose a historian claims that a new lens made a coastal lighthouse visible farther out to sea. A finding that ships began reporting the light from greater distances after the lens was installed *supports* the claim: it is what the claim predicts. A finding that the distances in ships' reports did not change *weakens* it. A finding that the lens was expensive does neither. On topic but neutral in effect, that kind of choice is the most common wrong answer on this question type.

Worked examples

Worked example 1: Hermit crabs (supporting a hypothesis)

Hermit crabs protect their soft bodies by living in empty snail shells, and they must move into larger shells as they grow. On one hypothetical beach, a marine biologist noticed that few large empty shells were available. She hypothesizes that the shortage of large shells is limiting how big the beach's hermit crabs can grow.

Which finding, if true, would most directly support the marine biologist's hypothesis?

- A) Hermit crabs on the beach prefer the shells of one snail species over the shells of others.
- B) The largest hermit crabs on the beach were found living in the largest shells.
- C) Many small empty shells were found scattered along the beach.
- D) When large empty shells were added to one part of the beach, crabs there grew larger over the next year than crabs elsewhere on the beach.

Thinking. The hypothesis links a cause (too few large shells) to an effect (limited growth). Prediction: remove the shortage and the crabs should grow bigger. D tests exactly that. **Answer: D.**

Why the traps fail. A is on topic but says nothing about shortage or growth. B would be expected whether or not a shortage exists, because bigger crabs need bigger shells in any case. C concerns small shells, not the large ones the hypothesis is about.

Worked example 2: The mill law (weakening a claim)

In a hypothetical state, the number of children employed in textile mills fell sharply between 1880 and 1890. A historian claims that the main cause of the decline was a law, passed in 1885, that banned mill work for children under twelve.

Which finding, if true, would most directly weaken the historian's claim?

- A) Most of the decline in child employment occurred between 1880 and 1884, before the law was passed.
- B) Inspectors reported that most mills obeyed the law after 1885.
- C) Several labor unions in the state campaigned for the law.
- D) Child employment in the mills continued to fall during the 1890s.

Thinking. A cause must come before its effect. If most of the decline had happened by 1884, a law passed in 1885 cannot be its main cause. **Answer: A.**

Why the traps fail. B points the wrong way: widespread obedience makes the law a more believable cause. C explains how the law passed, not what it did. D fits the claim, since a law still in force could keep reducing child employment.

Worked example 3: North and South (illustrating with a quotation)

In Elizabeth Gaskell's 1855 novel *North and South*, Margaret Hale loves Helstone, the village in southern England where her parents live. In one conversation, she admits that Helstone has drawbacks but still praises it highly.

Which quotation from *North and South* most effectively illustrates the claim?

- A) "Helstone is like a village in a poem—in one of Tennyson's poems."
- B) "All the other places in England that I have seen seem so hard and prosaic-looking, after the New Forest."
- C) "We have rain, and our leaves do fall, and get sodden: though I think Helstone is about as perfect a place as any in the world."
- D) "Leave Helstone, papa! But why?"

Thinking. The claim has two parts, drawbacks and high praise, so the quotation must show both. C names drawbacks (rain, sodden leaves) and then calls Helstone "about as perfect a place as any in the world." **Answer: C.**

Why the traps fail. A and B praise Helstone (B by calling other places dull, or *prosaic*, by comparison) but admit no drawback. D shows Margaret's surprise at the idea of leaving but contains neither a drawback nor praise.

Worked example 4: Jellyfish and warm water (completing a statement)

A student wondered whether jellyfish are more common in warmer water. After comparing summer surveys at three hypothetical beaches, the student concluded that jellyfish sightings were more frequent where the water was warmer, noting that _____

Jellyfish Sightings at Three Beaches (hypothetical data)

Beach	Average water temperature (°C)	Jellyfish sightings per survey
North Cove	18	4
Harbor Beach	22	11
South Point	25	19

Which choice most effectively uses data from the table to complete the statement?

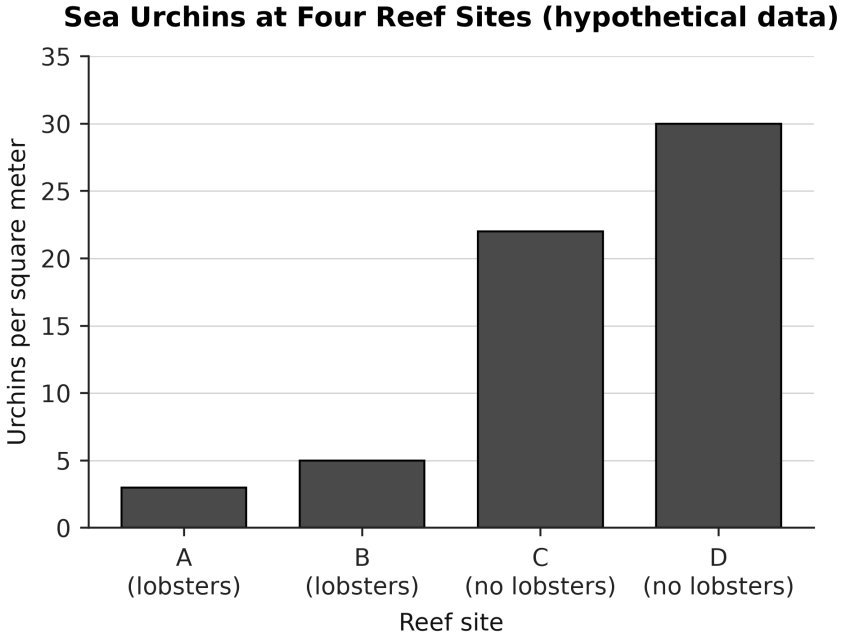
- A) Harbor Beach had an average of 11 jellyfish sightings per survey.
- B) South Point, the beach with the warmest water, had the most sightings per survey, and North Cove, the beach with the coolest water, had the fewest.
- C) South Point had both the coolest water and the most sightings per survey.
- D) North Cove had fewer sightings per survey than Harbor Beach did.

Thinking. The conclusion links two columns, temperature and sightings, so the completion must use both and show them rising together. **Answer: B.**

Why the traps fail. A is accurate but reports one beach's sightings with no temperature. C misreads the table: South Point had the warmest water, 25°C. D is accurate but leaves out temperature, so it cannot show the link the student claims.

Worked example 5: Lobsters and urchins (describing data that support a claim)

A marine biologist surveyed sea urchins at four hypothetical reef sites, two where spiny lobsters were present and two where they were absent. The biologist claims that where spiny lobsters are present, they eat enough urchins to keep urchin numbers low.



Which choice best describes data from the graph that support the marine biologist's claim?

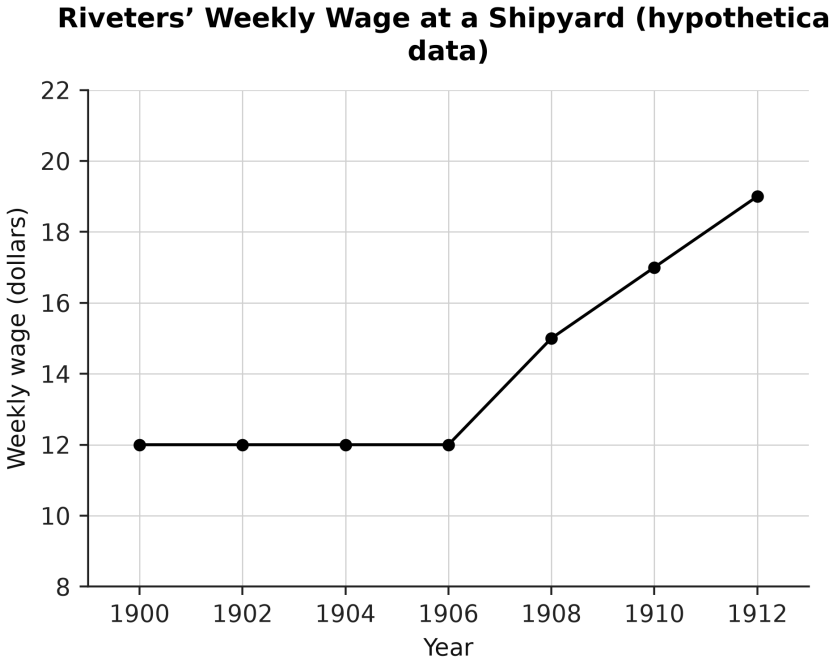
- A) Site D had more urchins per square meter than any other site.
- B) Site A had fewer urchins per square meter than Site B did.
- C) Sites C and D had about 22 and 30 urchins per square meter, respectively.
- D) Sites A and B, where lobsters were present, each had 5 or fewer urchins per square meter, while Sites C and D, where lobsters were absent, each had more than 20.

Thinking. The claim compares sites with lobsters to sites without them. Only one choice crosses that line.
Answer: D.

Why the traps fail. Every choice is accurate, which is what makes this question hard. A describes one bar. B compares two lobster sites with each other, which says nothing about the lobsters' effect. C describes only the sites without lobsters, leaving nothing to compare them with.

Worked example 6: Riveters' wages (harder: a before-and-after claim)

A labor historian studying a hypothetical shipyard argues that a strike by riveters in 1906 marked a turning point: their pay had not changed for years before the strike, but it rose steadily in the years that followed.



Which choice most effectively uses data from the graph to illustrate the historian's claim?

- A) Riveters earned $12 a week in every year shown from 1900 through 1906, and their weekly wage then rose to $15 in 1908, $17 in 1910 and $19 in 1912.
- B) Riveters' weekly wage rose from $12 in 1900 to $19 in 1912.
- C) The largest two-year increase in riveters' weekly wage, $3, occurred between 1906 and 1908.
- D) Riveters earned $17 a week in 1910.

Thinking. A turning point needs two facts: no change before 1906 and a steady rise after it. Only A shows both. **Answer: A.**

Why the traps fail. B is accurate but blends the two periods into one rise, hiding the flat stretch that makes 1906 a turning point. C shows a jump after the strike but says nothing about the years before or the continued rise. D is a single point.

Common traps

Trap	What it looks like	How to spot it
On topic, no effect	A finding about the subject that would not change your view of the claim	Ask whether it makes the claim more or less likely
Wrong direction	A supporter on a weaken question, or the reverse	Reread the stem's verb before you choose
Half the claim	Evidence for one part of a two-part claim	List the claim's parts and check each one

Trap	What it looks like	How to spot it
Expected anyway	A result that would occur whether or not the claim is true	Ask whether it would also appear if the claim were false
Misread data	Wrong row, series, year or unit	Check every number against the graphic
Accurate but beside the point	True data that skips the claim's comparison	Name the comparison first, then look for it
Shared words	A quotation that echoes the claim's mood but not its point	Match the meaning of every part of the claim

Harder-module variations

- **Two-part claims,** where only one choice addresses both parts.
- **Weaken questions** that include a choice supporting the claim.
- **Graphs with two series,** where you must compare the right series in the right categories.
- **Before-and-after claims,** as in Worked example 6, where the timing of a change is the point.
- **Choices that are all accurate,** so relevance alone decides.
- **Literary claims about attitude,** with several quotations that share the claim's mood but only one that matches its exact point.

Check yourself

1. What does *if true* tell you to do with a finding?
2. Name the three things a finding can do to a claim.
3. On a data question, which two tests must the correct choice pass?
4. A claim says something changed after an event. What two facts must the evidence show?
5. Why can an accurate description of a graph still be the wrong answer?
6. **Answers:** (1) Accept it as fact and ask only what it does to the claim. (2) Support it, weaken it or have no bearing on it. (3) It must be accurate, and it must make the claim's comparison. (4) What things were like before the event and how they changed after it. (5) It may describe data the claim does not depend on.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

Command of Evidence: Textual
skillbyskillprep.com/skills/command-of-evidence-textual

Command of Evidence: Quantitative
skillbyskillprep.com/skills/command-of-evidence-quantitative

Practice Set

20 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C0901

> Seagrass meadows are often called nurseries because young fish can hide among the long, grasslike blades. In a hypothetical survey, a student volunteer counted juvenile fish at three kinds of coastal sites and found support for this idea: the average number of juvenile fish counted per survey was _____

Juvenile Fish Counts at Three Coastal Sites (hypothetical data)

Site	Average juvenile fish per survey
Seagrass meadow	46
Rocky reef	31
Sandy bottom	9

Which choice most effectively uses data from the table to complete the statement?

A higher at the rocky reef (31) than at the sandy bottom (9).

B higher at the rocky reef (46) than at the seagrass meadow (31).

C higher at the seagrass meadow (46) than at either the rocky reef (31) or the sandy bottom (9).

D about the same at all three sites.

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2 RW-C0902

> As a hypothetical comet moved closer to the Sun, observers noticed that its glowing tail grew much longer. A team of astronomers hypothesizes that the Sun's heat caused ice on the comet's surface to turn directly into gas, and that this escaping gas carried dust away from the comet to form the tail.

Which finding, if true, would most directly support the astronomers' hypothesis?

A Measurements show that the comet released more gas as its surface grew warmer.

B The comet takes about 75 years to complete one orbit around the Sun.

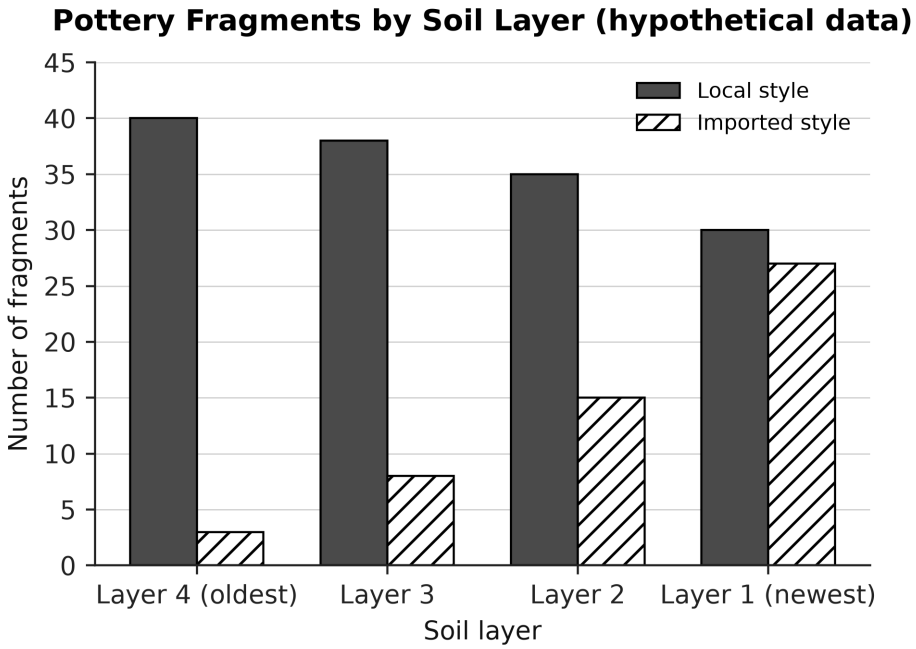
C The comet's tail was shortest when the comet was closest to the Sun.

D The comet's tail always pointed away from the Sun.

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3 RW-C0903

> At a hypothetical hilltop settlement, archaeologists sorted pottery fragments by style and by the soil layer in which they were found; deeper layers are older. A student reviewing their results claims that trade with distant regions grew over the settlement's history, since pottery made in distant regions became more common over time.



Which choice best describes data from the graph that support the student's claim?

A Local fragments outnumbered imported fragments in every layer.

B The number of imported fragments fell from 27 in the oldest layer to 3 in the newest layer.

C The number of local fragments was highest in the newest layer.

D The number of imported fragments rose from 3 in the oldest layer to 27 in the newest layer.

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4 RW-C0904

> Charles Dickens's 1854 novel *Hard Times* is set in Coketown, a fictional industrial town. The narrator suggests that Coketown's public buildings look so much alike that their designs give no clue to what each building is used for.

Which quotation from *Hard Times* most effectively illustrates the claim?

A "It had a black canal in it, and a river that ran purple with ill-smelling dye"

B "The jail might have been the infirmary, the infirmary might have been the jail, the town-hall might have been either, or both, or anything else, for anything that appeared to the contrary in the graces of their construction."

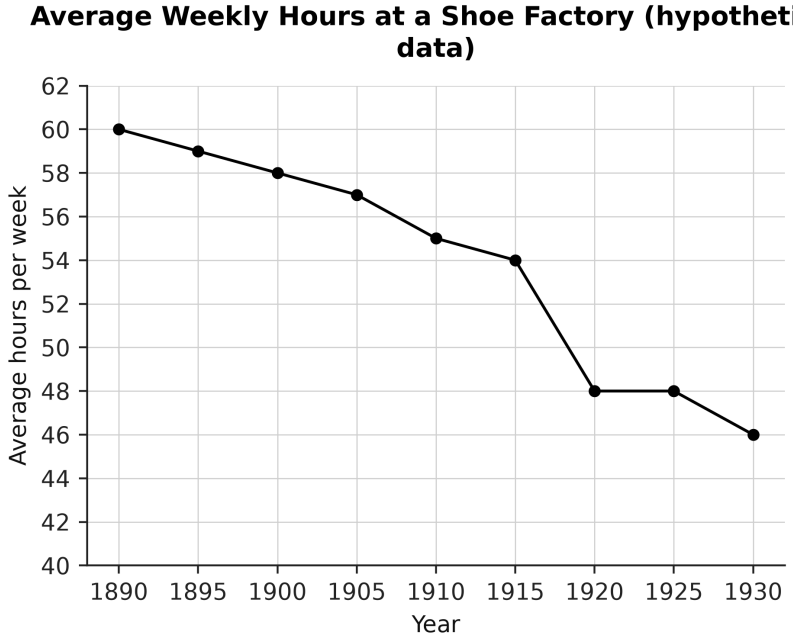
C "It was a town of machinery and tall chimneys, out of which interminable serpents of smoke trailed themselves for ever and ever, and never got uncoiled."

D "You saw nothing in Coketown but what was severely workful."

---

5 RW-C0905

> A hypothetical set of records from a shoe factory shows the average number of hours its employees worked each week from 1890 to 1930. A labor historian studying the records claims that the workweek at the factory became substantially shorter over this period.



Which choice most effectively uses data from the graph to illustrate the claim?

A The average workweek fell from 60 hours in 1890 to 46 hours in 1930.

B The average workweek rose from 46 hours in 1890 to 60 hours in 1930.

C The average workweek was 48 hours in both 1920 and 1925.

D The average workweek was longest in 1920.

---

6 RW-C0906

> Before 1880, goods reached a hypothetical prairie town by wagon over rough roads. A historian studying the records of the town's general store hypothesizes that the arrival of a railroad line in 1880 lowered the prices of goods shipped to the town from distant cities, because railroads moved freight more cheaply than wagons did.

Which finding, if true, would most directly support the historian's hypothesis?

A The town's population grew steadily during the 1870s.

B The store's prices for goods from distant cities rose in the years just after 1880.

C The store's prices for factory-made goods from distant cities fell in the years just after 1880.

D The railroad station was the largest building in the town.

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7 RW-C0907

> Young stars are often surrounded by disks of gas and dust from which planets may form. In a hypothetical survey, astronomers measured the percentage of stars with such disks in four star clusters of different ages. They concluded that most of these disks disappear within a star's first 10 million years, noting that the percentage of stars with disks was _____

Stars with Dusty Disks in Four Clusters (hypothetical data)

Cluster	Age (millions of years)	Stars with disks (%)
P	2	64
Q	5	31
R	10	12
S	30	3

Which choice most effectively uses data from the table to complete the statement?

A 12% in the 10-million-year-old cluster and 3% in the 30-million-year-old cluster.

B 64% in the 2-million-year-old cluster but only 12% in the 10-million-year-old cluster.

C 31% in the 5-million-year-old cluster.

D 64% in the 30-million-year-old cluster but only 3% in the 2-million-year-old cluster.

8 RW-C0908

> Excavations at a hypothetical coastal site uncovered a large mound made of thousands of discarded shells. One archaeologist claims that the mound grew slowly over many generations as a small group of residents ate shellfish there. Another archaeologist argues that the mound was built up quickly during a few large gatherings, when many people came together for short feasts.

Which finding, if true, would most directly weaken the first archaeologist's claim?

A The mound contains shells from several different species of shellfish.

B Thin layers of soil and ash separate many bands of shells, indicating repeated visits over a long period.

C Similar shell mounds have been found at other coastal sites.

D Radiocarbon dates from shells at the bottom and at the top of the mound are nearly identical.

9 RW-C0909

> Thomas Hardy's 1874 novel *Far from the Madding Crowd* introduces Gabriel Oak, who tends a flock of sheep on a hillside at night. In one scene, Hardy suggests that Oak values the night sky both as a practical tool and as a thing of beauty.

Which quotation from *Far from the Madding Crowd* most effectively illustrates the claim?

A "he stood still after looking at the sky as a useful instrument, and regarded it in an appreciative spirit, as a work of art superlatively beautiful."

B "he stood and carefully examined the sky, to ascertain the time of night from the altitudes of the stars."

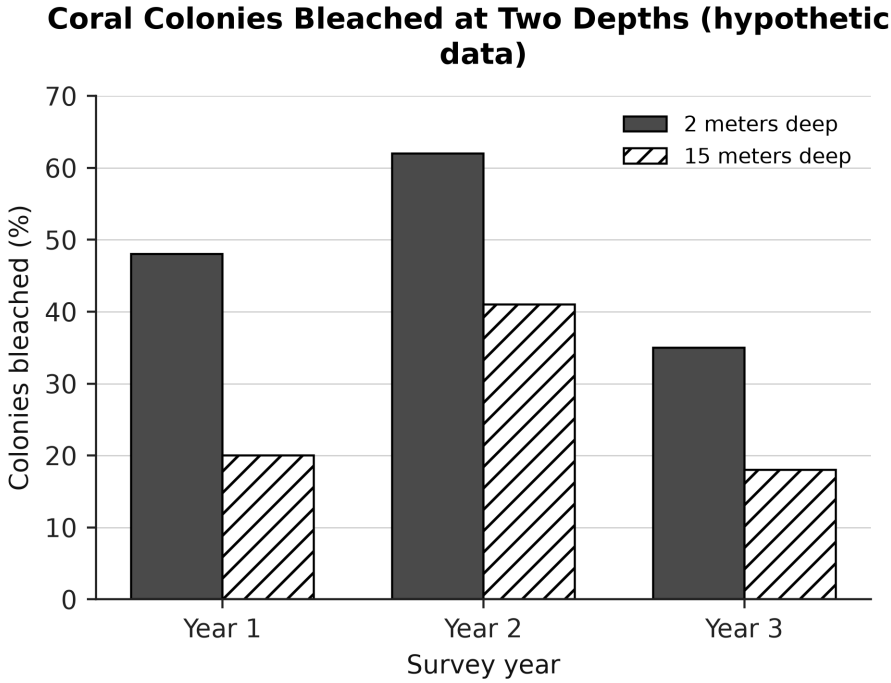
C "Castor and Pollux with their quiet shine were almost on the meridian"

D "When Farmer Oak smiled, the corners of his mouth spread till they were within an unimportant distance of his ears"

---

10 RW-C0910

> During marine heat waves, many corals expel the algae living in their tissues and turn pale, a response called bleaching. In a hypothetical survey, a team of marine biologists recorded bleaching at reef sites at two depths over three years. The team concluded that deeper water gave corals some protection, noting that _____



Which choice most effectively uses data from the graph to complete the statement?

A the percentage of colonies that bleached at 2 meters rose from 48% in Year 1 to 62% in Year 2.

B in every year, more than half of the colonies at 15 meters bleached.

C in every year, a smaller percentage of colonies bleached at 15 meters than at 2 meters.

D in every year, about the same percentage of colonies bleached at both depths.

---

11 RW-C0911

> In the late nineteenth century, some American factory owners built entire towns for their workers, including houses, stores and churches. Such towns could offer steady housing, but they also gave employers unusual power over employees' daily lives. A historian studying a hypothetical company town hypothesizes that the company used its ownership of workers' housing to discourage them from joining labor unions.

Which finding, if true, would most directly support the historian's hypothesis?

A The company owned every house in the town and set all of the rents itself.

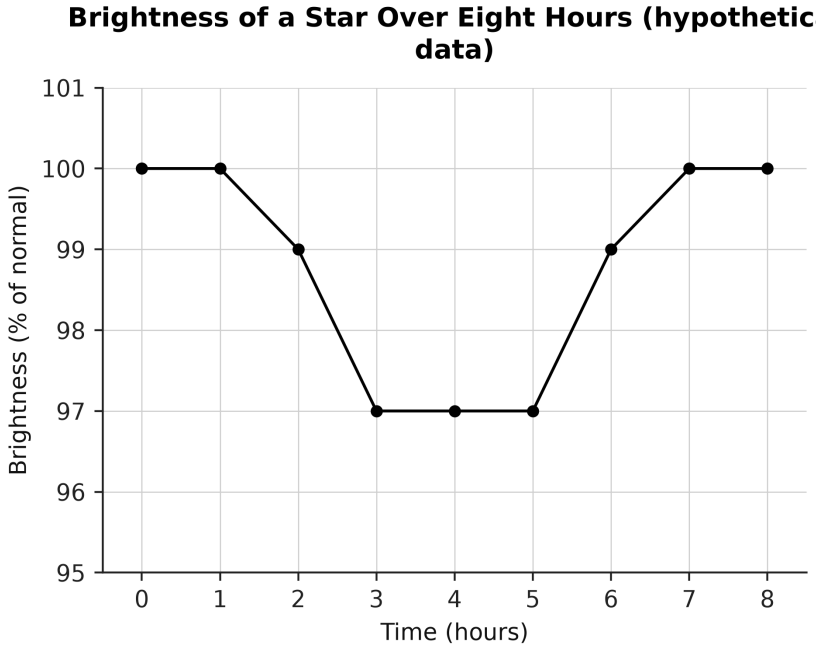
B Workers who rented company houses joined unions at higher rates than workers who lived outside the town.

C The company's houses were larger than most workers' houses in nearby cities.

D Company records show that workers who joined a union were often evicted from company-owned houses soon afterward.

12 RW-C0912

> When a planet passes between its star and Earth, it blocks a small fraction of the star's light, an event called a transit. An astronomer recorded the brightness of a hypothetical star every hour for eight hours and claims that a transit occurred, during which a planet blocked about 3 percent of the starlight.



Which choice most effectively uses data from the graph to illustrate the claim?

A The star's brightness was 100% of normal at hours 0, 1, 7 and 8.

B The star's brightness dropped from 100% of normal to 97%, stayed at 97% for several hours, and then returned to 100%.

C The star's brightness fell steadily for all eight hours, ending at 97% of normal.

D The star's brightness rose from 97% of normal to 100% and then stayed at 100%.

13 RW-C0913

> Octopuses can change the color and pattern of their skin in less than a second. A marine biologist planning a hypothetical field study proposes that one octopus species uses this ability mainly to hide from predators and only rarely to signal to other octopuses. The biologist plans to film the octopuses in the wild over several months.

Which finding, if true, would most directly support the marine biologist's hypothesis?

A The octopuses usually matched their surroundings when a predator approached but seldom changed color when another octopus came near.

B The octopuses usually matched their surroundings when a predator approached, and they also changed color often when another octopus came near.

C The octopuses seldom changed color when another octopus came near, and they also seldom changed color when a predator approached.

D The octopuses changed color more often during the day than at night.

---

14 RW-C0914

> Using hypothetical records from a fictional city, an economic historian compared carpenters' daily wages with the price of a loaf of bread from 1850 to 1880. The historian argues that carpenters were better off in 1880 than in 1850, even though bread had become more expensive. For example, _____

Carpenters' Wages and Bread Prices in a Fictional City (hypothetical data)

Year	Daily wage (cents)	Price of one loaf (cents)	Loaves one day's wage could buy
1850	150	5	30
1860	175	7	25
1870	240	8	30
1880	280	7	40

Which choice most effectively uses data from the table to complete the statement?

A a day's wage bought 30 loaves in both 1850 and 1870.

B the daily wage rose from 150 cents in 1850 to 280 cents in 1880.

C a loaf of bread cost less in 1880 than in 1850.

D a day's wage bought 40 loaves in 1880, compared with 30 in 1850, even though a loaf cost 7 cents in 1880 and only 5 cents in 1850.

---

15 RW-C0915

> The icy surface of a hypothetical moon orbiting a giant planet is crossed by long, straight cracks. A planetary scientist claims that the cracks are young features, formed in the recent past as tides from the giant planet repeatedly stretched and squeezed the moon's crust. Older surfaces in the solar system tend to be more heavily cratered, because they have been exposed to impacts for longer.

Which finding, if true, would most directly weaken the planetary scientist's claim?

A The moon's orbit is slightly oval, which makes the strength of the planet's tides change over each orbit.

B The longest cracks are found near the moon's equator.

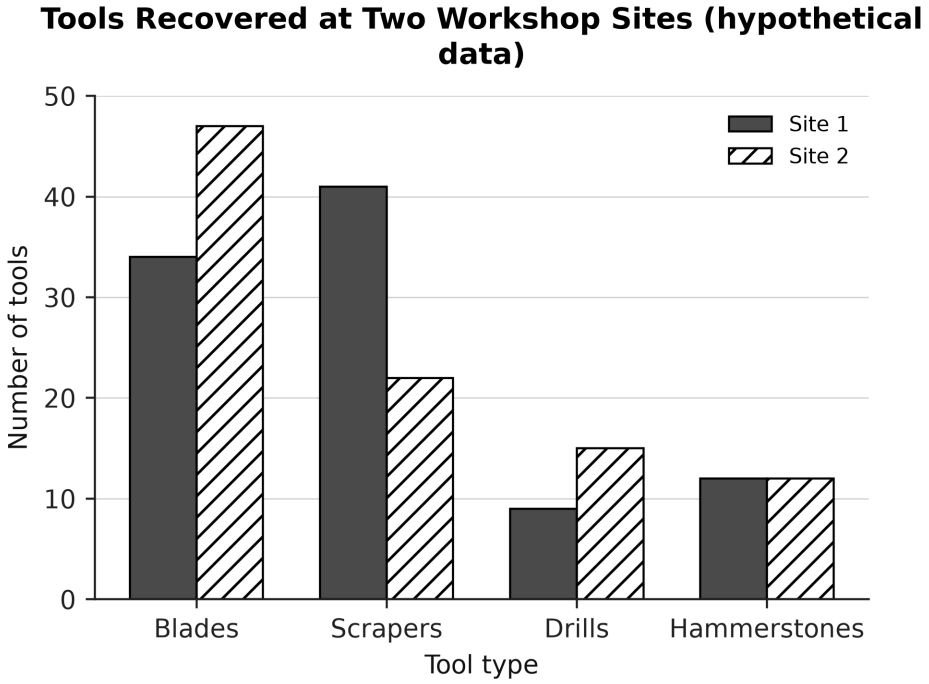
C The cracks are covered by many impact craters, about as many as the oldest parts of the moon's surface.

D Cracks on a different moon have been shown to be young.

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16 RW-C0916

> Stone tools can reveal what tasks people performed at a site. Scrapers were often used to clean animal hides, while blades were typically used for cutting. An archaeologist counted every complete tool recovered from two hypothetical workshop sites and sorted the tools by type. Based on the counts, the archaeologist hypothesizes that hide preparation was the more common of these two tasks at Site 1, whereas cutting was the more common task at Site 2.



Which choice best describes data from the graph that support the archaeologist's hypothesis?

A Site 2 had more blades (47) than Site 1 had scrapers (41).

B At Site 1, scrapers (41) outnumbered blades (34), whereas at Site 2, blades (47) outnumbered scrapers (22).

C Site 2 had more drills (15) than Site 1 had (9).

D Both sites had the same number of hammerstones (12).

---

17 RW-C0917

> In Herman Melville's 1851 novel *Moby-Dick*, the narrator, Ishmael, explains that whenever he feels gloomy, he goes to sea. He then suggests that this pull toward the ocean is not his alone: almost everyone shares it, he claims, including people who are not aware that they do.

Which quotation from *Moby-Dick* most effectively illustrates the claim?

A "It is a way I have of driving off the spleen and regulating the circulation."

B "Look at the crowds of water-gazers there."

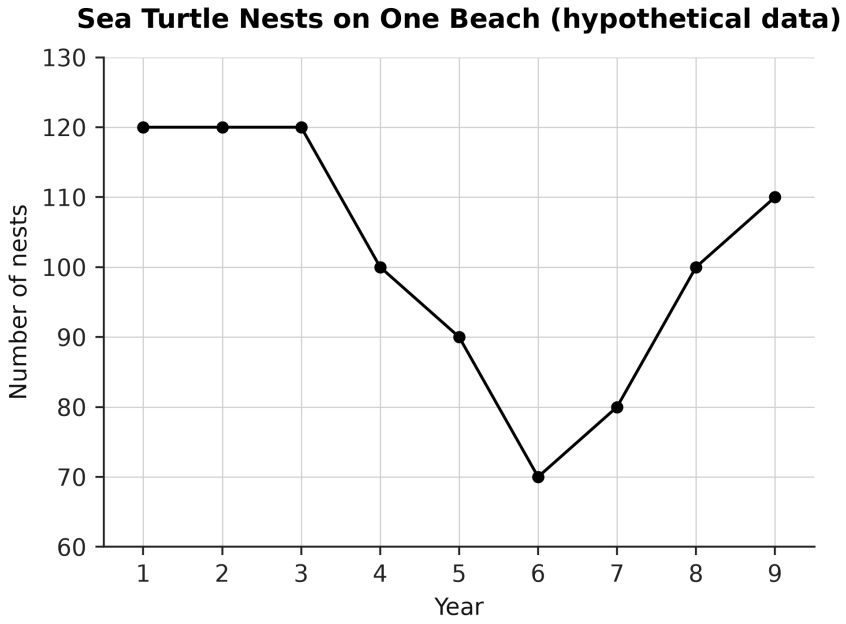
C "Yes, as every one knows, meditation and water are wedded for ever."

D "If they but knew it, almost all men in their degree, some time or other, cherish very nearly the same feelings towards the ocean with me."

---

18 RW-C0918

> Bright artificial light can discourage sea turtles from nesting on a beach. On a hypothetical beach, floodlights were installed at a new resort early in Year 4, and in Year 7 the resort replaced them with shielded fixtures that keep light off the sand. A marine biologist who tracked nesting on the beach for nine years claims that the number of turtle nests fell after the floodlights were installed and then recovered after the lights were shielded.



Which choice most effectively uses data from the graph to illustrate the claim?

A The number of nests rose from 70 in Year 6 to 110 in Year 9.

B The number of nests fell from 120 in Year 3 to 70 in Year 6 and then rose to 110 by Year 9.

C The number of nests stayed at 120 from Year 1 through Year 3.

D The number of nests fell from 120 in Year 2 to 110 in Year 9.

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19 RW-C0919

> At a hypothetical site occupied about 9,000 years ago, archaeologists uncovered dozens of grinding stones, tools used to crush seeds into flour. The team hypothesizes that the people who used the site ground mainly wild barley rather than barley they had domesticated, and that they stayed at the site only during a short harvest season each year instead of living there year-round. Wild barley has brittle seed heads that shatter easily when ripe, whereas domesticated barley has sturdy seed heads that hold their seeds until harvest.

Which finding, if true, would most directly support the team's hypothesis?

A Charred barley beside the stones has the brittle seed heads typical of wild barley, and the floors hold only thin layers of debris from short, repeated stays.

B Charred barley beside the stones has the brittle seed heads typical of wild barley, and the floors hold thick layers of debris from year-round living.

C Charred barley beside the stones has the sturdy seed heads typical of domesticated barley, and the floors hold only thin layers of debris from short stays.

D Grinding stones much like these have been found at many other sites from the same period.

---

20 RW-C0920

> Meteor showers occur when Earth passes through trails of debris, usually left by comets, and the debris in different trails strikes the atmosphere at different speeds. Using hypothetical observations of four showers, a student claims that showers with faster meteors tend to produce a greater share of especially bright meteors. The student notes, for instance, that _____

Four Meteor Showers (hypothetical data)

Shower	Peak rate (meteors per hour)	Average speed (km/s)	Bright meteors (% of total)
W	110	35	4
X	60	71	12
Y	25	58	9
Z	90	21	2

Which choice most effectively uses data from the table to complete the statement?

A Shower W, with the highest peak rate (110 meteors per hour), had a smaller share of bright meteors (4%) than Shower X did (12%).

B Shower Z had a higher peak rate (90 meteors per hour) than Shower X did (60 meteors per hour).

C Shower X, with the fastest meteors (71 km/s), had the highest share of bright meteors (12%), while Shower Z, with the slowest (21 km/s), had the lowest (2%).

D Shower Y, with faster meteors (58 km/s) than Shower X (71 km/s), had a smaller share of bright meteors.

Practice Set Explanations

1. Correct answer: C RW-C0901

Command of Evidence: Quantitative · Information and Ideas · Easier

C *higher at the seagrass meadow (46) than at either the rocky reef (31) or the sandy bottom (9).*

The claim is that seagrass meadows shelter more young fish than other habitats. The table shows an average of 46 juvenile fish per survey at the seagrass meadow, compared with 31 at the rocky reef and 9 at the sandy bottom. Choice C accurately compares the seagrass meadow with the other sites and supports its role as a nursery.

A These values are accurate, but the comparison leaves out the seagrass meadow, so it says nothing about whether seagrass shelters more young fish.

B This misreads the table. The seagrass meadow averaged 46 juvenile fish and the rocky reef 31, not the other way around.

D The averages range from 9 to 46, so they are far from equal, and equal counts would not support the nursery idea anyway.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

2. Correct answer: A RW-C0902

Command of Evidence: Textual · Information and Ideas · Easier

A *Measurements show that the comet released more gas as its surface grew warmer.*

The hypothesis links heat to gas release: as the comet warms, its ice turns to gas that carries dust into the tail. A finding that the comet released more gas as its surface grew warmer directly supports that link, which is why this choice offers the strongest support.

B The length of the comet's orbit says nothing about whether heating releases gas, so this finding is irrelevant to the hypothesis.

C This would weaken the hypothesis, since more heat near the Sun should release more gas and dust and make the tail longer, not shorter.

D The direction of the tail does not show that heat turns ice into gas, so this finding does not address the hypothesis.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

3. Correct answer: D RW-C0903

Command of Evidence: Quantitative · Information and Ideas · Easier

D *The number of imported fragments rose from 3 in the oldest layer to 27 in the newest layer.*

The claim is that imported pottery became more common over time. Because deeper layers are older, moving from Layer 4 to Layer 1 tracks time. Imported fragments rose from 3 in the oldest layer to 27 in the newest, which directly shows the increase the student describes.

A This is true, but it compares the two styles within each layer rather than showing that imported pottery became more common over time.

B This reverses the layers. The graph shows 3 imported fragments in the oldest layer and 27 in the newest layer.

C This misreads the graph: local fragments were most numerous in the oldest layer (40) and least numerous in the newest layer (30).

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

4. Correct answer: B RW-C0904

Command of Evidence: Textual · Information and Ideas · Easier

B *"The jail might have been the infirmary, the infirmary might have been the jail, the town-hall might have been either, or both, or anything else, for anything that appeared to the contrary in the graces of their construction."*

The claim has a specific focus: buildings whose designs do not reveal their uses. This quotation says the jail "might have been the infirmary" and the town hall "might have been either, or both, or anything else," judging by "their construction." It is the only choice about buildings that cannot be told apart.

A This describes Coketown's polluted water, not its buildings, so it does not show that the buildings' designs hide their purposes.

C This quotation pictures smoke pouring from chimneys. It conveys gloom and industry but says nothing about buildings that cannot be told apart.

D This stresses that everything in Coketown served work, but it does not say that different buildings look interchangeable.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

5. Correct answer: A RW-C0905

Command of Evidence: Quantitative · Information and Ideas · Easier

A *The average workweek fell from 60 hours in 1890 to 46 hours in 1930.*

The claim concerns change across the whole period: a substantially shorter workweek. The graph begins at 60 hours in 1890 and ends at 46 hours in 1930, a clear decline. The choice that compares those two endpoints illustrates the claim directly, while the others misread the graph or show no change.

B This reverses the trend. The graph shows 60 hours in 1890 and 46 hours in 1930, so the workweek shrank rather than grew.

C This is accurate, but it describes a period with no change at all, so it does not show the workweek becoming shorter.

D The graph shows 48 hours in 1920, one of the lowest values. The longest workweek, 60 hours, came in 1890.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

6. Correct answer: C RW-C0906

Command of Evidence: Textual · Information and Ideas · Easier

C *The store's prices for factory-made goods from distant cities fell in the years just after 1880.*

The hypothesis predicts that once the railroad arrived in 1880, goods shipped from distant cities became cheaper at the store. A finding that the store's prices for those goods fell soon after 1880 matches that prediction exactly, so it most directly supports the hypothesis.

A Population growth before the railroad arrived says nothing about prices after 1880, so this finding does not bear on the hypothesis.

B Rising prices after the railroad arrived would weaken the hypothesis, which predicts that prices for shipped goods fell.

D The size of the station is irrelevant to whether shipping costs or store prices changed after the railroad arrived.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

7. Correct answer: B RW-C0907

Command of Evidence: Quantitative · Information and Ideas · Medium

B *64% in the 2-million-year-old cluster but only 12% in the 10-million-year-old cluster.*

The conclusion concerns the first 10 million years. The table shows that 64% of stars in the 2-million-year-old cluster had disks, compared with only 12% in the 10-million-year-old cluster, a drop showing that most disks are gone by about 10 million years. Only this choice compares ages within that window.

A These values are accurate, but they track the decline after 10 million years; they do not show the drop from many disks to few within the first 10 million years.

C A single value shows no change over time, so it cannot illustrate disks disappearing as stars grow older.

D This reverses the table. The 2-million-year-old cluster had 64% and the 30-million-year-old cluster had 3%.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

8. Correct answer: D RW-C0908

Command of Evidence: Textual · Information and Ideas · Medium

D *Radiocarbon dates from shells at the bottom and at the top of the mound are nearly identical.*

The first archaeologist claims the mound grew slowly across many generations. If shells from the bottom and the top had nearly identical radiocarbon dates, that would suggest a shorter period of accumulation. That evidence undercuts slow growth and fits the second archaeologist's view instead.

A The variety of species says nothing about how quickly the mound formed, so this finding neither supports nor weakens either claim.

B Evidence of repeated visits over a long period would support the first archaeologist's claim of slow growth, not weaken it.

C The existence of other mounds does not show how quickly this particular mound was built, so the finding is irrelevant.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

9. Correct answer: A RW-C0909

Command of Evidence: Textual · Information and Ideas · Medium

A *"he stood still after looking at the sky as a useful instrument, and regarded it in an appreciative spirit, as a work of art superlatively beautiful."*

The claim has two parts: Oak values the sky as a practical tool and as a thing of beauty. This quotation shows both, since he looks at the sky "as a useful instrument" and then regards it "as a work of art superlatively beautiful." Each of the other choices addresses only one part of the claim or neither part.

B This shows Oak using the sky as a practical tool to tell time, but it says nothing about his appreciating its beauty, so it covers only half the claim.

C This is the narrator's description of where two stars appear. It does not reveal how Oak himself regards the sky.

D This describes Oak's smile, not his attitude toward the night sky, so it is unrelated to the claim.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

10. Correct answer: C RW-C0910

Command of Evidence: Quantitative · Information and Ideas · Medium

C *in every year, a smaller percentage of colonies bleached at 15 meters than at 2 meters.*

The conclusion is that deeper water gave corals some protection, so the data must show less bleaching at 15

meters than at 2 meters. The graph shows exactly that in Year 1 (20% versus 48%), Year 2 (41% versus 62%) and Year 3 (18% versus 35%). Choice C is the only accurate comparison of the two depths offered.

A This is accurate, but it compares two years at a single depth, so it says nothing about whether deeper corals fared better.

B The graph shows at most 41% bleaching at 15 meters, so in no year did more than half of the deeper colonies bleach.

D The bars differ widely in every year, such as 48% versus 20% in Year 1, so the two depths did not show similar bleaching.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

11. Correct answer: D RW-C0911

Command of Evidence: Textual · Information and Ideas · Medium

D *Company records show that workers who joined a union were often evicted from company-owned houses soon afterward.*

The hypothesis says the company used its ownership of housing to discourage union membership. Evictions that followed soon after workers joined a union would show the company using housing as a penalty for union activity, directly linking its control of housing to discouraging unions.

A This confirms the company controlled housing, but not that it used that control against union members, so it supports only part of the hypothesis.

B Different membership rates do not show whether the company used housing to discourage unions. Other differences between the groups could explain those rates.

C The size of the houses has no bearing on whether the company used housing to discourage union membership.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: B RW-C0912

Command of Evidence: Quantitative · Information and Ideas · Medium

B *The star's brightness dropped from 100% of normal to 97%, stayed at 97% for several hours, and then returned to 100%.*

A transit should produce a temporary dip in brightness, and the claim specifies about 3 percent. The graph shows the brightness falling from 100% to 97%, holding there from hour 3 to hour 5, and returning to 100% by hour 7. Only this choice describes the full dip and its size.

A These readings are accurate, but they show only the star's normal brightness, not the temporary dip that a transit would cause.

C The graph shows the brightness recovering to 100% by hour 7, so it did not fall steadily or end at 97%.

D This describes only the recovery at the end of the dip. Without the earlier drop, it does not show light being blocked and then restored.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: A RW-C0913

Command of Evidence: Textual · Information and Ideas · Medium

A *The octopuses usually matched their surroundings when a predator approached but seldom changed color when another octopus came near.*

The hypothesis has two parts: color change is used mainly to hide from predators, and only rarely to signal to other octopuses. A finding that the octopuses blended in when predators approached but seldom changed color near other octopuses supports both parts at once, which no other choice does.

B The first half fits the hypothesis, but frequent color changes around other octopuses would contradict the claim that signaling is rare.

C Rare signaling fits the hypothesis, but rarely changing color near predators contradicts the claim that hiding is the main use.

D Time of day does not show whether color changes serve hiding or signaling, so this finding addresses neither part of the hypothesis.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

14. Correct answer: D RW-C0914

Command of Evidence: Quantitative · Information and Ideas · Medium

D *a day's wage bought 40 loaves in 1880, compared with 30 in 1850, even though a loaf cost 7 cents in 1880 and only 5 cents in 1850.*

The historian's claim has two parts: carpenters were better off in 1880, and bread cost more then. The table shows that a day's wage bought 40 loaves in 1880 versus 30 in 1850, while a loaf rose from 5 to 7 cents. Only this choice uses data for both parts of the claim.

A This is accurate, but it shows no improvement and does not involve 1880, so it cannot support the comparison the historian makes.

B Higher wages alone do not show carpenters were better off, because this choice ignores bread prices, which also rose.

C The table shows a loaf cost 7 cents in 1880 and 5 cents in 1850, so bread cost more in 1880, not less.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

15. Correct answer: C RW-C0915

Command of Evidence: Textual · Information and Ideas · Medium

C *The cracks are covered by many impact craters, about as many as the oldest parts of the moon's surface.*

The text notes that older surfaces collect more craters because they have been exposed to impacts for longer. If the cracks carried as many craters as the moon's oldest terrain, they would likely be old, not recent. That finding directly undercuts the claim that the cracks are young features.

A Changing tidal strength would help explain how tides could crack the crust, so this finding fits the scientist's claim rather than weakening it.

B The location of the cracks says nothing about their age, so this finding does not bear on whether they formed recently.

D Evidence about another moon's cracks does not reveal the age of these cracks, so it neither weakens nor directly supports the claim.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

16. Correct answer: B RW-C0916

Command of Evidence: Quantitative · Information and Ideas · Harder

B *At Site 1, scrapers (41) outnumbered blades (34), whereas at Site 2, blades (47) outnumbered scrapers (22).*

The hypothesis compares tasks within each site: hide preparation (scrapers) was more common at Site 1, and cutting (blades) was more common at Site 2. The graph shows 41 scrapers versus 34 blades at Site 1 and 47 blades versus 22 scrapers at Site 2. Only this choice makes both within-site comparisons.

A Both numbers are accurate, but comparing one site's blades with the other site's scrapers does not show which task was more common at either site.

C This is accurate, but drills are not tied to either task in the hypothesis, so the comparison is irrelevant to the claim.

D This is accurate but irrelevant, because hammerstones are not linked to hide preparation or cutting in the text.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

17. Correct answer: D RW-C0917

Command of Evidence: Textual · Information and Ideas · Harder

D *"If they but knew it, almost all men in their degree, some time or other, cherish very nearly the same feelings towards the ocean with me."*

The claim has two parts: almost everyone shares Ishmael's pull toward the ocean, and some do so without knowing it. This quotation covers both, since "almost all men" have "very nearly the same feelings towards the ocean with me," and "If they but knew it" signals that many do not realize this.

A This explains what going to sea does for Ishmael himself. It says nothing about other people sharing his feelings toward the ocean.

B This shows that many people are drawn to the water, but it does not link their feelings to Ishmael's or suggest that they are unaware of them.

C This says everyone already knows the link between water and thought, which clashes with the claim that people may share the feeling without realizing it.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

18. Correct answer: B RW-C0918

Command of Evidence: Quantitative · Information and Ideas · Harder

B *The number of nests fell from 120 in Year 3 to 70 in Year 6 and then rose to 110 by Year 9.*

The claim has two parts: nests declined after floodlights arrived in Year 4, and they recovered after shielding in Year 7. The graph shows 120 nests in Year 3, a fall to 70 by Year 6 and a rise to 110 by Year 9. Only this choice captures both the decline and the recovery.

A This shows the recovery after shielding but not the earlier drop after the floodlights went up, so it illustrates only half of the claim.

C These values are accurate, but they describe the years before the floodlights, so they show neither the decline nor the recovery.

D Both values are accurate, but comparing those two years hides the sharp drop and rebound, so the choice does not show the claim's two changes.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

19. Correct answer: A RW-C0919

Command of Evidence: Textual · Information and Ideas · Harder

A *Charred barley beside the stones has the brittle seed heads typical of wild barley, and the floors hold only thin layers of debris from short, repeated stays.*

The hypothesis makes two claims: the barley was mainly wild, and the site was used only seasonally. Brittle seed heads point to wild barley, as the text explains, and thin debris layers from short stays point to seasonal use. Only this finding supports both parts; each of the two close choices contradicts one part.

B The barley evidence fits the hypothesis, but year-round living contradicts the claim that people stayed only during a short harvest season.

C Short stays fit the hypothesis, but domesticated barley contradicts the claim that the people ground mainly wild barley.

D Similar stones elsewhere reveal nothing about which barley was ground here or how long people stayed, so this finding addresses neither part.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

20. Correct answer: C RW-C0920

Command of Evidence: Quantitative · Information and Ideas · Harder

C *Shower X, with the fastest meteors (71 km/s), had the highest share of bright meteors (12%), while Shower Z, with the slowest (21 km/s), had the lowest (2%).*

The claim links two variables: meteor speed and the share of bright meteors. The table shows that Shower X had the fastest meteors (71 km/s) and the highest share of bright meteors (12%), while Shower Z had the slowest (21 km/s) and the lowest share (2%). Choice C accurately relates both variables; choice D also mentions both but misreads which shower is faster.

A The numbers are accurate, but the choice is built on peak rate rather than speed, so it does not address the relationship the student claims.

B This is accurate, but peak rate is not part of the claim, which concerns meteor speed and the share of bright meteors.

D This misreads the table. Shower X's meteors (71 km/s) were faster than Shower Y's (58 km/s), so the stated comparison is false.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

CHAPTER 10

Inferences

An Inferences question gives you a short passage that stops in the middle of a thought. A blank sits where the conclusion should be, and you pick the choice that most logically completes the text. These questions usually come last in the Information and Ideas group, and they can feel like the hardest in it, because the answer is not written in the passage; it has to follow from what is.

That is the whole skill: separating what the passage *supports* from what is merely possible. Some conclusions are certain; others are probable. The right answer claims exactly as much as the evidence allows.

What the question looks like

The stem is nearly always the same: "Which choice most logically completes the text?" The passage reads like a compact argument: it gives background, presents a finding or a contrast, and leads into the blank with words such as "This suggests that _____," "Therefore, _____" or "it can most reasonably be concluded that _____." Each choice is a full idea, usually a clause that finishes the final sentence.

Passages come from science, history and social studies, and the humanities, including literary criticism. Some describe a study; others state a general idea, such as how a map's purpose decides what it leaves out, and ask you to apply it.

How to recognize it

Several question types end with a blank, so check the stem and the choices.

What you see	Question type
Blank completed by a single word or short phrase; "most logical and precise word or phrase"	Words in Context (Chapter 5)
Blank completed by data from a table or graph	Command of Evidence: Quantitative (Chapter 9)
Blank at the end; choices are full ideas; "most logically completes the text"	Inferences (this chapter)
Blank between sentences filled by a linking word such as *however* or *therefore*	Transitions (Chapter 13)
Choices differ only in punctuation or grammar; "conforms to the conventions of Standard English"	Boundaries or Form, Structure, and Sense (Chapters 11 and 12)

The method

1. **Read the passage as an argument.** Label each sentence for yourself: background, evidence, contrast or limit. Use Highlights & Notes to mark the key finding and any qualifier.
2. **Read the words just before the blank.** *Therefore* asks for a conclusion; *however* asks for something that runs against what came before; *unless* or *only if* asks for a condition.
3. **Predict a conclusion the evidence supports.** Say it in your own words before you look at the choices.

4. **Test each choice:** does the evidence support this conclusion at its stated level of certainty? Eliminate guesses without support.
5. **Favor modest wording.** Correct answers often say *may*, *likely*, *at least some* or *cannot be determined*. Wrong answers often say *always*, *all*, *proves* or *the only*.
6. **Reread the completed passage.** It should read as one continuous argument, with nothing added from outside.

Supported, merely possible, goes too far

Most wrong answers fail in a few predictable ways. Take a tiny passage: "Every fishing boat in the harbor returned before the storm. The *Marlin* is one of the harbor's fishing boats. Therefore, _____"

Kind of choice	Example	Verdict
Must be true	the *Marlin* returned before the storm.	Correct: it follows from the two premises
Could be true	the *Marlin*'s crew noticed the storm clouds early.	Wrong: possible, but nothing requires it
Goes too far	every boat on the coast returned before the storm.	Wrong: claims more than the evidence covers
Repeats a premise	every fishing boat in the harbor returned before the storm.	Wrong: restates rather than concludes
Contradicts	the *Marlin* was still at sea when the storm arrived.	Wrong: the premises rule it out

This example gives a certain conclusion. Test passages also support qualified conclusions, such as "probably" or "may." Choose the strongest supported answer; do not demand a formal proof or rely on an unsupported possibility.

Common logical patterns

- **Trade-off:** a resource spent on one thing is not available for another.
- **Ruling out alternatives:** when the passage eliminates the usual explanations, the remaining one becomes likely.
- **Applying a principle:** the passage states a general rule and a case; the blank applies the rule.
- **Broken expectation:** X normally produces Y, but here Y did not happen, so something else must be at work.
- **Two causes tangled together:** two possible causes appear together, so the evidence cannot say which one mattered.

Worked examples

Worked example 1: The sea stars (trade-off)

Many sea stars can regrow an arm they have lost. In a hypothetical aquarium study, sea stars that were regrowing an arm ate more food than sea stars with all their arms, yet over the same months they grew less in overall body size. These results suggest that _____

Which choice most logically completes the text?

- A) sea stars with all their arms eat less because they are rarely hungry.
- B) regrowing an arm uses energy that would otherwise go toward overall growth.
- C) sea stars cannot survive the loss of more than one arm.
- D) many sea stars can regrow an arm they have lost.

Thinking. The regrowing sea stars took in more food but grew less overall. The extra energy went somewhere, and the passage offers one candidate: the new arm. **Answer: B.**

Why the traps fail. A invents a reason (hunger) the study never measured. C goes far beyond a study of animals regrowing a single arm. D repeats the opening premise rather than concluding anything.

Worked example 2: The bakery's bad year (ruling out alternatives)

In a hypothetical mill town, a bakery's sales fell by a third in 1893. The bakery had not raised its prices or changed what it sold, and no rival bakery had opened nearby. That year the town's largest employer, a woolen mill, closed and laid off most of its workers. These facts suggest that _____

Which choice most logically completes the text?

- A) the mill closed because the bakery's sales had fallen.
- B) the laid-off mill workers all moved away from the town.
- C) the drop in the bakery's sales was probably connected to the closing of the mill.
- D) the bakery would have doubled its sales if the mill had stayed open.

Thinking. The second sentence rules out three common reasons for falling sales: prices, products and competition. The third supplies a likely remaining cause: many customers suddenly lost their wages. **Answer: C.** Notice the modest word *probably*.

Why the traps fail. A reverses cause and effect; nothing suggests a bakery could close a mill. B could be true but is not required; workers who stayed and had less money would explain the drop just as well. D attaches an exact figure the passage gives no basis for.

Worked example 3: What a map leaves out (applying a principle)

A map that showed every tree, stone and blade of grass would be as large as the land itself and useless for finding one's way. Mapmakers must therefore choose what to leave out, and their choices depend on the map's purpose: a sailor's chart marks depths and hazards but ignores roads, while a road map does the reverse. It follows that a map should not be judged inaccurate simply because it omits a great deal; instead, _____

Which choice most logically completes the text?

- A) the best maps are the ones that leave out the fewest details.
- B) sailors' charts are more useful than road maps.
- C) mapmakers should show every feature that a traveler might need.
- D) it should be judged by whether its omissions suit the purpose it was made for.

Thinking. The passage states a principle (every map must omit, and purpose decides what) and applies it to judging maps. The word *instead* asks for the standard that replaces "how much it omits." **Answer: D.**

Why the traps fail. A contradicts the passage's central point. B compares two kinds of maps the passage treats as equally reasonable for different purposes. C slides back toward the impossible map of the first sentence.

Worked example 4: The dockworkers (harder: a broken expectation)

When employers compete for a limited number of workers, wages would normally be expected to rise. In a hypothetical port city during the 1870s, ship owners repeatedly complained that they could not hire enough dockworkers, and ships sometimes waited days to be unloaded. Yet dockworkers' daily wages stayed almost unchanged throughout the decade. If these records are accurate, _____

Which choice most logically completes the text?

- A) the ship owners must have exaggerated the shortage of dockworkers.
- B) dockworkers must have preferred steady wages to higher ones.
- C) something other than the number of available workers was keeping wages from rising.
- D) dockworkers' wages must have risen sharply during the 1880s.

Thinking. The first sentence sets an expectation: shortage leads to higher wages. The records show a shortage and no rise, so if the shortage was real, some other force held wages down. The passage does not name that force, so the answer should not either. **Answer: C.**

Why the traps fail. A ignores the condition "if these records are accurate," which asks you to accept the shortage as real. B names one specific cause the passage never mentions; it could be true but is not required. D jumps to a decade the passage does not discuss.

Worked example 5: Parrotfish and seaweed (harder: two causes tangled together)

On coral reefs, fast-growing seaweed can crowd out young corals. A hypothetical survey found that reefs with many parrotfish, which graze on seaweed, had less seaweed and more young corals than reefs with few parrotfish. However, every reef with many parrotfish lay inside a protected area where fishing is banned, and the ban also protects other grazing fish and spares corals from damage by nets and anchors. Based on this survey alone, _____

Which choice most logically completes the text?

- A) parrotfish are the main reason that young corals survive on protected reefs.
- B) banning fishing has no effect on the amount of seaweed on a reef.
- C) reefs outside protected areas have no parrotfish at all.
- D) it is not possible to tell how much of the difference between the reefs is due to parrotfish rather than to other effects of the fishing ban.

Thinking. Parrotfish and the fishing ban appear together on the same reefs, so the survey cannot separate their effects. "Based on this survey alone" signals that the answer is about the limits of the evidence. **Answer: D.**

Why the traps fail. A is exactly the conclusion the survey cannot support. B contradicts the passage, which describes ways the ban could reduce seaweed. C turns "few parrotfish" into none.

Common traps

Trap	What it looks like	How to spot it
Merely possible	A sensible idea without evidence in the passage	Ask what evidence supports it and how strongly
Goes too far	A conclusion broader or more certain than the evidence	Look for *all*, *always*, *proves*, *the only* or exact figures
Repeats a premise	A choice that restates the passage	A conclusion must add something that follows
Reversed cause	Effect presented as cause	Check which event the passage says came first
Ignored signal word	A choice that fits *therefore* when the blank follows *however*	Reread the words just before the blank
Broken condition	A choice that rejects a stated assumption	Treat qualifiers such as "if these records are accurate" as rules, however reasonable the choice sounds
Outside knowledge	True in the world but not in the passage	Use only what the passage gives you

Harder-module variations

- **Longer chains of reasoning,** with three or four premises that must be combined.
- **Qualifiers that limit the conclusion,** such as "based on this survey alone."
- **Correct answers about what cannot be known,** as in Worked example 5.

- **Abstract humanities passages** that state a principle in general terms and apply it to a case.
- **Choices that are all plausible,** separated only by how much they claim.
- **Negatives and conditions,** such as *unless*, *only if* and *not necessarily*, that reverse the logic if you misread them.

Inferences questions reward slowing down at the blank. If two choices remain, keep the one you could defend using nothing but the passage.

Check yourself

1. What single question should you ask of every choice?
2. Why is a "could be true" choice wrong even when it is likely in real life?
3. What do the words just before the blank tell you?
4. A passage says "based on this study alone." What kind of answer should you expect?
5. Name two words that often signal a choice goes too far.
6. **Answers:** (1) Does the evidence support this conclusion at its stated level of certainty? (2) Mere possibility is insufficient without supporting evidence. (3) The kind of statement the blank needs, such as a conclusion, a contrast or a condition. (4) An answer about the limits of the evidence. (5) Any two of *all*, *always*, *proves* and *the only*.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/inferences

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C1001

> Barnacles spend their adult lives cemented to one spot, so they cannot move toward food. They feed by sweeping feathery legs through the water to catch tiny drifting organisms. On a hypothetical stretch of shoreline, a marine biologist observed that barnacles on rocks facing strong currents grew larger than barnacles in a sheltered cove nearby. Because currents carry more drifting food past a fixed spot, the biologist concluded that _____

Which choice most logically completes the text?

A the barnacles in the cove moved to calmer water to find more food.

B the barnacles facing strong currents probably had more food available to them than the sheltered barnacles did.

C strong currents make it harder for barnacles to catch any food at all.

D a barnacle's size has no connection to where it lives.

---

2 RW-C1002

> In the early 1800s, many households in a hypothetical New England village spun yarn and wove cloth at home to sell to local merchants. In 1825 a large water-powered mill opened nearby and began producing cloth faster and more cheaply than home weavers could. Merchants' account books from the village show that their purchases of homemade cloth dropped sharply within a few years. These records suggest that _____

Which choice most logically completes the text?

A the mill bought most of its yarn from village households.

B homemade cloth became more valuable after the mill opened.

C many village households lost income they had earned by selling cloth.

D merchants in the village stopped keeping account books after 1825.

---

3 RW-C1003

> Astronomers can estimate a star's surface temperature from its color. The hottest stars look blue-white, stars like the Sun look yellow-white, and cooler stars look orange or red. In a hypothetical star catalog, Star A is listed as orange and Star B as blue-white. Judging only by the two stars' colors, an astronomer would conclude that _____

Which choice most logically completes the text?

A Star B has a hotter surface than Star A does.

B Star A is closer to Earth than Star B is.

C Star A has a hotter surface than Star B does.

D the two stars have about the same surface temperature.

---

4 RW-C1004

> Philosophers sometimes distinguish a discovery from an invention. A discovery reveals something that already existed before anyone knew about it, such as a moon circling a distant planet. An invention brings into being something that did not exist until someone made it, such as a new kind of telescope. By this distinction, an astronomer who first observes a comet that has been orbiting the Sun for millions of years has made _____

Which choice most logically completes the text?

A an invention, since no one had ever seen the comet before.

B an invention, since a telescope was needed to observe the comet.

C neither a discovery nor an invention, since comets cannot be observed directly.

D a discovery, since the comet existed long before it was observed.

---

5 RW-C1005

> Beads made from the shell of a particular ocean snail have turned up at a hypothetical inland site hundreds of kilometers from the nearest coast. Excavators found many finished beads there but none of the shell scraps, broken drills or half-made beads that bead-making usually leaves behind. Since the snail lives only in the ocean, and since the beads apparently were not made at the site, the beads most likely _____

Which choice most logically completes the text?

A were made at the site from shells gathered in nearby rivers.

B were left by people who lived at the site for only a short time.

C reached the site already finished, perhaps through exchange with people living nearer the coast.

D were valued less than other objects found at the site.

---

6 RW-C1006

> Many deep-sea animals produce their own light, a trait called bioluminescence. Some deep-sea shrimp, when attacked, release a glowing cloud into the water. In a hypothetical laboratory test, predatory fish that encountered this cloud in a dark tank often struck at the glowing cloud while the shrimp swam off. In the dark deep sea, where a sudden flash of light is easy to see, the glowing cloud likely helps the shrimp _____

Which choice most logically completes the text?

A escape by drawing a predator’s attention away from its body.

B attract prey by lighting up the water around it.

C see predators that are approaching in the dark.

D hide by making the water around it completely dark.

---

7 RW-C1007

> In chapter 32 of Herman Melville's 1851 novel *Moby-Dick*, the narrator, Ishmael, sets out to classify whales. He notes that the naturalist Linnaeus had separated whales from fish, and he lists the features that set whales apart, including their lungs and warm blood. Ishmael nonetheless insists that the whale is a fish, defining it as "a spouting fish with a horizontal tail." Because Ishmael knows the evidence Linnaeus relied on and sets it aside anyway, readers can conclude that _____

Which choice most logically completes the text?

A Melville himself had never read the work of Linnaeus.

B Ishmael's classification reflects a deliberate choice rather than ignorance of Linnaeus's reasons.

C Linnaeus classified animals mainly by the shape of their tails.

D naturalists of Melville's time had abandoned Linnaeus's view of whales.

---

8 RW-C1008

> In a hypothetical mining town in the 1880s, the mining company paid part of each worker's wages in scrip, paper notes that could be spent only at the company's own store. Letters written by workers in the town complain that prices at the company store were much higher than prices in a neighboring town. Workers paid largely in scrip, however, had little choice about where to shop. Taken together, these details suggest that the scrip system _____

Which choice most logically completes the text?

A led most workers to do their shopping in the neighboring town.

B kept prices at the company store lower than prices elsewhere.

C was adopted because workers preferred paper notes to cash.

D let the company take back part of the wages it paid by charging high prices at its store.

---

9 RW-C1009

> An asteroid is visible because it reflects sunlight. If its surface reflects light evenly, a roughly round asteroid looks about equally bright as it turns, whereas an elongated one can look brighter when its broad side faces Earth. Observers tracking a hypothetical asteroid with an evenly reflecting surface found that its brightness rose and fell by a large amount in a regular cycle as it rotated. These observations suggest that the asteroid _____

Which choice most logically completes the text?

A is elongated rather than roughly round.

B produces its own light, which brightens and fades.

C is roughly round, since it is rotating.

D is moving steadily closer to Earth.

---

10 RW-C1010

> Coins can help archaeologists date a building, but only in one direction. A coin sealed beneath a building's original floor must have been dropped before the floor was laid, so the floor cannot be older than the coin. At a hypothetical Roman-era site, the most recently minted coin found sealed beneath a bathhouse floor dates to the year 160. Based on this evidence alone, archaeologists can conclude that the floor was laid _____

Which choice most logically completes the text?

A in 160 or at some earlier date.
B in exactly 160, the year the coin was minted.
C in 160 or at some later date.
D shortly before the bathhouse was abandoned.

11 RW-C1011

> Many fish use a gas-filled organ called a swim bladder to hold their position in the water without constant swimming. Sharks have no swim bladder, but many have large livers filled with oil, which is less dense than seawater. In a hypothetical study, marine biologists found that among several shark species, those with proportionally larger livers needed to swim less to keep from sinking. This finding suggests that a shark's oily liver _____

Which choice most logically completes the text?

A makes the shark denser than the surrounding seawater.
B allows the shark to breathe air at the water's surface.
C works only in sharks that also have a swim bladder.
D helps keep the shark from sinking, partly doing the job that a swim bladder does in other fish.

12 RW-C1012

> In a preface written in 1895, Thomas Hardy explained that his 1882 novel *Two on a Tower* grew from a wish to set "the emotional history of two infinitesimal lives against the stupendous background of the stellar universe." One of those lives belongs to a young man who studies the stars from an old tower. Yet Hardy also recalled that when the novel first appeared, readers seemed "less struck" with these aims than with their own opinions: that the book's morals were improper and that it was meant as a satire on England's established church. Hardy's account therefore suggests that the novel's early reception _____

Which choice most logically completes the text?

A showed that readers had rejected Hardy's plan to contrast human lives with the universe.
B focused on matters other than the purpose that Hardy said had guided him in writing it.
C matched the purpose Hardy described, since readers noticed the novel's astronomical setting.
D persuaded Hardy to remove the young astronomer from later editions.

13 RW-C1013

> A planet's temperature depends partly on how much starlight it absorbs. A planet covered by bright clouds reflects much of the light that reaches it and, all else being equal, stays cooler than a darker planet. Two hypothetical planets, P and Q, orbit the same star at the same distance. Measurements show that Planet P reflects far more of the star's light than Planet Q does, yet the two planets have nearly the same average surface temperature. If these measurements are accurate, then _____

Which choice most logically completes the text?

A some factor besides the amount of starlight absorbed must also be affecting at least one planet's temperature.

B Planet P must have a thicker atmosphere that traps more heat than Planet Q's atmosphere does.

C Planet P must orbit closer to the star than Planet Q does.

D the amount of starlight a planet reflects has no effect on its temperature.

---

14 RW-C1014

> Historians studying a hypothetical set of nineteenth-century labor disputes noticed that strikes held during busy seasons, when employers had large orders to fill, ended in workers' favor more often than strikes held during slow seasons. One explanation is that halting production costs an employer the most when orders are waiting. The same records show, however, that workers struck during busy seasons mainly in industries where their unions were already strong. Because strong unions and busy-season timing so often went together, the records by themselves _____

Which choice most logically completes the text?

A prove that busy-season timing was the main reason those strikes succeeded.

B show that unions were weakest in industries with busy seasons.

C suggest that employers preferred strikes to happen during busy seasons.

D cannot show whether timing or union strength better explains why those strikes succeeded more often.

---

15 RW-C1015

> Suppose a skilled modern potter makes a copy of an ancient vase so exact that even experts cannot tell the two apart by looking. Some philosophers of art argue that the original would still be more valuable, because part of what people prize in an ancient object is its history: it was shaped by a particular person, in a particular place, thousands of years ago. On this view, the copy lacks something that no amount of visual accuracy could supply. A person holding this view would most likely say that a museum visitor who unknowingly admired the copy _____

Which choice most logically completes the text?

A had the same experience as a visitor who admired the original.

B would have valued the copy more if it had been less accurate.

C appreciated its appearance but missed the connection to the past that only the original can offer.

D proved that the original vase has no special value.

Practice Set Explanations

1. Correct answer: B RW-C1001

Inferences · Information and Ideas · Easier

B *the barnacles facing strong currents probably had more food available to them than the sheltered barnacles did.*

The text gives two premises: barnacles cannot move toward food, and currents carry more food past a fixed spot. Combined with the observation that barnacles facing strong currents grew larger, the logical conclusion is that those barnacles probably had more food available than the sheltered ones did.

A The text says adult barnacles are "cemented to one spot," so they cannot move anywhere, which makes this conclusion impossible.

C The text says currents carry more food past a fixed spot, so this choice reverses the reasoning that leads to the blank.

D The observation links size to location, since barnacles facing currents grew larger, so this choice contradicts the evidence.

Review: Chapter 10, Inferences (page 150)

2. Correct answer: C RW-C1002

Inferences · Information and Ideas · Easier

C *many village households lost income they had earned by selling cloth.*

Households earned money by selling homemade cloth to merchants. After the mill began making cheaper cloth, merchants bought much less of it. If merchants bought less cloth, the households that sold it earned less, so the logical completion is that many households lost income from cloth sales.

A Nothing in the text describes the mill buying yarn. The records concern merchants buying cloth, so this conclusion is unsupported.

B Merchants bought much less homemade cloth after the mill opened, which points to falling demand, not rising value.

D The account books themselves show the drop in purchases after 1825, so the merchants clearly kept records after that year.

Review: Chapter 10, Inferences (page 150)

3. Correct answer: A RW-C1003

Inferences · Information and Ideas · Easier

A *Star B has a hotter surface than Star A does.*

The text states that blue-white stars are the hottest and that orange or red stars are cooler. Star B is blue-white and Star A is orange, so judging by color alone, an astronomer would conclude that Star B has the hotter surface.

B In the text, color indicates surface temperature, not distance, so nothing about the stars' colors shows which one is closer.

C This reverses the pattern: blue-white stars are the hottest and orange stars are cooler, so Star B is the hotter of the two.

D The stars have different colors, orange and blue-white, which the text links to quite different surface temperatures.

Review: Chapter 10, Inferences (page 150)

4. Correct answer: D RW-C1004

Inferences · Information and Ideas · Easier

D *a discovery, since the comet existed long before it was observed.*

A discovery, as the text defines it, reveals something that "already existed before anyone knew about it." The comet has been orbiting for millions of years, so it existed long before the astronomer saw it. The observation is therefore a discovery, not an invention.

A Being unseen is not the same as not existing. By the text's definitions, revealing something that already existed is a discovery.

B The text offers a telescope as an example of an invention, but the comet is the thing found, and it already existed.

C The text describes the astronomer observing the comet, so the claim that comets cannot be observed contradicts the passage.

Review: Chapter 10, Inferences (page 150)

5. Correct answer: C RW-C1005

Inferences · Information and Ideas · Medium

C *reached the site already finished, perhaps through exchange with people living nearer the coast.*

Two premises lead to the blank: the shells could not have come from near the site, and the beads were not made there. If the beads were neither collected nor made locally, they most likely arrived as finished objects from elsewhere, for example through exchange with communities nearer the coast.

A The text says the snail lives only in the ocean and that no bead-making debris was found, so both parts of this choice are contradicted.

B The text says nothing about how long people stayed at the site, so this conclusion does not follow from the evidence.

D The text never compares the beads with other objects, so it gives no basis for judging their relative value.

Review: Chapter 10, Inferences (page 150)

6. Correct answer: A RW-C1006

Inferences · Information and Ideas · Medium

A *escape by drawing a predator's attention away from its body.*

When attacked, the shrimp release a glowing cloud, and in the test the fish "struck at the glowing cloud while the shrimp swam off." Because a flash is easy to see in the dark, the cloud most likely protects the shrimp by drawing the predator's strike away from its body.

B The test involved shrimp under attack, and the fish struck at the cloud. Nothing in the text suggests the cloud is used to catch prey.

C The cloud is released after an attack begins, and the test shows fish striking at it, not shrimp using its light to spot threats.

D The cloud glows, so it brightens the water rather than darkening it. This choice contradicts the text.

Review: Chapter 10, Inferences (page 150)

7. Correct answer: B RW-C1007

Inferences · Information and Ideas · Medium

B *Ishmael's classification reflects a deliberate choice rather than ignorance of Linnaeus's reasons.*

The text stresses that Ishmael knows the features Linnaeus cited, such as lungs and warm blood, yet he "sets it aside anyway." Someone who knows the evidence and still rejects it is making a choice, not acting from ignorance. That is the conclusion the passage's "Because" clause sets up.

A The text shows Ishmael discussing Linnaeus's reasoning in detail, so nothing supports the claim that Melville was unfamiliar with that work.

C Tail shape belongs to Ishmael's own definition. The features the text connects with Linnaeus's view are lungs and warm blood, not tail shape.

D The text describes only Ishmael's response to Linnaeus and gives no information about what naturalists of Melville's time believed.

Review: Chapter 10, Inferences (page 150)

8. Correct answer: D RW-C1008

Inferences · Information and Ideas · Medium

D *let the company take back part of the wages it paid by charging high prices at its store.*

Scrip could be spent only at the company store, that store charged much more than stores elsewhere, and workers had little choice. Wages paid in scrip therefore flowed back to the company at inflated prices, so the system let the company recover part of what it paid out.

A Workers paid in scrip "had little choice about where to shop," and scrip worked only at the company store, so they could not easily shop elsewhere.

B The workers' letters say the company store's prices were much higher than those in the neighboring town, so this choice contradicts the text.

C The text gives no reason for adopting scrip and says nothing about what workers preferred; their complaints suggest they did not favor the system.

Review: Chapter 10, Inferences (page 150)

9. Correct answer: A RW-C1009

Inferences · Information and Ideas · Medium

A *is elongated rather than roughly round.*

The passage rules out uneven surface reflectivity as the cause of the brightness changes. Under that condition, large changes during rotation fit an elongated shape, which presents alternately broader and narrower sides to Earth. The evidence therefore supports A.

B The passage attributes the visible brightness to reflected sunlight, not to light produced by the asteroid itself.

C With an evenly reflecting surface, a roughly round asteroid would stay about equally bright as it turned, unlike the observed large changes.

D A steady approach would not produce a repeating rise and fall. The text links regular brightness changes to shape, not to distance.

Review: Chapter 10, Inferences (page 150)

10. Correct answer: C RW-C1010

Inferences · Information and Ideas · Medium

C *in 160 or at some later date.*

The text explains that a coin sealed beneath a floor was there before the floor was laid, so "the floor cannot be older than the coin." With the newest coin dated 160, the floor was laid in 160 or later. The evidence sets only an earliest possible date, not an exact year.

A This reverses the logic. A coin sealed beneath the floor was there before the floor, so the floor cannot predate the coin's year, 160.

B The coin could have circulated for years before it was dropped, so the evidence sets an earliest possible date, not an exact one.

D The text gives no information about when the bathhouse was abandoned, so this conclusion goes beyond the evidence.

Review: Chapter 10, Inferences (page 150)

11. Correct answer: D RW-C1011

Inferences · Information and Ideas · Medium

D *helps keep the shark from sinking, partly doing the job that a swim bladder does in other fish.*

Oil is less dense than seawater, and sharks with proportionally larger oily livers needed to swim less to stay up. Since sharks lack swim bladders, the oily liver most likely helps keep them from sinking, filling part of the role a swim bladder plays in other fish.

A Oil is less dense than seawater, so a larger oily liver would make a shark less dense, not denser. This choice reverses the text.

B The text discusses the liver only in connection with density and sinking. Nothing suggests that it helps sharks breathe air.

C The text states that sharks have no swim bladder, so the liver's role cannot depend on one.

Review: Chapter 10, Inferences (page 150)

12. Correct answer: B RW-C1012

Inferences · Information and Ideas · Harder

B *focused on matters other than the purpose that Hardy said had guided him in writing it.*

Hardy states his aim, setting two lives against "the stellar universe," and then recalls that early readers were "less struck" with it than with their views about the book's morals and its treatment of the church. The logical conclusion is that early responses centered on other concerns, without implying that readers considered and rejected his aim.

A Hardy says readers were "less struck" with his aims, meaning they paid them less attention, not that they weighed his plan and rejected it.

C Hardy reports that readers attended to morals and religion rather than to his stated aims, so their response did not match his purpose.

D The text says nothing about Hardy revising the novel, and the 1895 preface still describes the story's two lives, so this is unsupported.

Review: Chapter 10, Inferences (page 150)

13. Correct answer: A RW-C1013

Inferences · Information and Ideas · Harder

A *some factor besides the amount of starlight absorbed must also be affecting at least one planet's temperature.*

All else being equal, the more reflective Planet P should be cooler. Because the two planets have nearly the same temperature, all else must not be equal: some other factor is influencing at least one of them. The text supports that general conclusion but does not identify any specific cause.

B A heat-trapping atmosphere is one possible explanation, but the text gives no evidence about atmospheres, so this specific cause does not have to be true.

C The text states that the two planets orbit the same star at the same distance, so this choice contradicts the passage.

D This overreaches. Reflection matters "all else being equal"; the equal temperatures show only that something else differs between the planets.

Review: Chapter 10, Inferences (page 150)

14. Correct answer: D RW-C1014

Inferences · Information and Ideas · Harder

D *cannot show whether timing or union strength better explains why those strikes succeeded more often.*

Busy-season strikes succeeded more often, but those strikes also tended to occur where unions were strong. When two possible causes appear together, records alone cannot reveal which one, or both, made the difference. The completion must acknowledge that limit rather than credit either cause.

A Because timing and union strength appeared together, the records cannot separate their effects, so "prove" claims far more than the evidence allows.

B The records tie busy-season strikes to industries where unions were already strong, so nothing in them suggests that unions were weakest in industries with busy seasons.

C The text implies that busy-season strikes cost employers the most, so nothing suggests employers preferred them.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: C RW-C1015

Inferences · Information and Ideas · Harder

C *appreciated its appearance but missed the connection to the past that only the original can offer.*

The view locates part of an ancient object's value in its history, which "no amount of visual accuracy could supply." A visitor admiring the copy enjoys how it looks, the one thing it shares with the original, but misses the historical connection that only the original carries.

A The view holds that the copy lacks the original's history, so admiring the copy cannot offer the same experience as admiring the original.

B The view concerns history, not accuracy. Making the copy less exact would not give it the history it lacks.

D This contradicts the view, which holds that the original keeps a special value that a visually identical copy cannot match.

Review: Chapter 10, Inferences (page 150)

CHAPTER 11

Boundaries

Boundaries questions test the punctuation at the edges of sentences, clauses and phrases. The answer choices usually differ in punctuation. A focused set of rules handles most of these questions, so Boundaries is one of the most learnable skills on the test.

Boundaries belongs to the Standard English Conventions domain, together with Form, Structure, and Sense (Chapter 12). The two skills make up about 26 percent of the operational Reading and Writing questions, roughly 11 to 15 questions per section.

What the question looks like

Each question gives a short passage with a blank and this stem: "Which choice completes the text so that it conforms to the conventions of Standard English?" A typical set of choices looks like this:

- A) layer, it
- B) layer. It
- C) layer it
- D) layer? It

The only questions are which mark belongs at the blank, whether any mark belongs there, and sometimes where the mark goes. In each module, the conventions questions come after the Information and Ideas questions and before Expression of Ideas. Boundaries and Form, Structure, and Sense questions are mixed together and arranged from easiest to hardest.

How to recognize it

Chapter 12 uses the same stem, so read the four choices before you read the passage closely.

- If the words are identical and only the punctuation changes (perhaps with one capital letter after a period), it is a Boundaries question.
- If the choices focus on verb forms, pronouns or word order, it is usually a Form, Structure, and Sense question (Chapter 12).
- If the choices are different linking words, such as "however," "therefore" and "for example," all punctuated the same way, it is a Transitions question (Chapter 13).

Key terms

You need only a few grammar terms.

Term	What it means	Example
Independent clause	A subject plus a finite verb (a verb with a tense) that can stand alone as a sentence	The river flooded.

Term	What it means	Example
Dependent clause	A clause that begins with a word such as because, although, when, which or that, so it cannot stand alone	because the river flooded
Phrase	A group of words without its own subject-verb pair	after the heavy rain
Coordinating conjunction	One of the seven FANBOYS: for, and, nor, but, or, yet, so	…, but the bridge held.
Conjunctive adverb	A linking word such as however, therefore, consequently, instead or for example; it cannot join two clauses by itself	…; however, the bridge held.
Supplement	Extra information that the sentence could lose without breaking	The bridge, built of stone, held.

Two more ideas matter for supplements. An **appositive** is a noun phrase that renames the noun next to it ("Ana Ruiz, a chemist, …"). A **restrictive** (essential) element tells you *which* person or thing is meant, so it takes no commas: "Students who finished early left." A **nonrestrictive** (nonessential) element adds detail about something already identified, so it is set off: "My brother, who finished early, left."

The boundary rules

Situation	Correct punctuation	Common wrong choice
Two independent clauses, no conjunction	Period or semicolon	A comma alone (comma splice) or nothing (fused sentence)
Two independent clauses joined by FANBOYS	Usually a comma before the conjunction	An unnecessary comma separating the conjunction from its clause
Conjunctive adverb opening the second independent clause	Semicolon or period between the clauses; comma after the introductory adverb	Only a comma between the clauses (comma splice)
Dependent clause, then main clause	Comma after the dependent clause	Semicolon, colon or period (fragment)
List, example or explanation after a complete clause	Colon	Colon after an incomplete clause ("include:")
Supplement in the middle of a sentence	A matching pair: two commas, two dashes or two parentheses	Mixed marks; one mark missing
Restrictive (essential) element	No punctuation	Commas around it
Simple series	Commas between items	Semicolons or colons between one-word items
Series whose items contain commas	Semicolons between items	Commas, which blur where items end
Subject and verb; verb and object	No punctuation	One comma, dash or colon between them
Direct question	Question mark	Period

Situation	Correct punctuation	Common wrong choice
Reported (indirect) question	Period	Question mark

- **Period and semicolon do the same job** between two independent clauses. If both appear as choices at the same spot, they cannot both be right, so check whether each side of the blank really stands alone.
- **The words before a colon must form a complete sentence.** What follows can be a list, a phrase or a full clause that explains, expands on or emphatically contrasts with the first. A conjunction after the colon is not automatically wrong.
- **A comma before FANBOYS is the usual pattern.** Long clauses with internal commas may instead use a semicolon before the conjunction. Judge the full structure rather than rejecting a mark solely because a conjunction follows it.
- **Conjunctive adverbs can move.** Compare "The dam held; however, the road flooded" with "The dam held; the road flooded, however." The semicolon stays at the clause boundary. Commas set off a parenthetical adverb; they do not replace that boundary.
- **A single dash** can introduce an explanation at the end of a sentence, much like a colon. In the middle of a sentence, dashes work in pairs.
- **A reported question** ("She asked whether the dam would hold.") is a statement about a question, so it ends with a period. Only a direct question ("Will the dam hold?") takes a question mark.

The method

1. **Confirm the type.** Same words and different marks mean Boundaries.
2. **Read the whole sentence,** and the next one if the blank sits near a sentence break.
3. **Bracket each side of the blank.** Label it: independent clause, dependent clause, phrase, or the end of a supplement.
4. **Look back for an opening mark.** If a comma, dash or parenthesis opened a supplement earlier, the blank probably closes it with the same mark.
5. **Apply the rule from the table.** Eliminate every choice that creates a comma splice, a fused sentence, a fragment, a mismatched pair or a mark between a subject and its verb.
6. **Read your answer back** into the full sentence before you move on.

TIP Use the period test. Imagine a period at the blank and read each side. If both sides can stand alone, common options are a period, a semicolon, or a comma plus FANBOYS. A colon can introduce an explanation or emphatic development. If either side cannot stand alone, a period is wrong, and so is a semicolon unless it separates series items that contain commas.

Worked examples

Worked example 1: Two complete sentences

The lighthouse in the fictional harbor town of Gull Rock still uses its original glass _____ bends the light from a single lamp into a beam that sailors can see far out at sea.

- A) lens, it
- B) lens it
- C) lens. It
- D) lens, it,

Think: Bracket the two sides. "The lighthouse ... still uses its original glass lens" can stand alone. "It bends the light from a single lamp into a beam" can stand alone too. Two independent clauses with no conjunction need a period or a semicolon.

Answer: C.

Why the others fail: A is a comma splice. B fuses the two sentences with no mark at all. D keeps the splice and adds a comma between the subject "it" and its verb "bends."

Worked example 2: A FANBOYS conjunction

Volunteers at a fictional community garden planted tomatoes in the raised _____ planted peppers along the sunny south fence.

- A) beds and, they
- B) beds, and they
- C) beds, and, they
- D) beds and: they

Think: "Volunteers ... planted tomatoes in the raised beds" is an independent clause, and so is "they planted peppers along the sunny south fence." In this sentence, a comma before "and" correctly joins the clauses; no extra mark is needed after the conjunction.

Answer: B.

Why the others fail: A puts an unnecessary comma after "and." C adds the same error after the correct clause boundary. D puts a colon after "and," leaving the conjunction stranded before the clause it introduces.

TRAP When "and" joins two predicates with the same subject, a comma is normally unnecessary. In "The volunteers planted tomatoes and watered the peppers," the second predicate shares the subject "volunteers," so no comma is needed. A list of three or more items is different: a comma before the final "and" is acceptable there.

Worked example 3: A conjunctive adverb

The steel cables of a fictional suspension bridge lengthen slightly on hot _____ built the deck with joints that let it shift a few inches without cracking.

- A) days, therefore, engineers
- B) days; therefore, engineers
- C) days therefore engineers
- D) days; therefore; engineers

Think: Both sides are independent clauses. "Therefore" introduces the response to the cables' lengthening: engineers provided movable joints. Here the adverb opens the second clause. A semicolon separates the clauses, and a comma follows the introductory adverb.

Answer: B.

Why the others fail: A is a comma splice; commas around "therefore" do not join two clauses. C runs everything together. D isolates "therefore" between two semicolons, but the adverb alone is not an independent clause.

Worked example 4: Matching the opening mark

The fictional town's oldest fire engine—a red pumper built in the 1920s and restored at least four _____ still leads the parade every summer.

- A) times,
- B) times—
- C) times
- D) times;

Think: A dash after "engine" opened a supplement, so a second dash must close it and reconnect the subject with its verb, "still leads."

Answer: B.

Why the others fail: A mixes a dash with a comma. C never closes the supplement, so it runs into the verb. D puts a semicolon between the subject and its verb.

Worked example 5: When no punctuation is correct

Students who complete the fictional science museum's two-hour workshop on early _____ build a working crystal radio to take home.

- A) radios,
- B) radios
- C) radios—
- D) radios;

Think: The subject is "Students who complete … early radios," and the verb is "build." The "who" clause is restrictive: it tells *which* students build a radio, so it takes no commas. Nothing goes between a subject and its verb.

Answer: B.

Why the others fail: A places one comma between the subject and the verb. C does the same with a single dash. D uses a semicolon, which needs an independent clause on each side.

Worked example 6: Harder-module example (check every mark)

Early trials of a fictional company's solar water pump went well. The pump ran for six hours a day in full _____ stopped within minutes whenever clouds passed overhead, so engineers added a small battery.

- A) sun; however, it,
- B) sun; however, it
- C) sun, however, it
- D) sun however, it

Think: "The pump ran for six hours a day in full sun" and "it stopped within minutes whenever clouds passed overhead" are both independent clauses. "However" marks the contrast between them. A and B both supply a semicolon at the clause boundary and a comma after the introductory adverb, so check the rest of each choice too.

Answer: B.

Why the others fail: A adds a comma between the subject "it" and its verb "stopped." C leaves a comma splice between the two independent clauses. D also fails to supply a valid clause boundary.

TIP A choice can start with the right punctuation and still end with an error. Read every mark back into the completed sentence.

Common traps

Trap	How to spot it
Comma splice	Both sides of a lone comma could stand as sentences. Apply the period test.
Comma splice beside a conjunctive adverb	Two independent clauses joined only by a comma, even if an adverb is set off correctly. Put the sentence boundary between the clauses.
Unnecessary comma after FANBOYS	A comma cuts "and" or "but" off from its clause without introducing a parenthetical. In the ordinary pattern, the comma belongs before the conjunction.
Mismatched pair	A supplement opened with a dash, comma or parenthesis. Scan back, find the opener and match it.
Colon after an incomplete clause	"include:", "such as:", "consists of:". The words before a colon must form a full sentence.
Semicolon before a phrase	One side of the semicolon cannot stand alone.
The pause comma	A comma where you would pause in speech, often after a long subject. Long subjects still take no comma before the verb.
Commas around essential information	Removing the words would change which person or thing is meant.

Trap	How to spot it
Question mark after a reported question	"asked whether," "wondered if," "wanted to know how." Reported questions end with a period.

Harder-module variations

- **Long supplements with commas inside.** The closing mark must match the opening mark, not the nearest comma inside the supplement.
- **Two boundaries in one blank.** A choice may need a period between two clauses *and* a pair of commas around "however" inside the new sentence. Check every mark in a choice, not just the first one.
- **Meaning still matters.** Check which event a time phrase describes and which ideas a transition connects. Adverbs such as "however" can occupy several positions within a clause; their location alone does not make a choice wrong.
- **Lists inside lists.** When series items contain commas, semicolons separate the items, and every separator in that series must match.
- **Essential names.** "The chemist Ana Ruiz" takes no commas when the name tells you which chemist. "My only sister, Ana, ..." takes a pair because "my only sister" already identifies her.

CHECK On harder questions, read well beyond the blank in both directions. The deciding clue, such as an opening dash or a contrast, may sit several words away.

Check yourself

1. Which marks can join two independent clauses without a conjunction?
2. When "but" joins two independent clauses, where does the comma go?
3. When "however" opens the second of two independent clauses, what punctuation usually comes before and after it?
4. A supplement opens with a dash. What must close it?
5. Name two places where no punctuation belongs, even in a long sentence.
6. **Answers:** (1) A period or a semicolon; a colon can introduce an explanation or emphatic development. (2) Before "but" in the ordinary comma-plus-conjunction pattern. (3) A semicolon or period at the clause boundary and a comma after the introductory "however." (4) A second dash. (5) Between a subject and its verb, between a verb and its object, and around restrictive (essential) clauses or names.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/boundaries

Practice Set

20 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C1101

> Visitors to the fictional Copper Ridge Trail pass a cliff striped with red and gray bands of rock. Each band is a layer of mud that hardened at the bottom of an ancient lake. The red layers formed when the lake was shallow and the mud was exposed to _____ gray layers formed when the water was deep.

Which choice completes the text so that it conforms to the conventions of Standard English?

A air? The **B** air. The **C** air, the **D** air the

---

2 RW-C1102

> The head baker at Juniper Loaf, a fictional bakery, tested two ways of proofing bread dough, the stage when shaped dough rests and rises before it goes into the oven. Dough proofed in a warm room rose in about an _____ dough proofed overnight in a refrigerator developed a deeper, slightly sour flavor.

Which choice completes the text so that it conforms to the conventions of Standard English?

A hour but, **B** hour but: **C** hour, but, **D** hour, but

---

3 RW-C1103

> Streetcar drivers in the early 1900s often had to open their windows to see during rain or snow. Mary Anderson patented a solution in 1903. Her device, a lever inside the car that moved a spring-loaded arm with a rubber _____ let the driver clear the windshield from inside the vehicle.

Which choice completes the text so that it conforms to the conventions of Standard English?

A blade, **B** blade **C** blade; **D** blade—

---

4 RW-C1104

> Students in a fictional cooking class baked three batches of the same cookie recipe, using butter in the first batch, oil in the second, and shortening in the third. A student's notes record how the batches differed in _____ texture, and color after ten minutes in the same oven at the same temperature.

Which choice completes the text so that it conforms to the conventions of Standard English?

A height; **B** height: **C** height, **D** height

---

5 RW-C1105

> Engineering students at a fictional college entered a contest to build the lightest bridge that could hold a 20-kilogram load for one minute. Every team received the same budget for materials. The bridge made of thin balsa strips and glued _____ held the load easily and weighed less than a paperback book.

Which choice completes the text so that it conforms to the conventions of Standard English?

A joints, **B** joints; **C** joints— **D** joints

---

6 RW-C1106

> A hypothetical study followed twenty soccer players through a week of summer training. Practices lasted two hours, and temperatures often passed 30 degrees Celsius. Because the players lost water through sweat during each _____ the coaches weighed them before and after practice to estimate how much fluid each player needed to drink.

Which choice completes the text so that it conforms to the conventions of Standard English?

A session; **B** session, **C** session. **D** session:

7 RW-C1107

> Seismographs at a fictional observatory in the Andes recorded two earthquakes on the same morning. The first, a small tremor, struck just before _____ a second, much stronger earthquake shook the region and rattled windows in towns more than 100 kilometers away. After comparing the two records, the observatory's staff concluded that the small tremor had been a foreshock, a smaller quake that comes before a larger one in the same area.

Which choice completes the text so that it conforms to the conventions of Standard English?

A sunrise; twenty minutes later,
B sunrise, twenty minutes later,
C sunrise twenty minutes later,
D sunrise, twenty minutes later;

8 RW-C1108

> A sports scientist in a hypothetical experiment timed sprinters over 40 meters after two kinds of warm-up, testing each runner on two separate days so that fatigue would not affect the results. After a warm-up of easy jogging, the runners' times were close to their usual _____ after a warm-up that added three short, fast sprints, most runners finished about a tenth of a second sooner. The scientist plans to repeat the test with longer races.

Which choice completes the text so that it conforms to the conventions of Standard English?

A results, however, **B** results however, **C** results; however, **D** results; however;

9 RW-C1109

> In 1942, the Hollywood film actress Hedy Lamarr and composer George Antheil received a patent for a frequency-hopping communication system. The system was designed to make radio-controlled torpedoes harder to detect or jam. Their idea—switching a radio signal among many frequencies at split-second _____ made the transmission sound like mere noise to anyone listening in. A receiver that hopped frequencies in step with the sender, however, picked up the signal clearly.

Which choice completes the text so that it conforms to the conventions of Standard English?

A intervals— **B** intervals, **C** intervals **D** intervals;

10 RW-C1110

> Engineers in a fictional port city planned a new seawall to protect the harbor from storm waves, which had damaged several docks the previous winter. Before construction began, the team wanted to know whether a curved wall would turn waves back more effectively than a straight _____ built two scale models and tested both in a long tank, sending the same series of waves at each and filming the splashes.

Which choice completes the text so that it conforms to the conventions of Standard English?

A one? They **B** one, they **C** one they **D** one. They

11 RW-C1111

> A student in a hypothetical kitchen experiment cut an apple into slices, brushed half of them with lemon juice, and left the rest plain to see which would brown more slowly. The slices sat side by side on one white plate at room temperature. After an hour, the student _____ the treated slices had stayed noticeably paler than the untreated ones, which had turned a deep tan around their edges.

Which choice completes the text so that it conforms to the conventions of Standard English?

A observed, that **B** observed that **C** observed: that **D** observed; that

12 RW-C1112

> A hypothetical survey asked 300 high school cross-country runners what helped them most during the final mile of a race, when fatigue is usually greatest. Answers ranged from a favorite song to a sip of water, but one response appeared more often than any _____ teammate running close enough to talk to. Coaches who read the results began pairing runners of similar speed during long practice runs, hoping the habit would carry over to meets.

Which choice completes the text so that it conforms to the conventions of Standard English?

A other; a **B** other. A **C** other: a **D** other a

13 RW-C1113

> Danish seismologist Inge Lehmann spent much of her career managing seismograph stations in Denmark and Greenland. In 1936, she published a paper describing a three-shelled model of Earth's interior. The model, which placed a distinct inner core inside Earth's outer _____ helped explain unexpectedly strong seismic waves that stations in two Russian cities had recorded after a large earthquake in New Zealand. Later observations with more sensitive instruments strengthened the evidence for an inner core.

Which choice completes the text so that it conforms to the conventions of Standard English?

A core; **B** core, **C** core **D** core—

14 RW-C1114

> A fictional dance band of the 1920s recorded its songs by playing into a large metal horn connected to a needle that cut grooves into a spinning wax disc. The band could not adjust the volume of any single instrument once a recording was made. Quiet instruments barely registered on the _____ stood closest to the horn while the loud brass players stood near the back of the small room.

Which choice completes the text so that it conforms to the conventions of Standard English?

A disc, so the violinist **B** disc so, the violinist **C** disc, so the violinist, **D** disc, so, the violinist

15 RW-C1115

> Twelve amateur cyclists in a hypothetical training study rode the same set of routes three times a week for two months. Each rider wore a heart-rate monitor (a thin sensor strap fastened around the _____ during every ride. The study's coaches then compared each rider's average heart rate on flat routes with the same rider's average on hilly routes of equal length. The hilly routes produced higher averages for every rider.

Which choice completes the text so that it conforms to the conventions of Standard English?

A chest; **B** chest **C** chest). **D** chest)

16 RW-C1116

> The fictional Delmar Conservatory traces its history to 1887, when a group of church musicians began offering evening lessons in a rented hall above a hardware store. The school's most prized possession—a hand-built pipe organ whose 1,200 pipes, some taller than a two-story house and others no longer than a pencil, were shipped in crates from three different _____ still plays at every spring commencement. Students who restored the organ decades later found the builders' pencil notes inside several of its wooden pipes, including the date each pipe was tuned.

Which choice completes the text so that it conforms to the conventions of Standard English?

A workshops, **B** workshops; **C** workshops— **D** workshops

17 RW-C1117

> On June 15, 1933, the Chicago Symphony Orchestra performed Symphony No. 1 in E Minor, a work that had won an award in a Chicago composing competition the year before. With that performance, the Arkansas-born _____ Price became the first African American woman composer to have a symphony premiered by a major American orchestra. Price had studied organ and piano teaching at the New England Conservatory of Music in Boston before settling in Chicago with her family in 1927. Over a career spanning three decades, she wrote more than 300 musical compositions.

Which choice completes the text so that it conforms to the conventions of Standard English?

A composer Florence **B** composer, Florence **C** composer: Florence **D** composer—Florence

18 RW-C1118

> Engineers designing a footbridge for a fictional riverside park chose a lightweight aluminum deck to keep construction costs low and to avoid renting a large crane. The deck panels were built in a nearby workshop and trucked to the site. The lighter deck saved money and could be lifted into place in a single _____ also vibrated noticeably when groups of people walked across it in step, as crowds often did during weekend festivals. To solve the problem, the team installed dampers, devices that absorb motion, beneath the walkway, and the shaking dropped sharply.

Which choice completes the text so that it conforms to the conventions of Standard English?

A day; however, it, **B** day; however, it **C** day, however, it **D** day however, it

19 RW-C1119

> Geologists mapping a fictional valley in Montana found a boulder of pink granite, about the size of a small car, resting on a plain of gray shale, miles from any granite bedrock. The boulder's minerals pointed to a distant _____ remained a puzzle until the team noticed long, parallel scratches in the bedrock nearby, all pointing in the same direction. Such scratches typically form when a glacier drags embedded stones across the ground, so the boulder had most likely been carried there by ice. Geologists call such out-of-place boulders glacial erratics.

Which choice completes the text so that it conforms to the conventions of Standard English?

A source, how it got there, however,
B source; how it got there however,
C source how it got there, however,
D source. How it got there, however,

20 RW-C1120

> For a hypothetical taste test at a culinary school, a food scientist served panelists three versions of the same tomato soup, each seasoned differently: the first, with salt _____ second, with salt and vinegar; and the third, with salt, vinegar, and a pinch of sugar. The soups were served at the same temperature in identical white bowls, and the panelists did not know how each one had been seasoned. Most panelists rated the third version the most balanced, describing its sourness as noticeable but not sharp. A few preferred the plainer first version.

Which choice completes the text so that it conforms to the conventions of Standard English?

A alone, the **B** alone: the **C** alone; the **D** alone the

Practice Set Explanations

1. Correct answer: B RW-C1101

Boundaries · Standard English Conventions · Easier

B *air. The*

Both sides of the blank are independent clauses: "The red layers formed when the lake was shallow and the mud was exposed to air" and "The gray layers formed when the water was deep." Two independent clauses with no joining word must be separated, and a period does that job. Choice B ends the first statement and starts a new sentence with "The."

A A question mark ends a direct question, but the first clause is a statement, so it needs a period instead.

C A comma alone cannot join two independent clauses; this choice creates a comma splice.

D With no punctuation at all, the two independent clauses run together into a fused sentence.

Review: Chapter 11, Boundaries (page 164)

2. Correct answer: D RW-C1102

Boundaries · Standard English Conventions · Easier

D *hour, but*

The blank joins two independent clauses: "Dough proofed in a warm room rose in about an hour" and "dough proofed overnight in a refrigerator developed a deeper, slightly sour flavor." Choice D supplies the ordinary comma-before-"but" pattern. No parenthetical or other structure calls for a mark after "but" here.

A The comma belongs before "but," not after it; a comma after the conjunction cuts "but" off from the clause it introduces.

B The colon follows "but," stranding the conjunction before the clause it introduces. The words before this colon do not form a complete sentence.

C The comma before "but" is correct, yet the extra comma after "but" wrongly separates the conjunction from its clause.

Review: Chapter 11, Boundaries (page 164)

3. Correct answer: A RW-C1103

Boundaries · Standard English Conventions · Easier

A *blade,*

The phrase "a lever inside the car that moved a spring-loaded arm with a rubber blade" is an appositive: it renames "Her device" and adds detail the sentence could do without. A comma opens the appositive after "device," so a matching comma must close it before the verb "let." Choice A completes the pair.

B Without a closing comma, the appositive runs straight into the verb "let," and the opening comma is left unmatched.

C A semicolon cannot close a supplement opened by a comma, and it would cut the subject "Her device" off from its verb.

D A supplement opened with a comma must close with a comma; pairing a comma with a dash mixes two kinds of marks.

Review: Chapter 11, Boundaries (page 164)

4. Correct answer: C RW-C1104

Boundaries · Standard English Conventions · Easier

C *height,*

The blank falls inside a series of three items: "height," "texture," and "color." Items in a simple series are separated by commas, as the comma after "texture" already shows. Choice C supplies the comma that separates the first item from the second.

A Semicolons separate series items only when the items themselves contain commas; these one-word items need ordinary commas.

B A colon may introduce a list but never splits one; here it would cut the first item off from the other two.

D Without a comma, "height texture" runs two separate items together, so the reader cannot tell where the first item ends.

Review: Chapter 11, Boundaries (page 164)

5. Correct answer: D RW-C1105

Boundaries · Standard English Conventions · Easier

D *joints*

The subject of the sentence is the long phrase "The bridge made of thin balsa strips and glued joints," and its verb is "held." No punctuation belongs between a subject and its verb, even when the subject is long, so D is correct. The phrase "made of … joints" identifies which bridge, so it is not set off.

A A single comma separates the subject from its verb "held"; commas could set off the description only as a pair, and none opens it.

B A semicolon must join two independent clauses, but the words before the blank are only the subject of the sentence.

C A single dash splits the subject from its verb; a dash could set off the description only as one of a matching pair.

Review: Chapter 11, Boundaries (page 164)

6. Correct answer: B RW-C1106

Boundaries · Standard English Conventions · Easier

B *session,*

The sentence opens with a dependent clause, "Because the players lost water through sweat during each session," which cannot stand alone. When a dependent clause comes before the main clause, a comma marks where it ends. Choice B places that comma right before the independent clause "the coaches weighed them before and after practice."

A A semicolon needs an independent clause on each side, but the "Because" clause before the blank cannot stand alone.

C A period would leave "Because the players lost water through sweat during each session" as a fragment, since a "because" clause needs a main clause.

D A colon must follow a complete independent clause; the "Because" clause before the blank is dependent.

Review: Chapter 11, Boundaries (page 164)

7. Correct answer: A RW-C1107

Boundaries · Standard English Conventions · Medium

A *sunrise; twenty minutes later,*

"The first, a small tremor, struck just before sunrise" is an independent clause, and so is "twenty minutes later, a second, much stronger earthquake shook the region." A semicolon can join two independent clauses. Because "twenty minutes later" tells when the second quake happened, it belongs after the semicolon, at the start of the second clause, as in A.

B A comma cannot join two independent clauses; placing commas on both sides of "twenty minutes later" still leaves a comma splice.

C With no mark after "sunrise," the two independent clauses run together, and "twenty minutes later" floats between them.

D The semicolon is misplaced: it puts "twenty minutes later" in the first clause, so the timing seems to describe the first tremor instead of the second quake.

Review: Chapter 11, Boundaries (page 164)

8. Correct answer: C RW-C1108

Boundaries · Standard English Conventions · Medium

C *results; however,*

Both sides of the blank are independent clauses, and "however" marks the contrast between ordinary times after jogging and faster times after sprints. Choice C places a semicolon at the clause boundary and a comma after "however," which opens the second clause in this version of the sentence.

A Commas on both sides of "however" still leave two independent clauses joined only by a comma, which is a comma splice.

B With no punctuation before "however," the two independent clauses run together into a fused sentence.

D The two semicolons isolate "however," but the adverb alone is not an independent clause. The second semicolon incorrectly divides the introductory adverb from its clause.

Review: Chapter 11, Boundaries (page 164)

9. Correct answer: A RW-C1109

Boundaries · Standard English Conventions · Medium

A *intervals—*

The words "switching a radio signal among many frequencies at split-second intervals" are a supplement that explains "Their idea." The supplement opens with a dash after "idea," so it must close with a matching dash before the verb "made." Choice A completes the pair and reconnects the subject "Their idea" to its verb.

B A supplement opened with a dash must close with a dash; a comma here creates a mismatched pair.

C Without a closing mark, the supplement runs into the verb "made," and the opening dash is left unmatched.

D A semicolon cannot close a dash supplement, and it would separate the subject "Their idea" from its verb "made."

Review: Chapter 11, Boundaries (page 164)

10. Correct answer: D RW-C1110

Boundaries · Standard English Conventions · Medium

D *one. They*

The first sentence reports what the team wanted to know: "whether a curved wall would turn waves back more effectively than a straight one." That is an indirect question, a statement about a question, so it ends with a period rather than a question mark. The period also correctly separates the two independent clauses, so D is correct.

A An indirect question that begins with "whether" is part of a statement, so it takes a period, not a question mark.

B A comma alone cannot join two independent clauses; this choice creates a comma splice.

C With no punctuation, the two independent clauses run together into a fused sentence.

Review: Chapter 11, Boundaries (page 164)

11. Correct answer: B RW-C1111

Boundaries · Standard English Conventions · Medium

B *observed that*

The verb "observed" takes the clause "that the treated slices had stayed noticeably paler than the untreated ones" as its object, the thing that was observed. No punctuation belongs between a verb and its object, and

that includes an object clause beginning with "that." Choice B keeps the verb and its object together.

A A comma between "observed" and its "that" clause separates the verb from the very thing the student observed.

C A colon cannot separate a verb from its object; the "that" clause is what the student observed, so it must follow "observed" directly.

D A semicolon must join two independent clauses, but "that the treated slices had stayed … paler" is a dependent clause.

Review: Chapter 11, Boundaries (page 164)

12. Correct answer: C RW-C1112

Boundaries · Standard English Conventions · Medium

C *other: a*

The clause before the blank, "one response appeared more often than any other," is complete, and the words after it name that response. A colon can follow an independent clause to introduce an explanation, example or list, and what follows a colon does not need to be a full sentence. Choice C correctly introduces "a teammate running close enough to talk to."

A A semicolon must join two independent clauses, but "a teammate running close enough to talk to" is a noun phrase with no main verb.

B A period would leave "A teammate running close enough to talk to" as a fragment, since the phrase has no main verb.

D Without punctuation, the explanation runs straight into the clause, producing the ungrammatical "any other a teammate."

Review: Chapter 11, Boundaries (page 164)

13. Correct answer: B RW-C1113

Boundaries · Standard English Conventions · Medium

B *core,*

The clause "which placed a distinct inner core inside Earth's outer core" adds nonessential information about "The model." A nonrestrictive clause is set off by a matching pair of marks. A comma opens this one after "model," so a comma must close it before the verb "helped." Choice B completes the pair.

A A semicolon cannot close a clause opened by a comma, and it would cut the subject "The model" off from its verb "helped."

C Without a closing comma, the "which" clause runs into the main verb "helped," and the opening comma is left unmatched.

D A nonessential clause that opens with a comma must close with a comma; a dash here creates a mismatched pair.

Review: Chapter 11, Boundaries (page 164)

14. Correct answer: A RW-C1114

Boundaries · Standard English Conventions · Medium

A *disc, so the violinist*

The blank joins two independent clauses: "Quiet instruments barely registered on the disc" and "the violinist stood closest to the horn." When a coordinating conjunction such as "so" links two independent clauses, a comma goes before the conjunction. No other comma belongs in this stretch of the sentence, so A is correct.

B The comma belongs before "so," not after it; a comma after the conjunction cuts "so" off from the clause it introduces.

C The comma before "so" is right, but the comma after "violinist" wrongly separates the subject from its verb "stood."

D The comma before "so" is right, but the extra comma after "so" separates the conjunction from its clause for no reason.

Review: Chapter 11, Boundaries (page 164)

15. Correct answer: D RW-C1115

Boundaries · Standard English Conventions · Medium

D *chest)*

The parenthetical "(a thin sensor strap fastened around the chest)" describes the monitor. Parentheses always come in pairs, so the supplement must close with ")" after "chest." The sentence then continues without a break, "during every ride," so no other mark belongs after the parenthesis. Choice D does both.

A A semicolon cannot close a parenthesis; this choice leaves the parenthesis open and cuts off "during every ride."

B Without a closing parenthesis the supplement never ends, so "during every ride" seems to describe the chest instead of when the monitor was worn.

C The period ends the sentence too early, leaving "during every ride" stranded as a fragment.

Review: Chapter 11, Boundaries (page 164)

16. Correct answer: C RW-C1116

Boundaries · Standard English Conventions · Harder

C *workshops—*

The supplement that renames "The school's most prized possession" opens with a dash after "possession." It contains its own pair of commas around "some taller than a two-story house and others no longer than a pencil,"

but that pair is already complete. The dash supplement still needs a closing dash before the verb "still plays," so C is correct.

A A comma is tempting because the supplement already contains commas, but a supplement opened by a dash must close with a dash.

B A semicolon cannot close a dash supplement, and it would separate the subject "The school's most prized possession" from its verb "still plays."

D Without a closing dash, the long supplement runs into the verb "still plays," leaving the opening dash unmatched.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: A RW-C1117

Boundaries · Standard English Conventions · Harder

A *composer Florence*

In "the Arkansas-born composer Florence Price," the name is essential: it identifies which composer the sentence means, so it is not set off by punctuation. The whole phrase is the subject of the verb "became," and no single mark may divide a subject. Choice A correctly leaves the phrase unpunctuated.

B A single comma splits the subject; even a nonessential name would need a second comma after "Price," and there is none.

C A colon must follow a complete independent clause, but "the Arkansas-born composer" is only part of the subject.

D A single dash divides the subject in two; dashes set off a supplement only as a pair, and the name is essential here.

Review: Chapter 11, Boundaries (page 164)

18. Correct answer: B RW-C1118

Boundaries · Standard English Conventions · Harder

B *day; however, it*

Each side of the blank is an independent clause. "However" signals the contrast between the deck's advantages (it saved money and went up in one day) and its problem (it vibrated). Choice B correctly separates the clauses with a semicolon and sets off the introductory "however" with a comma. It also keeps the subject "it" attached to its verb "vibrated."

A The semicolon and the comma after "however" are correct, but the final comma separates the subject "it" from its verb "vibrated." No parenthetical justifies that comma.

C Commas on both sides of "however" leave two independent clauses joined by only a comma, creating a comma splice.

D With no punctuation before "however," the two independent clauses run together into a fused sentence.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: D RW-C1119

Boundaries · Standard English Conventions · Harder

D *source. How it got there, however,*

"The boulder's minerals pointed to a distant source" is an independent clause, and so is "How it got there, however, remained a puzzle." Two independent clauses need a period or semicolon between them. Inside the new sentence, "however" interrupts the subject and verb, so it needs a comma on each side. Only D handles both boundaries.

A Commas cannot join two independent clauses; even with "however" set off correctly, this choice is a comma splice.

B The semicolon correctly separates the clauses, but "however" lacks its opening comma, so the lone comma after it splits the subject from "remained."

C With no punctuation after "source," the two independent clauses run together into a fused sentence.

Review: Chapter 11, Boundaries (page 164)

20. Correct answer: C RW-C1120

Boundaries · Standard English Conventions · Harder

C *alone; the*

The colon introduces a series of three items, and each item contains its own comma ("the first, with salt alone"). When series items contain commas, semicolons separate the items so readers can see where each one ends, as the semicolon before "and the third" shows. Choice C supplies the matching semicolon between the first and second items.

A A comma blurs the boundary between items because commas already appear inside each item; the series needs semicolons, as before "and the third."

B A colon would signal that what follows explains "the first, with salt alone," but the next words are simply the second item in the list.

D With no punctuation, "salt alone the second" runs two items together, so the reader cannot tell where the first item ends.

Review: Chapter 11, Boundaries (page 164)

CHAPTER 12

Form, Structure, and Sense

Form, Structure, and Sense questions test whether the words in a sentence fit together. Does the verb agree with its subject? Does the pronoun match the noun it stands for? Does the sentence have a real main verb, in the right tense? Does each describing phrase sit next to the thing it describes? Is each noun correctly plural or possessive? Unlike Boundaries questions (Chapter 11), these questions change words, not punctuation.

The skill shares the Standard English Conventions domain with Boundaries, and the two kinds of questions are mixed together and ordered from easiest to hardest. Six rule sets cover nearly everything the test asks.

What the question looks like

The stem is the same as for Boundaries: "Which choice completes the text so that it conforms to the conventions of Standard English?" The choices are different forms of a word or different versions of a clause:

- A) provides
- B) provide
- C) providing
- D) to provide

For modifier questions, each choice is often a full main clause that ends the passage.

How to recognize it

The choices tell you which rule set applies. Name it before you reread the passage.

What changes in the choices	Rule set being tested
Singular versus plural verbs (is/are, rises/rise)	Subject-verb agreement
it, its, they, them, their	Pronoun-antecedent agreement
-ing forms, to + verb, past-tense verbs	Verb finiteness
has, had, will, is + -ing	Tense and aspect
Whole clauses with different subjects after an opening phrase	Modifier placement
its/it's, their/there/they're, player's/players'	Plurals and possessives

The six rule sets

1. Subject-verb agreement

A singular subject takes a singular verb ("The bridge *sways*"), and a plural subject takes a plural verb ("The bridges *sway*"). The test hides the subject in four ways.

- **Phrases in between.** Cross out any prepositional phrase (a phrase that begins with a word such as "of," "in" or "with") between the subject and the verb: "The box [of old tools] *is* heavy."

- **Add-on phrases.** "Along with," "as well as," "together with" and "in addition to" do not make a subject plural. By contrast, two distinct subjects joined by "and" normally take a plural verb.
- **Inverted order.** In sentences that begin with "there," "here" or a place phrase, the subject follows the verb: "Beside the gate *stand* two oaks."
- **Tricky subjects.** "Each," "every," "one of" and "the number of" take singular verbs; "a number of" takes a plural verb. In standard American usage, a collective noun such as team, band or committee takes a singular verb when the group acts as one unit; a nearby "its" confirms this.

2. Pronoun-antecedent agreement

An antecedent is the noun a pronoun replaces. Singular things take "it" and "its"; plural things take "they," "them" and "their." For a person, use the appropriate personal pronouns; singular "they" is also standard. In standard American usage, a company, city, school or team acting as one unit takes "it" and "its." Two traps appear most often: a nearer noun of the other number, and a pronoun that could point to two different nouns, in which case the clearer choice names the noun itself.

3. Verb finiteness

A **finite verb** has a tense and can serve as a sentence's main verb: "built," "builds," "has built," "will build." **Nonfinite forms** cannot serve as the main verb on their own: the infinitive ("to build"), the -ing form ("building") and the past participle when it works as a describer, as in "Built in 1885, the lighthouse ...".

- Every sentence needs at least one finite verb in its main clause.
- A verb inside a relative clause (a "who," "which" or "that" clause that describes a noun), or after "because" or "although," belongs to that clause. It does not count as the main verb.
- If the sentence already has its main verb, an added action often needs a nonfinite form or a conjunction instead of a second finite verb.

4. Tense and aspect

Tense places an action in time; aspect shows whether it is ongoing or completed. Match the tense of the surrounding verbs unless a time signal says otherwise.

Form	Example	Use it for	Signal words
Simple past	built	A finished past action	in 1990, last year, ago
Present perfect	has built	A past action linked to now	since, so far, for the past decade
Past perfect	had built	The earlier of two past actions	by the time (+ past), before (+ past event)
Progressive	is building, was building	An action in progress	currently, while, at that moment
Simple future	will build	A later action	next year, soon

5. Modifier placement

A **modifier** is a word or phrase that describes something. An opening modifier, such as "Built in 1885," must be followed immediately by the noun it describes. If the wrong noun follows, the modifier is **dangling** ("Built in 1885, tourists admire the lighthouse" says the tourists were built in 1885). A **misplaced** modifier sits next to the wrong word elsewhere in the sentence. Watch two disguises: possessives ("Having trained for months, the runner's time improved" makes "time" the subject, not "runner") and passive voice ("the samples were tested by the chemist" makes "samples" the subject).

6. Plurals and possessives

A plural names more than one and takes no apostrophe (three players). A singular possessive adds 's (the player's shoe). A plural possessive ending in s adds only an apostrophe (the players' shoes); an irregular plural adds 's (the children's books). The possessive pronouns its, their, whose and your never take apostrophes. The apostrophe forms are contractions: it's (it is), they're (they are), who's (who is), you're (you are). "There" points to a place or begins "there is."

The method

1. **Name the rule set** from what changes in the choices.
2. **Find the controlling word:** the subject for agreement, the antecedent for pronouns, the main clause for finiteness, the time signal for tense, the opening phrase for modifiers, the owner for possessives.
3. **Strip the distractions:** cross out prepositional phrases, add-ons and relative clauses.
4. **Predict the form** before you read the choices closely.
5. **Read the whole sentence back** with your choice, checking that it still agrees with the sentences around it.

TIP For agreement questions, silently read the subject and verb together, with nothing between them: "the collection ... attracts." The mismatch is easier to hear without the middle words.

Worked examples

Worked example 1: Find the true subject

The collection of hand-painted maps in a fictional county library _____ visitors from across the state every summer.

- A) attract
- B) attracts
- C) are attracting
- D) have attracted

Think: Cross out "of hand-painted maps in a fictional county library." The subject is "The collection," which is singular.

Answer: B.

Why the others fail: A, C and D are plural forms that agree with "maps," a noun inside the prepositional phrase.

Worked example 2: Every sentence needs a main verb

The first version of a fictional student team's water filter, which forced muddy water through layers of sand and charcoal, _____ most of the sediment in a single pass.

- A) removing
- B) to remove
- C) removed
- D) having removed

Think: The subject is "The first version." The verb "forced" belongs to the "which" clause, so the sentence still needs its own finite verb.

Answer: C.

Why the others fail: A, B and D are nonfinite forms, so each leaves the sentence as a fragment with no main verb.

Worked example 3: Pronoun number with a decoy

The fictional harbor's ferries carry more passengers than any other boat in the fleet, so the city inspects _____ every spring.

- A) it
- B) its
- C) them
- D) this

Think: The city inspects the ferries, a plural noun, and the pronoun is the object of "inspects." The singular nouns "fleet" and "boat" sit closer to the blank, but they are decoys.

Answer: C.

Why the others fail: A and D are singular, so they point to "fleet" or "boat." B is a possessive with no noun to own.

Worked example 4: Tense from a time signal

By the time a fictional bakery opened its second shop in 2020, it _____ bread to local schools for a decade.

- A) delivers
- B) has delivered
- C) had been delivering
- D) will deliver

Think: "By the time ... opened ... in 2020" sets a past moment. The deliveries ran for ten years *before* that moment, so the sentence needs the past perfect. The progressive form "had been delivering" also shows that the action was ongoing.

Answer: C.

Why the others fail: A is present tense. B, the present perfect, connects the action to now, not to 2020. D points to the future.

Worked example 5: Put the describer next to the described

A fictional town commissioned a fountain for its central square. Carved from a single block of granite, _____

- A) the sculptor finished the fountain in just under two years.
- B) the fountain weighs more than a pickup truck.
- C) visitors often mistake the fountain for a natural boulder.
- D) the town council paid for the fountain with a local tax.

Think: What was carved from a block of granite? The fountain. So "the fountain" must come right after the comma.

Answer: B.

Why the others fail: A says the sculptor was carved from granite. C says the visitors were. D says the town council was. In each, the fountain appears too late to fix the modifier.

Worked example 6: Harder-module example (three traps in one sentence)

The committee of engineers and geologists that reviews the fictional dam's safety, along with two outside inspectors, _____ its findings every spring.

- A) publish
- B) publishes
- C) publishing
- D) have published

Think: Strip the distractions: "of engineers and geologists" is a prepositional phrase, "that reviews the fictional dam's safety" is a relative clause, and "along with two outside inspectors" is an add-on. The subject is "The committee," a group acting as one unit, and "its findings" confirms it is singular.

Answer: B.

Why the others fail: A and D are plural forms that agree with "geologists" or "inspectors." C is a participle, not a finite verb.

Common traps

Trap	How to spot it
Nearest-noun agreement	A noun right before the blank differs in number from the true subject. Cross out the phrase it sits in.
Add-on phrases	"Along with," "as well as," "together with." They never turn a singular subject plural.
Inverted order	The verb comes before its subject, often after "there" or an opening place phrase.
-ing fragment	The sentence has no finite verb outside its relative or dependent clauses.
Relative-clause verb mistaken for the main verb	A verb directly after "who," "which" or "that." It belongs to that clause.
Pronoun of the wrong number	"It" or "its" for a plural noun; "they" or "their" for a single company, city or team.
Unsupported tense shift	A choice changes tense with no time signal to justify it.
Dangling modifier	The noun after the opening phrase cannot perform the phrase's action.
Hidden subject	A possessive ("the runner's time") or passive voice ("were tested by") puts the wrong noun in the subject spot.
Apostrophe errors	An apostrophe in a plain plural, "it's" used for ownership, or "player's" when many players are meant.

Harder-module variations

- **Long distances.** The subject and verb may be twenty or more words apart, with a relative clause and an add-on in between. Bracket the extras until only the subject and the blank remain.
- **Two rules at once.** A choice can fail on finiteness and agreement together, so eliminate nonfinite forms first, then check number.
- **Tense clues in another sentence.** A date or "by the time" phrase may sit one sentence earlier. Read the whole passage, not just the sentence with the blank.
- **Modifier choices that all mention the right person.** Only one makes that person the grammatical subject. Passive voice and possessives are the usual disguises.
- **Pronouns with competing antecedents.** Ask what can logically do the action; then check number.

CHECK After choosing, read the full sentence with your answer and ask, "Who or what does this verb, and is it one or many?"

Check yourself

1. How do you find the true subject when a long phrase separates it from the verb?
2. Does "along with" make a subject plural?
3. What makes a verb finite, and why does every sentence need one?
4. Which tense shows the earlier of two past events?
5. What must come right after an introductory modifier?
6. Which is correct for ownership: "its" or "it's"?
7. **Answers:** (1) Cross out prepositional phrases, add-ons and relative clauses, then ask who or what does the action. (2) No; an add-on does not change the subject's number. (3) It has a tense and can serve as the main verb; without one, the sentence is a fragment. (4) The past perfect ("had built"). (5) The noun that the modifier describes. (6) "Its"; "it's" means "it is" or "it has."

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/form-structure-and-sense

Practice Set

20 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C1201

> Mud has collected at the bottom of a fictional lake in northern Canada for thousands of years. Every summer, a thin, pale layer settles on the lake bottom, and every winter, a darker layer follows. The pairs of layers in a single core sample _____ a year-by-year record of the lake's history, much like the rings of a tree.

Which choice completes the text so that it conforms to the conventions of Standard English?

A provides **B** is providing **C** provide **D** has provided

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2 RW-C1202

> A fictional pizza shop recently replaced an old gas oven with a wood-fired oven built by a local mason. The new oven reaches about 450 degrees Celsius, and _____ thick clay dome holds that heat for hours, so each pizza bakes in roughly ninety seconds. Customers can watch the cooks through a glass wall.

Which choice completes the text so that it conforms to the conventions of Standard English?

A its **B** it's **C** its' **D** their

---

3 RW-C1203

> Students in a fictional robotics club built a small rover to explore a gravel pit near their school. The rover, which carried a camera and a soil probe on a folding arm, _____ across the uneven ground at a steady walking pace. Its camera sent pictures back to a laptop every few seconds.

Which choice completes the text so that it conforms to the conventions of Standard English?

A rolling **B** to roll **C** having rolled **D** rolled

---

4 RW-C1204

> In the fictional mountain town of Bellbrook, the town band's two oldest drums date from the 1880s and are still played at every parade. Each spring, a local craftsman with forty years of experience stretches new calfskin heads over _____ before the season's first parade, using the same wooden hoops that the original maker carved.

Which choice completes the text so that it conforms to the conventions of Standard English?

A it **B** them **C** its **D** this

---

5 RW-C1205

> The coaches of a fictional high school track team began keeping logs of their runners' nightly sleep in 2021, after several runners complained of feeling tired at morning practices. Since then, the coaches _____ the sleep logs with race times, and they now schedule lighter workouts after nights when most runners slept poorly.

Which choice completes the text so that it conforms to the conventions of Standard English?

A have compared **B** compare **C** will compare **D** had compared

---

6 RW-C1206

> Cheddar sold at a fictional cheese shop in Wisconsin ages for different lengths of time, from three months to two years. The younger cheddars taste mild and creamy. The oldest wheels rest on wooden shelves in a cool, damp limestone cave. Wrapped in cloth and turned by hand every week, _____

Which choice completes the text so that it conforms to the conventions of Standard English?

A the shop's workers check each wheel for cracks and mold.

B a cave stores the cheese at a steady, cool temperature.

C the cheese develops a sharp flavor and a crumbly texture.

D customers can taste the difference that aging makes.

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7 RW-C1207

> Over millions of years, a river carved the deep canyon at the center of a fictional national park in Utah. The canyon walls reveal a long history of changing environments, from shallow seas to sandy deserts. Near the base of the walls, below layers of red sandstone that formed from ancient desert dunes, _____ a band of gray limestone packed with the fossil shells of small sea creatures that once lived in warm water.

Which choice completes the text so that it conforms to the conventions of Standard English?

A lie **B** lies **C** are lying **D** have lain

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8 RW-C1208

> During a hypothetical study of hydration, sports scientists weighed twelve volleyball players before and after a two-hour practice in a warm gym. The players drank water freely during practice, and the amounts were recorded. Because sweat accounts for most of the weight an athlete loses during exercise, the twelve _____ changes in body weight gave the scientists an estimate of how much fluid each player needed to replace before the next session.

Which choice completes the text so that it conforms to the conventions of Standard English?

A player's **B** players **C** players's **D** players'

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9 RW-C1209

> While working as a fur trader in Labrador between 1912 and 1917, Clarence Birdseye saw that people there often froze food in winter because fresh food was hard to obtain. After returning to the United States, he began to experiment with a method of his own. His process _____ packaged food rapidly between two refrigerated metal plates, a method that preserved the original taste of foods such as fish, fruits, and vegetables.

Which choice completes the text so that it conforms to the conventions of Standard English?

A freezing **B** to freeze **C** froze **D** having frozen

---

10 RW-C1210

> When the fictional Lindqvist Concert Hall opened in 1911, critics praised the clarity of its sound. The praise was no accident. Before the first concert, the hall's builders _____ the room's acoustics by firing a starter pistol on the empty stage, timing how long the echo took to fade, and adjusting the wooden ceiling panels until it faded quickly. Musicians who performed there that first season described hearing every instrument clearly.

Which choice completes the text so that it conforms to the conventions of Standard English?

A test **B** had tested **C** have tested **D** will test

11 RW-C1211

> Two wind turbines stand on a windy ridge above the fields of a fictional farming cooperative, about a kilometer from the nearest road. Before the turbines could supply any power, a crew had to connect _____ to an underground cable that runs down the hill to the cooperative's barns. Today the turbines generate most of the electricity the cooperative uses each year, and members sell the surplus to the regional grid.

Which choice completes the text so that it conforms to the conventions of Standard English?

A it **B** its **C** they **D** them

12 RW-C1212

> A hypothetical survey of youth soccer leagues in one state tracked how often players changed positions during a season. Coaches at more than forty clubs reported each player's positions after every game. According to the survey, the number of players who try at least three different positions _____ sharply when coaches rotate everyone through defense, midfield, and attack in practice games. Players in those leagues also reported enjoying practice more.

Which choice completes the text so that it conforms to the conventions of Standard English?

A rises **B** rise **C** have risen **D** are rising

13 RW-C1213

> Bartolomeo Cristofori, an Italian harpsichord maker who worked in Florence, is generally credited with inventing the piano, although his invention was not well known in his lifetime. He replaced the harpsichord's plucking mechanism with hammers, so a player could make notes soft or loud by changing how hard the keys were struck. The name he gave the instrument described this feature: a harpsichord that plays both soft and loud. Built in Florence in 1720, _____

Which choice completes the text so that it conforms to the conventions of Standard English?

A Cristofori made the instrument that is now the oldest of his three surviving pianos.

B the Metropolitan Museum of Art in New York City owns the oldest of Cristofori's three surviving pianos.

C visitors to New York City can see the oldest of Cristofori's three surviving pianos.

D the oldest of Cristofori's three surviving pianos belongs to the Metropolitan Museum of Art in New York City.

14 RW-C1214

> Near a fictional coastal town, a cliff of soft sandstone retreats a little farther from the sea after every major storm, and several houses now stand only a few meters from its edge. To protect those homes, the town hired a team of geologists. The team's first task was _____ how quickly the cliff had worn away over the past fifty years by comparing old aerial photographs with new ones.

Which choice completes the text so that it conforms to the conventions of Standard English?

A measured **B** to measure **C** it measured **D** they measured

---

15 RW-C1215

> During the 1890s, a fictional family of instrument makers in Vienna built violins for touring musicians. Each violin was varnished by hand in the family's workshop on the edge of the city. Touring players valued the instruments for a warm, even tone that carried well in large halls. Several of the family's violins survive today, and _____ paper labels, written in faded brown ink, record the year each instrument was finished.

Which choice completes the text so that it conforms to the conventions of Standard English?

A there **B** they're **C** their **D** theirs

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16 RW-C1216

> Bakers have long known that bread dough rises faster in a warm kitchen than in a cold one, because yeast becomes more active as the temperature climbs. To measure the effect precisely, a food scientist in a hypothetical experiment placed identical batches of dough in chambers held at different temperatures. The batch kept at 30 degrees Celsius, along with two smaller test batches stored beside it, _____ ready for the oven in less than half the time needed by the batch kept at 20 degrees. The scientist repeated the trial three times with similar results.

Which choice completes the text so that it conforms to the conventions of Standard English?

A was **B** were **C** are **D** have been

---

17 RW-C1217

> Planners in a fictional river town asked a team of inspectors whether the town's aging stone bridge, built more than a century earlier, could carry the heavier buses on a proposed new route. After a week of measuring cracks in the mortar and comparing them with records from the 1960s, the inspectors concluded that the bridge could safely handle the buses for now. Their report recommended that the town first repair the two arches nearest the east bank of the river, where the current runs fastest, because _____ showed the deepest cracks.

Which choice completes the text so that it conforms to the conventions of Standard English?

A it **B** that **C** its **D** they

---

18 RW-C1218

> Working at Columbia University in the 1950s, Marie Tharp turned sonar depth readings into hand-drawn maps of the ocean floor. At first, Tharp was barred from research ships because she was a woman, so her colleague Bruce Heezen collected readings at sea. One of their early maps depicted the North Atlantic seafloor, including its canyons, ridges, and mountains. By the time their map of the entire ocean floor was published in 1977, Tharp and Heezen _____ a map of the Indian Ocean floor as well.

Which choice completes the text so that it conforms to the conventions of Standard English?

A produce **B** have produced **C** had produced **D** will produce

19 RW-C1219

> Strong mountain gusts can make a long, light bridge sway, and the motion can grow uncomfortable or even dangerous for the people crossing it. The designers of a footbridge for a fictional mountain resort, who had studied how air flows around different shapes by testing small wooden models in a wind tunnel for several months, _____ a deck with a smoothly rounded underside that lets wind slip past instead of pushing against a flat edge. Visitors crossing the finished bridge on windy days report feeling only a gentle movement underfoot.

Which choice completes the text so that it conforms to the conventions of Standard English?

A chose **B** choosing **C** having chosen **D** who chose

20 RW-C1220

> Coaches often warn sprinters not to rise too quickly from the starting blocks, arguing that standing upright too early slows the first few strides. In a hypothetical study, a coach filmed ten sprinters with a high-speed camera and measured the angle of each runner's body during the first three steps of a race. Each sprinter ran five timed starts over two days, and the coach also recorded where each runner's front foot landed on every step. Comparing the footage with each sprinter's average time over the first 10 meters, _____

Which choice completes the text so that it conforms to the conventions of Standard English?

A the fastest starts came from runners who kept their bodies low and leaned forward.

B the coach found that runners who stayed low and leaned forward had the fastest starts.

C a low, forward-leaning body produced the fastest starts, according to the coach's analysis.

D runners who stayed low and leaned forward were found by the coach to have the fastest starts.

Practice Set Explanations

1. Correct answer: C RW-C1201

Form, Structure, and Sense · Standard English Conventions · Easier

C *provide*

The subject of the verb is "The pairs," not "layers" or "sample," which sit inside the phrase "of layers in a single core sample." A plural subject takes a plural verb, and the present tense matches the passage's general description. Choice C, "provide," agrees with "pairs."

A "Provides" is singular; it agrees with the nearby noun "sample" instead of the plural subject "The pairs."

B "Is providing" is a singular verb form, so it does not agree with the plural subject "The pairs."

D "Has provided" is singular, matching "sample" rather than the true subject, the plural "pairs."

Review: Chapter 12, Form, Structure, and Sense (page 180)

2. Correct answer: A RW-C1202

Form, Structure, and Sense · Standard English Conventions · Easier

A *its*

The blank shows that the thick clay dome belongs to the oven, so it needs a possessive pronoun. The possessive form of "it" is "its," with no apostrophe. Choice A also agrees in number with its singular antecedent, "The new oven."

B "It's" is a contraction of "it is" or "it has"; "it is thick clay dome" makes no sense here.

C "Its'" is not a word in Standard English; the possessive of "it" never takes an apostrophe.

D "Their" is plural, but it must refer to the singular noun "oven," so the pronoun does not agree in number.

Review: Chapter 12, Form, Structure, and Sense (page 180)

3. Correct answer: D RW-C1203

Form, Structure, and Sense · Standard English Conventions · Easier

D *rolled*

The subject of the sentence is "The rover," and the clause "which carried a camera and a soil probe on a folding arm" only describes it. The sentence still needs a main verb: a finite verb, one with a tense, that tells what the rover did. "Rolled" is the only finite verb among the choices, so D is correct.

A "Rolling" is a participle, not a finite verb; without a main verb, the sentence becomes a fragment.

B The infinitive "to roll" cannot serve as a sentence's main verb, so the sentence would have no predicate.

C "Having rolled" is a nonfinite participle phrase, so the sentence still lacks a main verb and is a fragment.

Review: Chapter 12, Form, Structure, and Sense (page 180)

4. Correct answer: B RW-C1204

Form, Structure, and Sense · Standard English Conventions · Easier

B *them*

The pronoun refers to "the town band's two oldest drums," a plural noun, so it must be plural. It also follows the preposition "over," so it must be an object form. "Them" is the plural object pronoun, making B correct.

A "It" is singular, but the antecedent "drums" is plural, so the pronoun does not agree in number.

C "Its" is a singular possessive; the sentence needs a plural object pronoun after "over," not a word showing ownership.

D "This" is singular and vague; it does not clearly point back to the two drums.

Review: Chapter 12, Form, Structure, and Sense (page 180)

5. Correct answer: A RW-C1205

Form, Structure, and Sense · Standard English Conventions · Easier

A *have compared*

"Since then" describes an action that began at a past point (2021) and continues into the present. That meaning calls for the present perfect tense, "have compared." Choice A also fits the present-tense "now schedule" later in the sentence.

B The simple present "compare" does not fit "Since then," which needs a verb linking a past starting point to the present.

C "Will compare" points to the future, but "Since then" describes an action that has been happening since 2021.

D "Had compared" is past perfect, which places an action before another past event; the sentence offers none, and "now" signals the present.

Review: Chapter 12, Form, Structure, and Sense (page 180)

6. Correct answer: C RW-C1206

Form, Structure, and Sense · Standard English Conventions · Easier

C *the cheese develops a sharp flavor and a crumbly texture.*

The opening phrase "Wrapped in cloth and turned by hand every week" must describe the noun that comes right after the comma. The cheese is what gets wrapped and turned, so "the cheese" must be the subject of the main clause. Choice C places it there.

A This makes "the shop's workers" the ones wrapped in cloth and turned by hand, which is illogical; the cheese is wrapped and turned.

B Placing "a cave" after the comma says the cave was wrapped in cloth and turned by hand, a dangling modifier.

D This says the customers were wrapped in cloth and turned every week; the cheese never appears as the subject.

Review: Chapter 12, Form, Structure, and Sense (page 180)

7. Correct answer: B RW-C1207

Form, Structure, and Sense · Standard English Conventions · Medium

B *lies*

This sentence is inverted: the verb comes before its subject. Put in normal order, it reads "a band of gray limestone ... lies near the base of the walls." The subject is the singular "band," not "walls," "layers" or "dunes," so the singular verb "lies" in B is correct.

A "Lie" is plural; it agrees with the nearby plural nouns "layers" or "dunes" instead of the true subject, "a band."

C "Are lying" is plural, so it does not agree with the singular subject "a band," which comes after the verb.

D "Have lain" is plural; the inverted order hides the singular subject "a band," but the verb must still agree with it.

Review: Chapter 12, Form, Structure, and Sense (page 180)

8. Correct answer: D RW-C1208

Form, Structure, and Sense · Standard English Conventions · Medium

D *players'*

The blank shows that the changes in body weight belong to the twelve players, so the word must be both plural and possessive. For a plural noun that already ends in s, the possessive adds only an apostrophe after the s. Choice D, "players'," is correct.

A "Player's" is a singular possessive, but "twelve" shows the sentence refers to many players, so the possessive must be plural.

B "Players" is plural but not possessive; the sentence needs a form showing that the changes belong to the players.

C "Players's" is not standard; a plural noun ending in s forms its possessive with an apostrophe alone.

Review: Chapter 12, Form, Structure, and Sense (page 180)

9. Correct answer: C RW-C1209

Form, Structure, and Sense · Standard English Conventions · Medium

C *froze*

The subject of the last sentence is "His process," and the sentence needs a finite main verb to say what the process did. "Froze" is a past-tense finite verb that matches the passage's past-tense account, so C is correct. The phrase after the comma, "a method that preserved ...," then renames that action.

A "Freezing" is a participle; without a finite verb, "His process freezing packaged food" is a sentence fragment.

B An infinitive such as "to freeze" cannot act as the main verb, so the sentence would lack a predicate.

D "Having frozen" is a nonfinite form, leaving "His process" without a main verb and the sentence incomplete.

Review: Chapter 12, Form, Structure, and Sense (page 180)

10. Correct answer: B RW-C1210

Form, Structure, and Sense · Standard English Conventions · Medium

B *had tested*

The passage describes two past events in order: the builders tested the acoustics, and later the hall opened in 1911. "Before the first concert" places the testing before another past event, and the past perfect, "had tested," marks that earlier time. Choice B is also the only past-tense option.

A The present tense "test" clashes with the passage's past-tense account of events from 1911.

C The present perfect "have tested" links a past action to the present, but this testing ended before the 1911 opening.

D "Will test" places the action in the future, contradicting a passage about testing completed before the first concert.

Review: Chapter 12, Form, Structure, and Sense (page 180)

11. Correct answer: D RW-C1211

Form, Structure, and Sense · Standard English Conventions · Medium

D *them*

The crew connected the turbines to the cable, so the pronoun must refer to the plural noun "turbines." The pronoun is also the object of the verb "connect," so it needs the object form. The plural object pronoun "them" in D meets both requirements.

A "It" is singular, so it cannot refer to the two turbines; it seems to point to the ridge or the cooperative instead.

B "Its" is a singular possessive, but the sentence needs a plural pronoun naming the turbines as the object of "connect."

C "They" is plural but is a subject form; after the verb "connect," the object form "them" is required.

Review: Chapter 12, Form, Structure, and Sense (page 180)

12. Correct answer: A RW-C1212

Form, Structure, and Sense · Standard English Conventions · Medium

A *rises*

The subject is "the number," which is singular; "of players who try at least three different positions" only describes it. The phrase "the number of" takes a singular verb, while "a number of" takes a plural one. Choice A, "rises," is singular and matches the present-tense verb "rotate."

B "Rise" is plural, agreeing with "players" or "positions" instead of the singular subject "the number."

C "Have risen" is plural; it agrees with "players" rather than with the singular subject "the number."

D "Are rising" is plural, so it does not agree with the singular subject "the number."

Review: Chapter 12, Form, Structure, and Sense (page 180)

13. Correct answer: D RW-C1213

Form, Structure, and Sense · Standard English Conventions · Medium

D *the oldest of Cristofori's three surviving pianos belongs to the Metropolitan Museum of Art in New York City.*

The opening phrase "Built in Florence in 1720" must describe the noun right after the comma. The thing built in 1720 is the piano, so the main clause must begin with the piano as its subject. Choice D does this: "the oldest of Cristofori's three surviving pianos" is what was built in Florence in 1720.

A Placing "Cristofori" after the comma says that Cristofori himself was built in Florence in 1720, a dangling modifier.

B This says the Metropolitan Museum of Art was built in Florence in 1720; the modifier attaches to the wrong noun.

C This makes "visitors" the ones built in Florence in 1720, so the opening phrase describes the wrong noun.

Review: Chapter 12, Form, Structure, and Sense (page 180)

14. Correct answer: B RW-C1214

Form, Structure, and Sense · Standard English Conventions · Medium

B *to measure*

The sentence already has its finite verb, "was." After this linking verb, the sentence needs a word group that names the task, and an infinitive phrase, "to measure how quickly the cliff had worn away," does that. Choice B is correct because it completes the sentence without adding a second finite verb.

A This creates "was measured," saying the task itself was measured; the sentence then never states what the task was.

C Adding the clause "it measured" after "was" stacks two finite verbs with nothing to connect them, so the sentence is ungrammatical.

D "They measured" is a complete subject and verb; placing it right after "was" stacks two finite verbs with no word connecting them.

Review: Chapter 12, Form, Structure, and Sense (page 180)

15. Correct answer: C RW-C1215

Form, Structure, and Sense · Standard English Conventions · Medium

C *their*

The labels belong to the surviving violins, so the blank needs a plural possessive pronoun that can come directly before a noun. "Their" is that form, and it agrees with the plural antecedent "violins." Choice C is correct.

A "There" points to a place or begins a phrase such as "there are"; it does not show ownership.

B "They're" is a contraction of "they are," which makes no sense before "paper labels."

D "Theirs" stands alone in place of a noun, as in "the violins are theirs"; it cannot come directly before "paper labels."

Review: Chapter 12, Form, Structure, and Sense (page 180)

16. Correct answer: A RW-C1216

Form, Structure, and Sense · Standard English Conventions · Harder

A *was*

The subject is "The batch," which is singular. The phrase "along with two smaller test batches stored beside it" is an add-on set off by commas; unlike "and," it does not make the subject plural. The singular past-tense "was" agrees with "batch" and fits the passage's past-tense account, so A is correct.

B "Were" is plural; it treats "along with" like "and," but an "along with" phrase does not change the number of the subject.

C "Are" is plural and present tense, so it fails to agree with "The batch" and clashes with the past-tense account.

D "Have been" is plural, agreeing with "batches" inside the add-on phrase instead of with the singular subject "The batch."

Review: Chapter 12, Form, Structure, and Sense (page 180)

17. Correct answer: D RW-C1217

Form, Structure, and Sense · Standard English Conventions · Harder

D *they*

The pronoun must name what "showed the deepest cracks," and only the arches fit: the report wants them repaired first because of their cracks. "The two arches" is plural, so the plural pronoun "they" in D is correct, even though the singular nouns "bank" and "current" sit closer to the blank.

A "It" is singular, so it points to a nearby singular noun such as "current" or "bank," and neither one could show cracks.

B "That" is singular and vague; it could point to the bank or the current, and it does not agree with the plural "arches."

C "Its" is a possessive that must come before a noun; "because its showed the deepest cracks" leaves the possessive owning nothing.

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: C RW-C1218

Form, Structure, and Sense · Standard English Conventions · Harder

C *had produced*

"By the time their map ... was published in 1977" sets a past reference point, and the Indian Ocean map was finished before it. An action finished before another past event takes the past perfect tense, so "had produced" in C is correct.

A The present tense "produce" clashes with the past time frame set by "By the time ... was published in 1977."

B "Have produced" is present perfect, which connects an action to the present; the sentence needs a tense that looks back from 1977.

D "Will produce" places the action in the future, but "By the time" with a past event requires an action completed before that event.

Review: Chapter 12, Form, Structure, and Sense (page 180)

19. Correct answer: A RW-C1219

Form, Structure, and Sense · Standard English Conventions · Harder

A *chose*

The subject is "The designers," and everything from "who had studied" to "wind tunnel" is a relative clause describing them. The verb "had studied" belongs to that clause, so the sentence still needs its own finite main verb. "Chose" supplies it, making A correct.

B "Choosing" is a participle, not a finite verb; the sentence would have a subject, "The designers," but no main verb.

C "Having chosen" is nonfinite, so the long subject never receives a main verb and the sentence becomes a fragment.

D "Who chose" adds a second relative clause describing the designers, so the sentence still lacks a main verb.

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: B RW-C1220

Form, Structure, and Sense · Standard English Conventions · Harder

B *the coach found that runners who stayed low and leaned forward had the fastest starts.*

The introductory phrase "Comparing the footage with each sprinter's average time over the first 10 meters" describes someone doing the comparing, and that person is the coach. The noun right after the comma must therefore be "the coach," as in B. Each other choice puts a different subject there, leaving the phrase dangling.

A Here the subject is "the fastest starts," so the sentence says the starts compared the footage with the times, which is illogical.

C The subject is "a low, forward-leaning body"; mentioning "the coach's analysis" at the end does not fix the modifier, which still attaches to "body."

D The passive voice makes "runners" the subject, so the runners seem to do the comparing; "by the coach" comes too late to fix it.

Review: Chapter 12, Form, Structure, and Sense (page 180)

CHAPTER 13

Transitions

A transition is a word or phrase that tells the reader how one idea connects to the next: whether the new idea adds to the last one, turns against it, results from it, illustrates it or follows it in time. Transitions questions ask you to choose that connector. The choices are familiar words, and a short, repeatable test separates the right one from the rest. This chapter teaches that test, the main families of transitions, and the traps built from words that sound smooth but point the wrong way.

What the question looks like

Each question gives you a short text, usually two to four sentences, with a blank where a connecting word belongs. The question asks: "Which choice completes the text with the most logical transition?" The four choices are transition words or phrases such as *however*, *for example*, *as a result* or *similarly*.

Transitions belong to the Expression of Ideas domain, which makes up about 20 percent of the section: roughly 8 to 12 scored questions across the two modules, shared with Rhetorical Synthesis (Chapter 14). Expression of Ideas questions come last in each module, and Transitions questions usually appear just before the Rhetorical Synthesis questions. Within the Transitions group, questions run from easier to harder.

How to recognize it

You are looking at a Transitions question when:

- the question asks for "the most logical transition";
- all four choices are connecting words or phrases;
- the blank sits at the start of a sentence, after a semicolon, or between commas inside a sentence.

Do not confuse this question type with Boundaries (Chapter 11). Boundaries choices usually differ in punctuation, even when each one contains the same transition word, and the question asks about "the conventions of Standard English." In a Transitions question, the choices usually all fit the grammar of the sentence, so the decision is about logic: which word states the true relationship between the ideas.

The method

Many wrong answers come from reading the choices too early: a smooth-sounding word settles into your ear, and you stop checking whether it makes sense. The method below keeps the choices out of the way until you know what you need.

1. **Hold off on the choices.** Read the text before you look at the four words.
2. **Read the sentence before the blank and the sentence with the blank.** In a longer text, read the whole thing, because the blank may respond to an idea two sentences back.
3. **Shrink each idea to a few words.** For example: "expected less traffic" and "traffic unchanged."
4. **Decide the direction.** Does the new idea continue in the same direction as the last one, or does it turn against it?

5. **Name the exact relationship and predict a transition.** Use the table below. Saying your own word ("but," "so," "for example") before you look is the most important step.
6. **Eliminate by family.** Cross out every choice from the wrong direction, then every choice from the wrong relationship. The Option Eliminator tool in Bluebook lets you do this on screen.
7. **Reread the sentence with your answer in place.** It should be logical, not just smooth.

Direction	Relationship	The second idea...	Common transitions
Turn	Contrast	differs from or opposes the first	however, by contrast, in contrast, on the other hand, conversely
Turn	Concession	holds true even though a point against it was just admitted	nevertheless, nonetheless, even so, still
Continue	Cause and effect	is a result of the first	therefore, thus, as a result, consequently, accordingly, hence
Continue	Addition	adds another point of the same kind	moreover, furthermore, in addition, additionally, also
Continue	Example	is one specific case of the first	for example, for instance, specifically, to illustrate
Continue	Sequence	happens later (or earlier) in time	then, next, subsequently, afterward, later, previously, earlier
Continue	Similarity	is parallel to the first in some way	similarly, likewise, in the same way, by the same token
Continue	Emphasis	makes the first point stronger	in fact, indeed, above all
Continue	Clarification	restates the first point more plainly	in other words, that is, put simply

Appendix B lists more transitions in each family, with an example and a trap for each.

TIP If two choices do exactly the same job in the sentence, such as *therefore* and *consequently*, neither can be the one best answer. Cross out both.

Worked examples

Example 1: Expectation versus result

Planners in a fictional coastal city expected a new bike lane on Front Street to reduce car traffic on that street. _____, a year after the lane opened, traffic counts on Front Street were nearly unchanged.

A	B	C	D
For instance	Likewise	However	Therefore

Think it through. Shrink the ideas: "expected less traffic" and "traffic unchanged." The second idea turns against the first, because what happened is not what was expected. Predict *but*. **Answer: C.**

Why the others fail. *For instance*, *likewise* and *therefore* all keep going in the same direction, so one direction check removes all three. *Therefore* is the most tempting because both sentences share a topic, but an expectation of less traffic cannot cause traffic to stay the same.

Example 2: Did the first idea cause the second?

A fictional bakery replaced its single large oven with three smaller ovens that can be set to different temperatures. _____, the bakers can now bake bread and pastries at the same time instead of waiting for one oven to heat or cool between batches.

A	B	C	D
In contrast	As a result	To illustrate	Likewise

Think it through. Idea one: "three ovens, different temperatures." Idea two: "two kinds of baking at once." Apply the cause-and-effect test: did the first fact make the second one possible? It did, so predict *so*. **Answer: B.**

Why the others fail. *In contrast* signals that the second idea differs from the first, but faster baking follows naturally from the new ovens. *To illustrate* would need the second sentence to be one case of a general claim, and the first sentence makes no general claim. *Likewise* needs two parallel situations, and there is only one.

TRAP Cause-and-effect words point forward: the earlier sentence is the cause and the later one is the result. If the later sentence gives the reason for the earlier one ("The bakery replaced its oven. The old one had broken twice."), *therefore* and *as a result* run backward and are wrong.

Example 3: A case of the claim, or a new claim?

Several features of a fictional school's new science wing are designed to save energy. _____, motion sensors switch off the lights in any classroom that has been empty for ten minutes.

A	B	C	D
In addition	Instead	Nevertheless	For example

Think it through. The first sentence is a general claim: some features save energy. The second describes one such feature. A specific case of a general claim calls for an example transition. **Answer: D.**

Why the others fail. *In addition* is the strong trap, but it would fit only if an earlier sentence had already named one feature. Here the motion sensors are the first case of the claim, so they illustrate it. *Instead* needs an alternative that is being rejected, and *nevertheless* signals a turn when the ideas agree.

CHECK To choose between example and addition, ask whether the second sentence could begin "One such…" (example) or "Another…" (addition).

Example 4: Which way does time run?

Engineers building a footbridge in a fictional river town first drilled test holes to confirm that the riverbank could support the bridge's weight. _____, they poured the concrete footings that anchor the bridge's two towers.

A	B	C	D
Previously	In contrast	Next	For instance

Think it through. The word *first* sets up an order. Drilling came first; pouring the footings came after. Predict *then*. **Answer: C.**

Why the others fail. *Previously* belongs to the right family but points the wrong way in time: it would place the footings before the test holes. *In contrast* treats two steps of one project as opposites. *For instance* treats pouring footings as an example of drilling.

TIP With sequence choices, check the verbs. A form such as "had tested" or "had built" often signals a step back in time, which calls for *earlier*, *previously* or *before that*.

Example 5: A blank inside the sentence

Nearly every room in a fictional county courthouse was remodeled during the 1980s. The second-floor courtroom, _____, still has its original 1915 wooden benches and hand-painted ceiling.

A	B	C	D
for instance	therefore	likewise	on the other hand

Think it through. Commas around a blank mean the transition sits inside the sentence, but it still links this whole sentence to the one before. "Nearly every room was remodeled" against "this room still has its originals" is a turn. **Answer: D.**

Why the others fail. *For instance* would make the courtroom one of the remodeled rooms, the opposite of what the sentence says. *Therefore* claims that remodeling the other rooms caused this one to keep its originals. *Likewise* suggests the courtroom matched the other rooms.

NOTE In Transitions questions the choices usually all fit the punctuation around the blank, so let the logic decide. Appendix B shows how transitions are punctuated at the start of a sentence, after a semicolon and between commas.

Example 6 (harder module): A concession across three sentences

Critics of a fictional city's new glass-walled train station argued that the building would waste energy, since its walls let in so much summer heat. The criticism has some basis: on the hottest afternoons, the waiting halls need heavy air-conditioning. _____, an energy audit found that the station uses less energy each year than the brick station it replaced, mainly because daylight through the glass reduces the need for electric lighting.

A	B	C	D
Indeed	As a result	Still	Similarly

Think it through. Three ideas: "critics say wasteful," "partly true: needs air-conditioning," and "audit shows less energy overall." The second sentence admits a point against the station, and the third holds its ground anyway. Admit-then-hold is a concession; predict *even so*. **Answer: C.**

Why the others fail. *Indeed* is the trap: it would intensify the criticism, but the third sentence weakens it. *As a result* claims that the air-conditioning causes the lower energy use, when the text credits daylight. *Similarly* needs a parallel point, yet the audit runs against the sentence before it.

Common traps

Trap	How to spot it
Wrong direction	The ideas turn, but the choice continues (or the reverse). Run the direction check before anything else.
Fake cause	*Therefore* or *as a result* joins two facts that merely sit side by side. Ask: did the first make the second happen or make it true?
Backward cause	The second sentence explains why the first happened. Result words point forward, so they fail.
Example versus addition	Ask "One such…" (example) or "Another…" (addition). An example needs a general claim before it.
Time flip	*Previously* where events run forward, or *subsequently* where the text steps back. Check verbs and time words.
Similarity without a match	*Similarly* or *likewise* with no two parallel things. Name both halves of the comparison.
Too strong	*In fact* or *indeed* when the second sentence adds a new point instead of strengthening the first.
Missing alternative	*Instead* or *rather* with no rejected option before it.

Harder-module variations

Harder Transitions questions use the same logic in denser texts. Expect these patterns:

- **The blank answers an earlier sentence.** A text may state a claim, add a detail, then return to the claim. Ask what the sentence with the blank responds to, not just what sits next to it.
- **Concession structures.** Admission phrases such as *admittedly*, *granted*, *to be sure* or "has some basis" set up a sentence that holds its ground anyway, which calls for *still*, *even so*, *nevertheless* or *nonetheless*.
- **Two choices from the same direction.** Harder questions often offer two "continue" words with different jobs, such as *therefore* and *in fact*, or two "turn" words, such as *however* and *instead*. Decide the exact relationship, not just the direction.
- **Negative wording.** Words such as *few*, *rarely*, *not* or *failed* can hide the direction. Preserve their exact meaning: "few critics expected the play to succeed" means only a small number predicted success, so "it sold out every night" is a turn. It does not mean every other critic predicted failure; some may have been unsure.
- **Less common transitions.** Know the job of *accordingly* (in response to that), *to that end* (to reach that goal), *in turn* (as the next link in a chain), *by the same token* (for a parallel reason), *that said* (a concession), *conversely* (the reverse case) and *in short* (a summary).

NOTE In harder texts, a technical term is often followed by a plain-language explanation. When the second sentence says the same thing in simpler words, choose a clarification word such as *that is* or *in other words*, not a result word.

Check yourself

1. What should you do before you read the four choices?
2. Which two relationships turn against the previous idea, and how do they differ?
3. How do you decide between *for example* and *in addition*?
4. Why can *therefore* be wrong even when two sentences are about the same subject?
5. Two choices do the same job in the sentence. What does that tell you?
6. Answers: (1) Read the text, shrink each idea to a few words, decide the direction and predict your own transition. (2) Contrast, where the ideas differ, and concession, where the second idea holds true even though a point against it was admitted. (3) Use *for example* when the second sentence is one case of a general claim before it; use *in addition* when it adds a separate point of the same kind. (4) A result word needs the first idea to cause the second; sharing a topic is not enough. (5) Neither can be the single best answer, so eliminate both.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/transitions

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C1301

> During a hypothetical two-month survey of her school's vegetable garden, a student found that most of the ladybugs she observed fed on aphids, tiny insects that suck sap from plant stems. One small yellow species, _____, ignored the aphids entirely and grazed on the white mildew that coats squash leaves in late summer.

Which choice completes the text with the most logical transition?

A for example **B** however **C** therefore **D** similarly

---

2 RW-C1302

> The architects of a fictional community library in Maple Falls designed its roof to collect rainwater, which is stored in underground tanks and used to flush the building's toilets. _____, they built the library's outer walls from thick blocks of packed earth, which keep the reading rooms cool in summer without air-conditioning.

Which choice completes the text with the most logical transition?

A Instead **B** For instance **C** Nevertheless **D** Additionally

---

3 RW-C1303

> Some insects that live in large colonies share information about food with one another. _____, the zoologist Karl von Frisch observed that a honeybee that has found nectar performs a kind of dance, and the dance shows other bees nearby where to find the nectar. Von Frisch later shared the 1973 Nobel Prize in Physiology or Medicine for discoveries about animal behavior.

Which choice completes the text with the most logical transition?

A For example **B** In contrast **C** Consequently **D** Afterward

---

4 RW-C1304

> The fictional river town of Alder Bend once drew its drinking water straight from the river, and outbreaks of typhoid fever, a disease often spread through contaminated water, struck nearly every summer. In 1912 the town built a plant that filtered its water through deep beds of sand. _____, typhoid cases in Alder Bend dropped sharply over the next three years.

Which choice completes the text with the most logical transition?

A However **B** Similarly **C** As a result **D** For instance

---

5 RW-C1305

> Each autumn, monarch butterflies from eastern North America migrate to mountain fir forests in Mexico, some flying as far as 3,000 miles. There they spend the winter clustered by the thousands on the trees. In spring, these butterflies leave their winter habitat, mate, and lay eggs along the northward route. _____, the monarchs that hatch from those eggs carry the migration farther north, and it takes three to four generations for monarchs to reach Canada.

Which choice completes the text with the most logical transition?

A Subsequently **B** Nonetheless **C** For example **D** In other words

6 RW-C1306

> For much of the nineteenth century, many physicians believed that cholera spread through miasma, a "bad air" thought to rise from decaying matter. The English physician John Snow, _____, reasoned that the disease was carried by contaminated water. During an 1854 outbreak in London's Soho district, he mapped cholera cases and traced them to a single public water pump on Broad Street. His findings, including his maps of the outbreak, appeared in a book he published the following year.

Which choice completes the text with the most logical transition?

A for instance **B** moreover **C** accordingly **D** by contrast

7 RW-C1307

> In a fictional theater company's production of a long historical play, twelve actors portray more than forty characters, so each actor plays several roles. The costume designer gave every character one signature item, such as a red scarf or a tall hat, which the actor puts on when switching roles so that audiences can tell the characters apart. _____, the lighting designer gave each of the play's three cities its own color of light, so viewers can tell at a glance where a scene takes place.

Which choice completes the text with the most logical transition?

A Nevertheless **B** For example **C** Similarly **D** As a result

8 RW-C1308

> For generations, families on a hypothetical island spoke two languages: an older island language at home and a widely used trade language in the markets. In the 1950s, a new ferry service linked the island to the mainland, and many young families moved to mainland cities for work, where their children grew up hearing only the trade language. _____, by 1990 fewer than a hundred people anywhere could still hold a full conversation in the island language.

Which choice completes the text with the most logical transition?

A For instance **B** Consequently **C** Similarly **D** Nonetheless

9 RW-C1309

> According to a historian's study of records from a fictional port city, the city's first board of health, founded in 1881, spent most of its early budget building sewers that carried waste away from crowded neighborhoods. _____, the board hired inspectors to test the milk and meat sold in the city's markets and to close stalls that sold spoiled food. Both efforts aimed to reduce the illnesses that spread most quickly in the summer months.

Which choice completes the text with the most logical transition?

A In contrast **B** Therefore **C** For example **D** Moreover

10 RW-C1310

> A film historian who analyzed a fictional studio's archive found that the studio's earliest films often borrowed techniques from the stage. _____, in one 1908 comedy from the archive, the camera stays in a single fixed position for the entire film, as though the viewer were seated in a theater watching actors cross a stage. Close-ups, which bring the viewer near an actor's face, do not appear in the studio's films until several years later.

Which choice completes the text with the most logical transition?

A For instance **B** Nevertheless **C** In other words **D** Afterward

11 RW-C1311

> Biologists observing a hypothetical population of mountain goats recorded every slip and fall on steep slopes during one summer. Young goats in their first year fell far more often than adults, usually after stepping onto loose stones. _____, the young goats rarely suffered serious injuries, largely because they did most of their climbing on low ledges, where a fall covered only a short distance and ended on grass or soft soil.

Which choice completes the text with the most logical transition?

A Therefore **B** Specifically **C** Nonetheless **D** Similarly

12 RW-C1312

> When two languages share a community, one may gradually replace the other as younger speakers stop using the older language at home. Linguists studying a hypothetical mountain community, however, describe its situation as "stable bilingualism." _____, nearly every resident speaks both the local language and the national language, and surveys taken thirty years apart show no decline in the share of children who learn the local language from their parents. Residents tend to use the national language at school and in government offices and the local language with family and neighbors.

Which choice completes the text with the most logical transition?

A Consequently **B** That is **C** Even so **D** Similarly

13 RW-C1313

> Today the fictional Corwin Hall, a concert venue that opened in 1962 in a busy river city, is admired for its curved ceiling of overlapping maple panels, which spreads sound evenly to all 1,800 seats. Yet the curved ceiling was the architect's second design for the hall. _____, she had proposed a flat plaster ceiling, but when engineers played recorded music inside a small wooden model of the hall, the flat surface produced harsh echoes near the stage. The plaster plan was dropped, and the maple panels replaced it in the final design.

Which choice completes the text with the most logical transition?

A Earlier **B** Subsequently **C** In addition **D** Therefore

14 RW-C1314

> A hypothetical field study tested whether planting strips of wildflowers along the edges of farm fields would increase the number of wild bees visiting nearby squash and melon crops. During the first year, the strips attracted large numbers of bees, but most of those bees stayed among the wildflowers rather than moving into the crops. _____, by the third year, bee visits to crops beside the strips were nearly double those on farms without strips, likely because the flowers had supported larger bee populations that eventually spread into the fields.

Which choice completes the text with the most logical transition?

A Indeed **B** Thus **C** Likewise **D** Even so

15 RW-C1315

> Health officials in a fictional rural county assumed during the 1950s that families would come to free vaccination clinics as long as the clinics were announced in local newspapers. Turnout at the first clinics, held in the county's largest town, was far lower than expected, and follow-up interviews showed that many farm families had no way to travel there on weekdays. _____, the county outfitted two buses as mobile clinics and sent them to farming communities on a weekly schedule. Within a year, vaccination rates in rural areas had more than tripled.

Which choice completes the text with the most logical transition?

A Nevertheless **B** Accordingly **C** Likewise **D** For instance

Practice Set Explanations

1. Correct answer: B RW-C1301

Transitions · Expression of Ideas · Easier

B *however*

The first sentence says most ladybugs ate aphids. The second describes a species that "ignored the aphids entirely" and ate mildew instead, so it turns against the general pattern. A contrast transition is needed, and "however" marks this species as the exception.

A "For example" would present the mildew-eating species as one case of the aphid-eating majority, but it is an exception to that pattern.

C "Therefore" signals a result, but the other ladybugs' diet does not cause this species to eat mildew.

D "Similarly" signals a parallel, yet this species behaves differently from the aphid-eating ladybugs described first.

Review: Chapter 13, Transitions (page 195)

2. Correct answer: D RW-C1302

Transitions · Expression of Ideas · Easier

D *Additionally*

The first sentence describes one design feature, a roof that collects rainwater. The second describes a separate feature, earth walls that keep the rooms cool. Because the second sentence adds another feature of the same kind, an addition transition fits, and "Additionally" presents the walls as one more design choice.

A "Instead" signals that the walls replaced the rainwater roof, but the library has both features, so nothing is being replaced.

B "For instance" needs a general claim before it, but the first sentence describes one specific feature, not a category the walls could illustrate.

C "Nevertheless" signals that something holds true despite an opposing point, but the two features do not conflict.

Review: Chapter 13, Transitions (page 195)

3. Correct answer: A RW-C1303

Transitions · Expression of Ideas · Easier

A *For example*

The first sentence makes a general claim: some insects that live in colonies share information about food. The second sentence gives one specific case of that behavior, the honeybee dance that von Frisch described. A specific case of a general claim calls for an example transition, so "For example" is correct.

B "In contrast" signals a difference, but the honeybee dance supports the claim about insects sharing food information rather than opposing it.

C "Consequently" signals a result, but the bee dance von Frisch observed is an instance of insects sharing information, not something that sharing caused.

D "Afterward" signals a later event, but the text sets up no sequence of events for von Frisch's observation to follow.

Review: Chapter 13, Transitions (page 195)

4. Correct answer: C RW-C1304

Transitions · Expression of Ideas · Easier

C *As a result*

The text links typhoid to contaminated water and then reports that the town began filtering its water. The drop in cases that follows is the outcome of that change, so a cause-and-effect transition is needed. "As a result" signals that the filtering plant led to fewer typhoid cases.

A "However" signals a contrast, but fewer cases is exactly what cleaner water would be expected to produce, not a surprise.

B "Similarly" needs a parallel situation, but the drop in cases is an outcome of the filtering plant, not a matching event.

D "For instance" would make the drop in cases an example of building the plant, which confuses an action with its effect.

Review: Chapter 13, Transitions (page 195)

5. Correct answer: A RW-C1305

Transitions · Expression of Ideas · Medium

A *Subsequently*

The text follows the migration in time order: the monarchs fly south in autumn, spend the winter in Mexico and begin traveling north and reproduce in spring. The final sentence describes what happens after the eggs hatch, the next stage of the migration. "Subsequently" signals that this stage comes later in the sequence.

B "Nonetheless" signals that something happens despite an opposing point, but the new generation's flight continues the migration rather than defying anything before it.

C "For example" would make the northward flight an instance of breeding and laying eggs, but it is the next stage, not an example.

D "In other words" signals a restatement, but the last sentence adds a new stage of the migration instead of rephrasing the previous sentence.

Review: Chapter 13, Transitions (page 195)

6. Correct answer: D RW-C1306

Transitions · Expression of Ideas · Medium

D *by contrast*

The first sentence describes the common belief that cholera spread through "bad air." The next sentence says Snow reasoned that contaminated water carried the disease, a view that opposed the common one. A contrast transition is needed, and "by contrast" marks Snow's idea as different from the miasma theory.

A "For instance" would make Snow one example of the physicians who believed in miasma, but he rejected that theory.

B "Moreover" adds a point that continues the previous idea, but Snow's water theory challenges the miasma belief instead of adding to it.

C "Accordingly" means "in response to that," but Snow's reasoning did not follow from the miasma belief; it opposed that belief.

Review: Chapter 13, Transitions (page 195)

7. Correct answer: C RW-C1307

Transitions · Expression of Ideas · Medium

C *Similarly*

The costume designer uses one simple visual signal, a signature item, to help audiences tell characters apart. The lighting designer uses another simple visual signal, a color for each city, to help audiences tell places apart. Because the two designers solve parallel problems in a parallel way, "Similarly" is the most logical transition.

A "Nevertheless" signals that something happens despite an opposing point, but the lighting plan serves the same goal as the costume plan.

B "For example" would make the lighting plan an instance of the costume plan, but colored light is a separate, parallel technique, not a costume.

D "As a result" claims the costume choices caused the lighting choices, but the text gives no sign that one decision led to the other.

Review: Chapter 13, Transitions (page 195)

8. Correct answer: B RW-C1308

Transitions · Expression of Ideas · Medium

B *Consequently*

The second sentence explains that children of the families who moved grew up hearing only the trade language, so they did not learn the island language. The third sentence reports the outcome: very few fluent speakers remained by 1990. Because the move led to the decline, "Consequently" is the most logical transition.

A "For instance" would present the 1990 speaker count as an example of families moving, but it is a result of that move, not a case of it.

C "Similarly" signals a parallel situation, but the drop in speakers is an effect of the move rather than a matching event.

D "Nonetheless" signals an unexpected outcome, but losing speakers is what the families' move would lead a reader to expect.

Review: Chapter 13, Transitions (page 195)

9. Correct answer: D RW-C1309

Transitions · Expression of Ideas · Medium

D *Moreover*

The first sentence describes one effort of the health board, building sewers. The second describes another effort, food inspections, and the last sentence says both served the same aim. Because the second sentence adds a further action of the same kind, "Moreover" is the logical transition.

A "In contrast" would set the inspections against the sewer work, but the text presents both as parallel efforts toward one goal.

B "Therefore" signals that the sewer spending caused the inspections, but nothing in the text says one decision led to the other.

C "For example" would make the food inspections an instance of building sewers, but they are a separate effort.

Review: Chapter 13, Transitions (page 195)

10. Correct answer: A RW-C1310

Transitions · Expression of Ideas · Medium

A *For instance*

The first sentence makes a general claim: the studio's early films often borrowed stage techniques. The second describes one film whose fixed camera imitates a seat in a theater, a specific case of that borrowing. "For instance" correctly introduces this example of the general claim.

B "Nevertheless" signals a contrast, but the fixed camera supports the claim about stage techniques rather than going against it.

C "In other words" signals a restatement, but the 1908 comedy is a specific new case, not a rewording of the general claim about borrowing.

D "Afterward" signals a later event, but the 1908 film is evidence for the claim, not a step that follows it in time.

Review: Chapter 13, Transitions (page 195)

11. Correct answer: C RW-C1311

Transitions · Expression of Ideas · Medium

C *Nonetheless*

The second sentence reports a fact that seems to predict harm: young goats fell far more often than adults. The

third says that, despite those falls, the young goats were rarely seriously hurt. A result that holds true in spite of the preceding point calls for a concession transition, and "Nonetheless" fits.

A "Therefore" claims the frequent falls caused the lack of injuries, but the text credits the low ledges, not the falls themselves.

B "Specifically" introduces a closer detail of the previous point, but the injury finding is a new point that runs against what the falls suggest.

D "Similarly" signals a parallel, but rare injuries do not match frequent falls; the two facts pull in opposite directions.

Review: Chapter 13, Transitions (page 195)

12. Correct answer: B RW-C1312

Transitions · Expression of Ideas · Harder

B *That is*

The second sentence introduces a technical term, "stable bilingualism," and the third explains what it means for this community: residents speak both languages, and the local language is not declining. A sentence that restates a term in plainer words calls for a clarification transition, and "That is" introduces this explanation.

A "Consequently" treats residents' speaking both languages as a result of stable bilingualism, but that situation is what the term means, not an effect of it.

C "Even so" signals a point that holds despite the previous one, but the third sentence explains the linguists' description rather than resisting it.

D "Similarly" signals a parallel case, but the third sentence describes the same community and term, not a second situation that resembles it.

Review: Chapter 13, Transitions (page 195)

13. Correct answer: A RW-C1313

Transitions · Expression of Ideas · Harder

A *Earlier*

The second sentence calls the curved ceiling the architect's "second design," and the next sentence, written with "had proposed," describes her first design. Because this sentence steps back to a time before the curved ceiling, a transition pointing to an earlier time is needed. "Earlier" fits that sequence.

B "Subsequently" points forward in time, but the plaster ceiling came before the curved design, as "second design" and "had proposed" make clear.

C "In addition" would present the plaster ceiling as one more feature of the hall, but it was rejected and never built.

D "Therefore" presents the plaster proposal as a conclusion drawn from the sentence before, but the sentence simply steps back in time to describe the first design.

Review: Chapter 13, Transitions (page 195)

14. Correct answer: D RW-C1314

Transitions · Expression of Ideas · Harder

D *Even so*

The second sentence reports a disappointing early result: the bees stayed in the wildflower strips instead of visiting crops. The third reports that, despite that start, crop visits nearly doubled by the third year. A later result that holds in spite of an earlier setback calls for a concession transition, and "Even so" signals exactly that.

A "Indeed" would strengthen the point that bees stayed out of the crops, but the third sentence reports the opposite outcome.

B "Thus" says the third sentence follows from the second, but bees avoiding the crops cannot explain why crop visits later rose; the text credits larger bee populations.

C "Likewise" signals a parallel result, but doubled crop visits do not match the first-year finding that bees avoided the crops.

Review: Chapter 13, Transitions (page 195)

15. Correct answer: B RW-C1315

Transitions · Expression of Ideas · Harder

B *Accordingly*

The interviews identified the problem: farm families could not travel to the clinics in town. The next sentence describes the county's response, bringing clinics to those families. Because the county acted in response to what the interviews revealed, a cause-and-effect transition is needed; "Accordingly," meaning "in response to that," fits.

A "Nevertheless" signals an action taken despite an obstacle, but the buses directly address the travel problem rather than ignoring it.

C "Likewise" signals a parallel action, but the mobile clinics are a response to the interview findings, not a similar step.

D "For instance" would make the mobile clinics an example of the travel problem, confusing the solution with the obstacle it was meant to fix.

Review: Chapter 13, Transitions (page 195)

CHAPTER 14

Rhetorical Synthesis

Rhetorical Synthesis questions give you a set of bullet-point notes and a goal, then ask which sentence uses the notes to reach that goal. *Synthesis* means combining information; *rhetorical* means using it for a purpose. These questions reward a reversal of normal reading habits: you read the goal first, and you may need only one or two of the notes. Handled that way, they are often among the quickest questions in the section.

What the question looks like

Each question opens with a short framing sentence, such as "While researching a topic, a student has taken the following notes:" Several bullet points follow, each a short fact about one topic. Then comes the question, which has two parts: a goal and a request. For example: "The student wants to emphasize a difference between the two bridges. Which choice most effectively uses relevant information from the notes to accomplish this goal?" Each of the four choices is a complete sentence built from the notes.

Rhetorical Synthesis belongs to the Expression of Ideas domain, along with Transitions (Chapter 13). These questions usually come last in each Reading and Writing module, and like the other skill groups, they run from easier to harder.

How to recognize it

The bullet-point notes are the giveaway; no other Reading and Writing question uses them. The question names the student's goal, usually in a sentence that begins "The student wants to…" Each common type of goal tells you what the right sentence must do.

The goal says...	The right sentence must...	Words that often signal it
emphasize a similarity	name both items and what they share	both, also, like, similarly
emphasize a difference	name both items and how they differ	whereas, while, unlike, but
introduce X to an unfamiliar audience	say who or what X is, with a short description	"X, a [description],"
present a study, its findings or its methods	give exactly the part asked for: results for findings, procedure for methods	found, showed; how many, how long, how measured
explain the significance of X	say why X mattered: an effect, an influence or what it made possible	led to, helped, made possible
give a specific example	name one concrete case, not a general statement	a named case, a number, a single scene

An *audience* condition matters. "An audience unfamiliar with X" means the sentence must identify X before saying anything else about it. "An audience already familiar with X" means the sentence can use X's name freely and should focus on the new information.

The method

1. **Read the goal first.** Skip the notes for now, and mark the task words in the goal with the Highlights & Notes tool or on scratch paper.
2. **Break the goal into parts.** Make a quick checklist: the task (similarity, difference, introduce, findings and so on), the subject (which items), and any condition (an audience, "the study's methods," "a specific example").
3. **Predict the shape of a correct sentence.** For a difference: "X does this, whereas Y does that." For an introduction: "X, a [description], did [something notable]."
4. **Test each choice against the checklist.** Eliminate any choice that misses a part, does a different task or talks about the wrong item.
5. **Check the survivor against the notes.** Glance at the bullet or two it uses to confirm the facts and the logic word.
6. **Choose.** If two choices remain, reread the goal and check each against the relevant notes. Pick the accurate choice that meets every part of the goal. A choice that leaves any part unmet is wrong, however well written.

Why you do not need to read every note. In this question type, the wrong choices are usually accurate. They fail because they do the wrong job: they state a similarity when the goal asks for a difference, a finding when it asks for a method, or a detail when it asks for an introduction. Reading every note first therefore costs time without helping you decide. Read the notes only to confirm your choice or to settle a close call.

TIP The average pace in Reading and Writing is about 71 seconds per question. A Rhetorical Synthesis question handled goal-first often takes less, and the time you save can go to questions you marked for review earlier in the module.

Worked examples

Example 1: A similarity

- In a hypothetical survey, a student compared two community gardens in a fictional city: Oak Street Garden and Riverside Garden.
- Oak Street Garden has 24 plots; Riverside Garden has 75.
- Both gardens collect rainwater in barrels for watering.
- Oak Street Garden opened in 2015; Riverside Garden opened in 2021.
- Both gardens give part of their harvest to a local food pantry.

Goal: The student wants to emphasize a similarity between the two gardens.

A) Oak Street Garden, which opened in 2015, has 24 plots, while Riverside Garden has 75.

B) Riverside Garden opened in 2021, six years after Oak Street Garden.

C) Like Oak Street Garden, Riverside Garden collects rainwater in barrels and gives part of its harvest to a local food pantry.

D) Riverside Garden, the larger of the two gardens, has 75 plots.

Think it through. Checklist: both gardens, plus something they share. Choice C names both and states two shared features, and "Like" signals the similarity. **Answer: C.** Choices A and B state differences, and D describes

only one garden, so it cannot compare anything. The goal identifies the strongest candidate; the notes confirm both shared features.

Example 2: A difference, but which one?

- A student researched two hypothetical bridges that cross the same river.
- The North Bridge (1928) is a stone arch bridge with five arches.
- The South Bridge (2010) is a suspension bridge hung from steel cables.
- Both bridges carry two lanes of car traffic and a footpath.
- The North Bridge took six years to build; the South Bridge took two.

Goal: The student wants to emphasize a difference in the two bridges' designs.

A) Both bridges carry two lanes of car traffic and a footpath across the river.

B) The North Bridge is a stone arch bridge, whereas the South Bridge is a suspension bridge hung from steel cables.

C) The North Bridge took six years to build, whereas the South Bridge took only two.

D) The South Bridge, completed in 2010, is a suspension bridge hung from steel cables.

Think it through. Checklist: both bridges, a difference, and the feature named in the goal, *design*. Choice B contrasts the two designs. **Answer: B.** Choice C is the trap: it states a true difference, but in building time, not design. Choice A is a similarity, and D describes only one bridge.

Example 3: Introducing a topic to an unfamiliar audience

- The Harrow County Map Room is a fictional public archive of local maps.
- It holds about 4,000 maps; the oldest was drawn in 1790.
- Many of the maps show farms, roads and streams that no longer exist.
- Volunteers have scanned about half of the collection so far.
- Students and local historians use the maps to trace how the county's towns grew.

Goal: The student wants to introduce the Harrow County Map Room to an audience unfamiliar with it.

A) Volunteers have scanned about half of the Harrow County Map Room's maps so far.

B) The oldest of the 4,000 maps was drawn in 1790.

C) Students and local historians use the maps to trace how the county's towns grew.

D) The Harrow County Map Room, a public archive of about 4,000 local maps dating back to 1790, helps students and historians trace how the county's towns grew.

Think it through. A reader who has never heard of the Map Room needs to learn what it is. Choice D names it, identifies it as a public archive, and says what it holds and why it is useful. **Answer: D.** Choice A names the archive but gives only a minor detail about scanning. Choices B and C say "the maps" as though the reader already knows which maps are meant.

CHECK For an unfamiliar audience, look for a category word right after the name, such as "a public archive," "a self-taught engineer" or "a desert shrub." A choice without one usually assumes too much.

Example 4: Findings or methods?

- In a hypothetical study, a team of education researchers asked whether classroom noise affects reading speed.
- Forty-eight students read the same passage: half in a quiet room and half in a room playing recorded hallway noise.
- Each student's reading time was measured with a stopwatch.
- Students in the quiet room finished, on average, 12 percent faster.
- The team plans to repeat the study with younger students.

Goal: The student wants to describe the study's method.

A) Students who read in a quiet room finished about 12 percent faster, on average, than students who heard hallway noise.

B) To test whether noise affects reading speed, researchers timed 48 students reading the same passage, half in a quiet room and half with recorded hallway noise.

C) The team plans to repeat its study of classroom noise with younger students.

D) Classroom noise may slow students' reading, a hypothetical study suggests.

Think it through. A method is how the study was done: who took part, what they did and what was measured. Choice B gives all three. **Answer: B.** Choice A is the strongest trap because it is accurate and specific, but it reports the finding, not the method. Choice D states a conclusion, and C describes a future plan.

Example 5 (harder module): Significance with a limit

- The fictional town of Bellmere built a seawall in 1966 to protect its harbor from storms.
- Before the wall, storms flooded the harbor district about once every five years.
- No storm has flooded the harbor district since the wall was completed.
- The wall's curved face turns waves back toward the sea.
- Engineers later used the curved design as a model in a guide for coastal towns across the region.
- The wall is 1.2 kilometers long.

Goal: The student wants to explain the significance of Bellmere's seawall beyond the town itself.

A) Since Bellmere completed its seawall in 1966, no storm has flooded the town's harbor district.

B) Bellmere's 1.2-kilometer seawall, built in 1966, has a curved face that turns waves back toward the sea.

C) Bellmere's seawall mattered beyond the town: engineers later used its curved, wave-turning design as a model in a guide for coastal towns across the region.

D) Before 1966, storms flooded Bellmere's harbor district about once every five years.

Think it through. This goal has two parts: *significance* (why the wall mattered) and a limit, *beyond the town itself*. Choice C names an effect outside Bellmere: the design became a model for coastal towns across the region. **Answer: C.** Choice A is the trap because it does explain the wall's significance, but only for Bellmere. Choice B describes the wall without saying why it mattered, and D gives background. In harder goals, a limiting phrase like this one often separates the two best choices.

Common traps

Trap	How to spot it
Accurate but off-goal	The sentence is true to the notes but does a different job, such as a finding when the goal asks for a method. Check the task word.
Half a comparison	A similarity or difference goal needs both items. A choice about only one item cannot compare.
The wrong difference	The goal names one feature (design, timing, cost), and the choice gives a true difference in another.
Assumes familiarity	For an unfamiliar audience, the choice says "the maps" or uses a last name alone without saying what or who it is.
General instead of specific	The goal asks for an example, and the choice restates the general claim.
Wrong scope	The goal sets a limit ("beyond the town," "according to the historian"), and the choice answers a narrower or different version.
Overstated claim	The choice says "proved" or "all" when the notes say "suggests," "may" or "most."
Unsupported link	The choice joins two notes with *because* or *so* when the notes never connect them that way.

Harder-module variations

Harder Rhetorical Synthesis questions keep the same format but demand more precision. Watch for these patterns:

- **Two-part goals.** "Present the study's aim and its findings" or "emphasize a difference and offer a possible explanation." Check off both halves; a choice that does one perfectly still fails.
- **A source attached to the goal.** "According to the historian" means the right choice must match that person's claim, not another effect listed in the notes.
- **Four choices about the right items.** When every choice mentions both items, compare them on the task word: does each one show a similarity, a difference or neither?
- **Hedges and links.** The right choice keeps a note's caution ("may" never becomes "proves") and joins notes with *because*, *whereas* or *although* only when the notes support that exact relationship.

Check yourself

1. What should you read first, and why?
2. What must a sentence contain to emphasize a difference?
3. How does "an audience unfamiliar with" change what the right answer must include?
4. A choice states a study's findings accurately, but the goal is to describe the method. Why is it wrong?
5. When should you read the notes closely?
6. Answers: (1) The goal in the question, because it decides which choice is right, while the notes mostly confirm facts. (2) Both items and the way they differ, in the feature the goal names. (3) The sentence must

identify the person, work or topic, usually with a short description, before giving other details. (4) It does a different job: findings are results, while a method is how the study was carried out. (5) Only to confirm the facts in your chosen sentence or to settle a close call between two choices.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/rhetorical-synthesis

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 RW-C1401

> While researching a topic, a student has taken the following notes:
>
> • Fallingwater is a weekend house near Mill Run in southwestern Pennsylvania.
>
> • The American architect Frank Lloyd Wright designed it in 1935 for the Kaufmann family.
>
> • The family had expected a house with a view of a waterfall on their land.
>
> • Wright instead placed the house directly above the waterfall.
>
> • The house was completed in 1937 and opened as a museum in 1964.

The student wants to introduce Fallingwater to an audience unfamiliar with the building. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Wright placed the house directly above a waterfall on the family's land, though the family had expected only a view of the falls.

B The Kaufmann family had expected a house with a view of a waterfall on their land.

C Fallingwater, a weekend house in southwestern Pennsylvania designed by the architect Frank Lloyd Wright, was built directly above a waterfall.

D In 1964, the house, which had been completed in 1937, opened to the public as a museum.

---

2 RW-C1402

> While researching a topic, a student has taken the following notes:
>
> • In a hypothetical survey of a city park, a student observed two ant colonies, Colony A and Colony B.
>
> • Colony A nested under a flat stone near the pond.
>
> • Colony B nested inside a fallen log near the walking path.
>
> • Both colonies sent out most of their foragers in the early morning.
>
> • Both colonies carried seeds back to their nests.
>
> • Colony A had about 400 workers; Colony B had about 1,500.

The student wants to emphasize a similarity between the two colonies. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Both colonies sent out most of their foragers in the early morning, and both carried seeds back to their nests.

B Colony A nested under a flat stone near the pond, while Colony B nested inside a fallen log near the walking path.

C Colony B, with about 1,500 workers, was much larger than Colony A, which had about 400.

D Colony A, which had about 400 workers, nested under a flat stone near the pond in the park.

---

3 RW-C1403

> While researching a topic, a student has taken the following notes:
>
> - In a hypothetical study, a team of biologists tested whether pigeons could learn to tell two shapes apart.
> - Twelve pigeons were shown a circle and a triangle side by side on a screen.
> - Pecking the triangle earned a food reward; pecking the circle did not.
> - After two weeks of training, ten of the twelve pigeons chose the triangle on more than 90 percent of trials.
> - The team plans to test whether the pigeons can also tell apart shapes of different sizes.

The student wants to present the study's findings. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A In a hypothetical study, biologists tested whether pigeons could learn to tell two shapes, a circle and a triangle, apart on a screen.

B The biologists plan to test whether the pigeons in the study can also tell apart shapes of different sizes.

C Each of the twelve pigeons in the study earned a food reward for pecking a triangle on a screen but not for pecking a circle.

D After two weeks of training, ten of the twelve pigeons in the study chose the triangle on more than 90 percent of trials.

---

4 RW-C1404

> While researching a topic, a student has taken the following notes:
>
> - Two fictional theater companies staged the same play about a lighthouse keeper in 2025.
> - The Brightwater Players performed the play on a bare stage with no scenery.
> - The Elm Street Company built a full-size lighthouse set with a working lamp.
> - Both productions ran for six weeks.
> - Both companies used a cast of five actors.
> - Critics praised the lead actors in both productions.

The student wants to emphasize a difference between the two productions. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Both the Brightwater Players and the Elm Street Company staged the play for six weeks with a cast of five actors.

B The Brightwater Players performed the play on a bare stage, whereas the Elm Street Company built a full-size lighthouse set.

C The Elm Street Company's production featured a full-size lighthouse set with a working lamp and ran for six weeks.

D Critics praised the lead actors in both the Brightwater Players' production and the Elm Street Company's production.

---

5 RW-C1405

> While researching a topic, a student has taken the following notes:
>
> • A team of linguists studied a hypothetical island dialect of a widely spoken language.
>
> • Speakers of the dialect use several words that are not found in the mainland variety of the language.
>
> • Most of these island words describe the sea or the weather.
>
> • One such word, "brellow," names a wind that suddenly changes direction.
>
> • Older islanders use the island words more often than younger islanders do.

The student wants to give a specific example of a word found only in the island dialect. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Speakers of the island dialect use several words, most of them about the sea or the weather, that are not found in the mainland variety.

B Older islanders use the words that are unique to their dialect more often than younger islanders do, the linguists found.

C A team of linguists studied the vocabulary of a hypothetical island dialect of a widely spoken language.

D The island word "brellow," which names a wind that suddenly changes direction, is not used in the mainland variety of the language.

---

6 RW-C1406

> While researching a topic, a student has taken the following notes:
>
> • Sequoyah (about 1775–1843) was a Cherokee silversmith who never learned to read or write English.
>
> • Around 1809, he began developing a way to write the Cherokee language.
>
> • He first experimented with picture symbols, then switched to symbols that each stood for a syllable.
>
> • By 1821, he had created 86 symbols representing all the syllables of Cherokee.
>
> • The system was easy to learn, and Cherokees soon published books and newspapers in their own language.

The student wants to introduce Sequoyah and his achievement to an audience unfamiliar with him. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A By 1821, a set of 86 symbols represented all the syllables of the Cherokee language.

B Sequoyah first experimented with picture symbols around 1809 and later switched to symbols that each stood for a single syllable.

C Sequoyah, a Cherokee silversmith, created a system of 86 symbols for writing all the syllables of the Cherokee language, completing it by 1821.

D Because the new writing system was easy to learn, Cherokees soon published books and newspapers in their own language.

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7 RW-C1407

While researching a topic, a student has taken the following notes:

- In 1893, the fictional city of Port Halden began requiring that all milk sold in the city be pasteurized, or heated to kill harmful bacteria.
- Before the rule, milk often traveled long distances from farms without refrigeration.
- City hospital records show that cases of milk-borne illness fell by about 60 percent within three years of the rule.
- By 1905, four neighboring cities had adopted similar rules, citing Port Halden's results.
- The rule was enforced by six city inspectors.

The student wants to explain the significance of Port Halden's 1893 milk rule. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Port Halden's 1893 rule required that all milk sold in the city, which often traveled long distances from farms, be pasteurized to kill harmful bacteria.

B Port Halden's 1893 milk rule was followed by a roughly 60 percent drop in milk-borne illness, and those results led four neighboring cities to adopt similar rules.

C Before 1893, milk sold in Port Halden often traveled long distances from farms without refrigeration.

D Port Halden's 1893 rule that all milk sold in the city be pasteurized was enforced by a team of six city inspectors.

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8 RW-C1408

While researching a topic, a student has taken the following notes:

- The fictional town of Ashford has two public libraries designed by different architects.
- Harbor Library (1958) has small, high windows that keep direct sunlight off the books.
- Hill Library (2019) has floor-to-ceiling windows made of glass that blocks ultraviolet light, which can fade book covers.
- Both libraries hold about 80,000 books.
- Harbor Library's reading room seats 40 people; Hill Library's seats 120.

The student wants to emphasize a difference in how the two libraries protect their books from sunlight. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Harbor Library shields its books from sunlight with small, high windows, while Hill Library uses glass that blocks ultraviolet light.

B Hill Library's reading room seats 120 people, while the reading room at Harbor Library seats only 40.

C Harbor Library and Hill Library were designed by different architects, but each holds about 80,000 books.

D Hill Library, completed in 2019, has floor-to-ceiling windows made of glass that blocks ultraviolet light, which can fade the covers of books.

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9 RW-C1409

While researching a topic, a student has taken the following notes:

- In a hypothetical study, a team of entomologists asked whether outdoor lighting affects how often fireflies flash.
- The team chose 20 suburban yards: ten where porch lights stayed on all night and ten where lights were turned off.
- In each yard, observers counted firefly flashes for 15 minutes, starting one hour after sunset.
- Fireflies in the dark yards flashed about three times as often as those in the lit yards.
- The team suggested that bright lights may make fireflies' signals harder for other fireflies to see.

The student wants to describe the method the team used in the study. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Fireflies in the yards where porch lights were turned off flashed about three times as often as fireflies in the yards where the lights stayed on all night.

B A hypothetical study suggests that bright outdoor lights may make fireflies' flashing signals harder for other fireflies to see.

C The team counted firefly flashes for 15 minutes, starting an hour after sunset, in ten yards with porch lights on and ten yards with lights off.

D A team of entomologists conducted a hypothetical study to learn whether outdoor lighting affects how often fireflies flash.

---

10 RW-C1410

While researching a topic, a student has taken the following notes:

- In a hypothetical garden observation, a student recorded two nectar-feeding animals during one summer day and evening.
- The first was a hummingbird, a bird; it fed in the late afternoon.
- The second was a hawk moth, an insect; it fed after sunset.
- Both hovered in front of flowers while feeding instead of landing on them.
- Both reached nectar deep inside tube-shaped flowers, the hummingbird with its long bill and the moth with its long tongue.

The student wants to emphasize a similarity in how the two animals fed. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A The hummingbird fed in the late afternoon, but the hawk moth did not begin feeding until after sunset.

B The hummingbird is a bird, whereas the hawk moth is an insect, yet both visited the same garden.

C The hawk moth, an insect, fed after sunset and reached nectar deep inside tube-shaped flowers with its long tongue.

D Both the hummingbird and the hawk moth hovered in front of flowers as they fed rather than landing on them.

---

11 RW-C1411

> While researching a topic, a student has taken the following notes:
>
> - Florence Nightingale (1820–1910) was a British nurse, statistician and social reformer.
> - During the Crimean War, she led a group of 38 women to a British army hospital at Scutari, across the Bosporus from Constantinople.
> - There she required basic standards of care, such as bathing, clean clothing and dressings, and adequate food.
> - Her night rounds among wounded soldiers earned her the nickname "the Lady with the Lamp."
> - In 1860, she opened a school of nursing at St. Thomas' Hospital in London.

The student wants to introduce Florence Nightingale to an audience unfamiliar with her. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Florence Nightingale, a British nurse and statistician, led 38 women to an army hospital during the Crimean War and later opened a nursing school in London.

B Nightingale's night rounds among the wounded soldiers at the army hospital in Scutari earned her the nickname "the Lady with the Lamp."

C At the army hospital in Scutari, Nightingale required basic standards of care for wounded soldiers, such as bathing, clean clothing and dressings, and adequate food.

D A school of nursing opened at St. Thomas' Hospital in London in 1860.

12 RW-C1412

> While researching a topic, a student has taken the following notes:
>
> - In a hypothetical study, a team of linguists examined a whistled form of a language used in a mountain village.
> - In the whistled form, speakers imitate the pitch and rhythm of spoken words by whistling.
> - The team wanted to know whether whistled words could be understood at a distance more reliably than shouted words.
> - At a distance of 1 kilometer, listeners correctly identified 85 percent of whistled words but only 30 percent of shouted words.
> - The team concluded that whistling lets villagers communicate across the valley more reliably than shouting does.

The student wants to present the study's aim and its main finding. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Linguists found that, at a distance of 1 kilometer, listeners correctly identified 85 percent of whistled words but only 30 percent of shouted words.

B To learn whether whistled words could be understood at a distance more reliably than shouted words, linguists tested listeners 1 kilometer away, who identified 85 percent of whistled words but only 30 percent of shouted ones.

C In the whistled form of a mountain village's language, speakers imitate the pitch and rhythm of spoken words, which the team concluded lets villagers communicate across the valley more reliably than shouting does.

D Linguists wanted to know whether whistled words could be understood at a distance more reliably than shouted words, and they concluded from their tests that a whistled language is easier for villagers to learn than a spoken one.

13 RW-C1413

While researching a topic, a student has taken the following notes:

- Until 1910, public drinking fountains in the fictional city of Larkport each had a single metal cup that everyone shared.
- That year, Larkport's health board replaced the shared cups with fountains that sent a stream of water upward so people could drink without a cup.
- Within two years, three nearby cities had installed the same kind of fountain.
- Larkport's newspapers printed the health board's notices explaining that illnesses could spread through shared cups.
- A historian argues that Larkport's change helped make the idea that diseases can spread through shared objects familiar to the general public.

The student wants to explain the significance of Larkport's 1910 change, according to the historian. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A In 1910, Larkport replaced the shared metal cups on its public fountains with fountains that sent a stream of water upward.

B Larkport's 1910 switch to cupless fountains mattered because, within two years, three nearby cities had installed the same kind of fountain.

C According to a historian, Larkport's 1910 switch to cupless fountains proved to the general public that diseases spread through shared objects such as metal drinking cups.

D According to a historian, Larkport's 1910 switch to cupless fountains helped make the public familiar with the idea that diseases can spread through shared objects.

14 RW-C1414

> While researching a topic, a student has taken the following notes:
>
> - A biologist compared the songs of city and forest populations of a hypothetical songbird species.
> - City birds sang at a higher average pitch than forest birds.
> - Forest birds sang longer songs than city birds.
> - Both populations began singing about 30 minutes before sunrise.
> - The biologist suggests that the city birds' higher pitch may help their songs be heard over low-pitched traffic noise.

The student wants to emphasize a difference between the two populations' songs and present a possible explanation for it. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A City birds sang at a higher average pitch than forest birds, perhaps because higher notes are easier to hear over low-pitched traffic noise.

B City birds sang at a higher pitch, while forest birds sang longer songs, though both populations began singing about 30 minutes before sunrise.

C Forest birds sang longer songs than city birds, possibly because their songs did not have to compete with traffic noise.

D Although city birds sang at a higher pitch than forest birds, both populations began singing about 30 minutes before sunrise.

---

15 RW-C1415

> While researching a topic, a student has taken the following notes:
>
> - A fictional 2019 thriller set in a harbor town is known for the way its director uses silence to build suspense.
> - In most scenes, a low musical score plays beneath the dialogue.
> - Before each of the film's three major plot twists, the score grows gradually quieter.
> - In the final scene, two characters search a dark warehouse for almost a minute with no music, dialogue or sound effects.
> - Critics also praised the film's use of reflections in windows and mirrors.

The student wants to give a specific example of how the film's director uses silence to build suspense. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A The director of the thriller is known for using silence throughout the film as a way to build suspense.

B Before each of the film's three major plot twists, its musical score grows gradually quieter.

C In the film's final scene, two characters search a dark warehouse for almost a minute with no music, dialogue or sound effects.

D In most of the film's scenes, a low musical score plays beneath the dialogue, and critics praised the film's use of reflections in windows and mirrors.

Practice Set Explanations

1. Correct answer: C RW-C1401

Rhetorical Synthesis · Expression of Ideas · Easier

C *Fallingwater, a weekend house in southwestern Pennsylvania designed by the architect Frank Lloyd Wright, was built directly above a waterfall.*

An audience unfamiliar with Fallingwater needs to learn what it is and why it stands out. Choice C names the building, identifies it as a weekend house in southwestern Pennsylvania, credits its architect and gives its most distinctive feature. None of the other choices names Fallingwater or says what kind of building it is.

A This choice gives a striking detail, but it never names Fallingwater or says where it is, and "Wright" and "the house" assume the reader already knows both.

B This choice describes the family's expectations without naming Fallingwater or its architect, so it does not introduce the building.

D This choice gives later facts about "the house" without naming it or saying what kind of building it is, so a new reader cannot tell which house is meant.

Review: Chapter 14, Rhetorical Synthesis (page 208)

2. Correct answer: A RW-C1402

Rhetorical Synthesis · Expression of Ideas · Easier

A *Both colonies sent out most of their foragers in the early morning, and both carried seeds back to their nests.*

The goal is a similarity, so the right choice must name what the two colonies share. Choice A states two shared behaviors from the notes: both colonies foraged mostly in the early morning, and both carried seeds home. The repeated word "both" makes the similarity clear.

B This choice contrasts the colonies' nesting sites, so it emphasizes a difference rather than the similarity the student wants.

C This choice compares the colonies' sizes, which highlights a difference rather than anything the two colonies share.

D This choice describes only Colony A, so it cannot show how the two colonies are alike.

Review: Chapter 14, Rhetorical Synthesis (page 208)

3. Correct answer: D RW-C1403

Rhetorical Synthesis · Expression of Ideas · Easier

D *After two weeks of training, ten of the twelve pigeons in the study chose the triangle on more than 90 percent of trials.*

Findings are what a study discovered. Choice D reports the result: after two weeks of training, ten of the twelve pigeons picked the rewarded triangle on more than 90 percent of trials. The other choices describe the study's question, its procedure or its future plans instead of what it found.

A This choice states the question the study asked, not the answer it found, so it does not present the findings.

B This choice describes a future plan, so it reports nothing the study has already found.

C This choice describes how the training worked, which is part of the study's method, not one of its results.

Review: Chapter 14, Rhetorical Synthesis (page 208)

4. Correct answer: B RW-C1404

Rhetorical Synthesis · Expression of Ideas · Easier

B *The Brightwater Players performed the play on a bare stage, whereas the Elm Street Company built a full-size lighthouse set.*

To emphasize a difference, a sentence must name both productions and show how they differ. Choice B sets the Brightwater Players' bare stage against the Elm Street Company's full lighthouse set, and "whereas" marks the contrast. The other choices describe shared features or mention only one company.

A This choice lists features the two productions share, so it emphasizes a similarity, the opposite of the student's goal.

C This choice describes only the Elm Street Company's production, so it cannot show how the two productions differ.

D This choice reports praise that both productions received, which is a similarity rather than a difference.

Review: Chapter 14, Rhetorical Synthesis (page 208)

5. Correct answer: D RW-C1405

Rhetorical Synthesis · Expression of Ideas · Medium

D *The island word "brellow," which names a wind that suddenly changes direction, is not used in the mainland variety of the language.*

A specific example must name one particular word. Choice D gives "brellow," explains what it means and notes that mainland speakers do not use it, so it is a clear example of a word unique to the island dialect. The other choices discuss the island words in general or describe the study.

A This choice describes the island words as a group but names no particular word, so it offers a general statement instead of a specific example.

B This choice compares how often older and younger islanders use the words, but it does not name any word as an example.

C This choice describes the study itself and mentions no word at all, so it cannot serve as an example.

Review: Chapter 14, Rhetorical Synthesis (page 208)

6. Correct answer: C RW-C1406

Rhetorical Synthesis · Expression of Ideas · Medium

C *Sequoyah, a Cherokee silversmith, created a system of 86 symbols for writing all the syllables of the Cherokee language, completing it by 1821.*

The goal has two parts: introduce Sequoyah to readers who do not know him and present his achievement. Choice C identifies him as a Cherokee silversmith and states what he accomplished, a complete system of 86 symbols for writing Cherokee by 1821. The other choices leave out either the person or the achievement.

A This choice states the achievement accurately but never mentions Sequoyah, so it does not introduce the person who created the system.

B This choice names Sequoyah but gives only steps in his process, without saying who he was or what the finished system accomplished.

D This choice describes the system's later effects without naming Sequoyah or explaining that he created it.

Review: Chapter 14, Rhetorical Synthesis (page 208)

7. Correct answer: B RW-C1407

Rhetorical Synthesis · Expression of Ideas · Medium

B *Port Halden's 1893 milk rule was followed by a roughly 60 percent drop in milk-borne illness, and those results led four neighboring cities to adopt similar rules.*

Explaining significance means showing why the rule mattered. Choice B gives what followed the rule: milk-borne illness fell by about 60 percent, and four neighboring cities, citing those results, adopted similar rules. The other choices describe what the rule required, conditions before it or how it was enforced, without saying what difference it made.

A This choice accurately explains what the rule required, but it says nothing about the rule's effects, so it does not show why the rule mattered.

C This choice gives background about conditions before the rule rather than explaining the rule's importance.

D This choice describes how the rule was enforced, a minor detail that does not explain its significance.

Review: Chapter 14, Rhetorical Synthesis (page 208)

8. Correct answer: A RW-C1408

Rhetorical Synthesis · Expression of Ideas · Medium

A *Harbor Library shields its books from sunlight with small, high windows, while Hill Library uses glass that blocks ultraviolet light.*

The goal names a specific difference: how each library protects its books from sunlight. Choice A contrasts Harbor Library's small, high windows with Hill Library's ultraviolet-blocking glass, so it covers both libraries and the exact feature the student wants to highlight.

B This choice presents a real difference between the libraries, but in seating, not in protection from sunlight, so it misses the goal's focus.

C This choice stresses something the libraries share, their collection size, so it emphasizes a similarity rather than a difference.

D This choice describes only Hill Library's windows and never mentions how Harbor Library handles sunlight, so it shows no difference.

Review: Chapter 14, Rhetorical Synthesis (page 208)

9. Correct answer: C RW-C1409

Rhetorical Synthesis · Expression of Ideas · Medium

C *The team counted firefly flashes for 15 minutes, starting an hour after sunset, in ten yards with porch lights on and ten yards with lights off.*

A study's method is how the work was done. Choice C explains the procedure: the team compared ten yards with porch lights on and ten with lights off, counting flashes in each for 15 minutes beginning an hour after sunset. That is a description of the method, which is exactly what the goal asks for.

A This choice accurately reports the study's main result, which makes it tempting, but a result is a finding, not a description of the method.

B This choice gives the team's suggested explanation for its results, not the procedure the team followed.

D This choice states only the question the team asked; it says nothing about how the team gathered data to answer it.

Review: Chapter 14, Rhetorical Synthesis (page 208)

10. Correct answer: D RW-C1410

Rhetorical Synthesis · Expression of Ideas · Medium

D *Both the hummingbird and the hawk moth hovered in front of flowers as they fed rather than landing on them.*

The goal asks for a similarity in how the animals fed. Choice D names both animals and states a shared feeding behavior from the notes: each hovered in front of flowers instead of landing. The word "both" makes the similarity explicit.

A This choice contrasts when the two animals fed, so it emphasizes a difference rather than a similarity in their feeding.

B This choice notes that both visited the garden, but it stresses that one is a bird and one an insect and says nothing about how they fed.

C This choice describes only the hawk moth's feeding, so it cannot show what the two animals have in common.

Review: Chapter 14, Rhetorical Synthesis (page 208)

11. Correct answer: A RW-C1411

Rhetorical Synthesis · Expression of Ideas · Medium

A *Florence Nightingale, a British nurse and statistician, led 38 women to an army hospital during the Crimean War and later opened a nursing school in London.*

An unfamiliar audience needs to know who Nightingale was and why she is remembered. Choice A gives her full name, identifies her as a British nurse and statistician, and summarizes two achievements from the notes: leading 38 women to an army hospital during the Crimean War and opening a nursing school in London.

B This choice offers a memorable nickname but uses only her last name and never says who she was, so it assumes the reader already knows her.

C This choice describes her standards of care but never identifies her or her role, so a new reader would not learn who she was.

D This choice mentions the nursing school but not Nightingale herself, so it does not introduce her to the audience.

Review: Chapter 14, Rhetorical Synthesis (page 208)

12. Correct answer: B RW-C1412

Rhetorical Synthesis · Expression of Ideas · Harder

B *To learn whether whistled words could be understood at a distance more reliably than shouted words, linguists tested listeners 1 kilometer away, who identified 85 percent of whistled words but only 30 percent of shouted ones.*

The goal has two parts: the study's aim and its main finding. Choice B states the aim, learning whether whistled words could be understood at a distance more reliably than shouted words, and then the finding: at 1 kilometer, listeners identified 85 percent of whistled words but only 30 percent of shouted ones. It is the only choice that accurately delivers both parts.

A This choice reports the main finding accurately but leaves out what the study set out to learn, so it completes only half of the student's goal.

C This choice explains how whistled speech works and restates the team's conclusion, but it never states the study's aim or the data it found.

D This choice states the aim, but its claim that a whistled language is easier to learn appears nowhere in the notes, so it misrepresents the finding.

Review: Chapter 14, Rhetorical Synthesis (page 208)

13. Correct answer: D RW-C1413

Rhetorical Synthesis · Expression of Ideas · Harder

D *According to a historian, Larkport's 1910 switch to cupless fountains helped make the public familiar with the idea that diseases can spread through shared objects.*

The goal asks for the change's significance as the historian sees it. The historian argues that the switch helped make the public familiar with the idea that diseases can spread through shared objects. Choice D states exactly that, naming the change and keeping the historian's cautious claim that it "helped make" the idea familiar.

A This choice describes what Larkport did in 1910 but not why it mattered, so it does not explain the change's significance.

B This choice points to nearby cities installing the same fountains, but the historian's point concerns public understanding of disease, not what other cities did.

C This choice overstates the historian's claim: the notes say the change helped make an idea familiar, not that it proved how diseases spread.

Review: Chapter 14, Rhetorical Synthesis (page 208)

14. Correct answer: A RW-C1414

Rhetorical Synthesis · Expression of Ideas · Harder

A *City birds sang at a higher average pitch than forest birds, perhaps because higher notes are easier to hear over low-pitched traffic noise.*

The goal has two parts: a difference between the songs and a possible explanation for it. Choice A states the pitch difference and then gives the biologist's suggested reason, that higher notes may be easier to hear over traffic noise. Its word "perhaps" also matches the cautious "may" in the notes.

B This choice lists two differences and a similarity but offers no explanation for any of them, so it completes only part of the goal.

C This choice pairs the song-length difference with a reason the notes never give; the biologist's explanation concerns pitch, not song length.

D This choice uses the pitch difference only to set up the shared singing time, so it emphasizes a similarity and gives no explanation.

Review: Chapter 14, Rhetorical Synthesis (page 208)

15. Correct answer: C RW-C1415

Rhetorical Synthesis · Expression of Ideas · Harder

C *In the film's final scene, two characters search a dark warehouse for almost a minute with no music, dialogue or sound effects.*

A specific example must point to one concrete moment in which the director uses silence. Choice C does that: in the final scene, two characters search a warehouse for almost a minute with no music, dialogue or sound effects, a single scene built on complete silence.

A This choice restates the general claim about the director's use of silence but points to no particular scene, so it is not a specific example.

B This choice describes music growing quieter, which is not silence, and it summarizes a pattern across three scenes rather than giving one example.

D This choice describes music beneath the dialogue and the film's visual reflections, neither of which shows how the director uses silence.

Review: Chapter 14, Rhetorical Synthesis (page 208)

PART III

Math

The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses

Linear Equations in One Variable

Linear Equations in Two Variables and Linear Functions

Systems of Two Linear Equations

Linear Inequalities

Equivalent Expressions

Nonlinear Equations and Systems

Nonlinear Functions

Ratios, Rates, Proportions, Units and Percentages

One-Variable and Two-Variable Data

Probability, Sampling, Inference and Evaluating Claims

Area, Volume, Lines, Angles and Triangles

Right Triangles and Trigonometry

Circles

CHAPTER 15

The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses

Before you study the Math skills, learn the three tools that come with every Math question: the entry rules for student-produced responses, the graphing calculator and the reference sheet. Used well, each one saves time and prevents mistakes that have nothing to do with how much math you know.

Read this chapter once now, then return to it whenever a later chapter's calculator tip or answer entry feels unfamiliar. Appendix C collects the calculator moves in one table.

What the Math section gives you

Feature	What to know
Questions and time	Two modules of 22 questions each, with 35 minutes per module. That averages about 95 seconds per question.
Formats	About 75% four-option multiple choice and about 25% student-produced response (SPR), where you type your own answer.
Calculator	Allowed on every Math question. The Bluebook testing app includes the Desmos graphing calculator, which you can move and resize. You may also bring a handheld calculator that meets the current College Board policy; CAS models are not allowed.
Reference sheet	Available throughout both Math modules. It lists area, volume and triangle facts (covered later in this chapter).
Order	Questions in each module run from easiest to hardest, with topics mixed together.

NOTE The Math directions say that, unless a question tells you otherwise, all variables and expressions stand for real numbers, figures are drawn to scale, figures lie in a plane, and the domain of a function f is every real x for which f(x) is a real number. "Drawn to scale" means you may use a figure to check whether an answer is reasonable.

Student-produced responses: the entry rules

An SPR question gives you no answer choices: on the digital test, you type your answer into a response field. In this book, write it on the line labeled Answer. Here are the College Board's entry rules in plain words:

1. If a question has more than one correct answer, enter just one of them.

2. A positive answer can use at most 5 characters. A negative answer can use at most 6, and the minus sign counts as one of the 6.
3. If a fraction is too long to fit, enter its decimal form instead.
4. If a decimal is too long to fit, cut it off or round it at the fourth digit, so that your entry fills the space.
5. Enter a mixed number as an improper fraction or a decimal. Three and one-half is 7/2 or 3.5, never 3 1/2.
6. Do not type a percent sign, a comma or a dollar sign. Units, spaces, $\sqrt{}$ and π cannot be typed either.

A "character" is any digit, decimal point, fraction bar or minus sign. So 7/2 uses 3 characters, 0.125 uses 5, and −3.5 uses 4.

Your answer	Acceptable entries	Not acceptable	Why
7/2	7/2 or 3.5	3 1/2	Mixed numbers are not allowed.
2/3	2/3, .6666, .6667, 0.666, 0.667	.66, .67, 0.66, 0.67	A repeating decimal must fill all the available spaces.
−1/3	−1/3, −.3333, −0.333	−.33	The minus sign takes one of 6 spaces; the rest must be filled.
8.5% (the question asks for p in p%)	8.5	8.5%, 0.085	Type the number the question names, without the sign.
$1,250	1250	$1250, 1,250	No dollar signs or commas.

NOTE The repeating-decimal examples match the College Board's Fall 2026 directions. Follow the current directions on test day.

Three habits keep your SPR entries safe:

- **Reduce fractions.** Reducing often shortens an entry, but the result still must fit the character limit.
- **Prefer the fraction when it fits.** Typing 2/3 is exact and avoids any question about how many decimal places are enough.
- **Reread the question's last line before typing.** Many SPR errors come from entering the right number for the wrong quantity, such as y when the question asks for x, or the total cost when it asks for the number of months.

If your work ends with a radical or π, stop and reread. A question that involves a circle or a special right triangle usually asks for a number you can type, such as the value of k when an area is written as $k\sqrt{3}$ or a volume as $k\pi$.

Example 1: An answer that does not fit as a fraction

Problem: $12x - 7 = 118$. What value of x is the solution to the given equation?

Thinking: Add 7 to both sides to get $12x = 125$, so $x = 125/12$. That fraction is already reduced, but it has 6 characters, one more than the 5 allowed for a positive answer. Switch to the decimal: $125 \div 12 = 10.41666\ldots$

Entry: Five characters are allowed, so enter 10.41 (cut off) or 10.42 (rounded). Either fills the space.

Traps: 10.4 does not fill the space, 10.42% contains a symbol, and 10 5/12 is a mixed number.

ONLINE (OPTIONAL) The website's guide to student-produced responses repeats these entry rules on one page.

skillbyskillprep.com/
student-produced-responses

The graphing calculator: nine moves, in words

The on-screen graphing calculator does far more than arithmetic. The moves below describe the built-in graphing calculator. Handheld models vary; some require equations solved for y and do not offer sliders. Practice each one before test day; a tool you have to figure out under time pressure is not a shortcut.

1. **Solve an equation by graphing both sides.** Enter the left side as one graph and the right side as another, each written as y = (that side). The x-coordinate of each intersection point is a solution.
2. **Solve a system.** Enter both equations exactly as written, even in a form such as 3x + 2y = 12, and select the intersection point to see its coordinates.
3. **Count solutions.** Count intersections, or x-intercepts when one side is 0. Adjust the window to look for hidden points, then use algebra to confirm the total.
4. **Use a table of values.** A function's table shows many inputs and outputs at once. It is a fast way to test several answer choices or to spot a pattern.
5. **Fit a model with regression.** Enter data in a two-column table. Use the table's column names in your model and a tilde (~) in place of the equals sign; for a line, enter $y_1 \sim mx_1 + b$. The calculator reports the values of the constants that fit the data best.
6. **Find the mean, median and standard deviation.** Type the data as a list inside the built-in mean, median or standard deviation function. No sorting by hand is needed.
7. **Check whether two expressions are equivalent.** Graph each as y = (expression). A visible mismatch rules out equivalence, but matching pictures alone do not prove it. Simplify algebraically and check excluded inputs.
8. **Use a slider for an unknown constant.** When an equation contains a letter such as k or c, the calculator can turn it into a slider. Drag the slider until the condition in the question happens: the lines become parallel, the graphs touch once, and so on.
9. **Check the angle mode.** Before any trigonometry, check whether the calculator is in degree or radian mode, and match the mode to the question.

TRAP The calculator gives decimals, but many questions want exact values. An intersection at x = 0.333 probably means 1/3, and a slider that stops near 2.67 may mean 8/3. Confirm the exact value with a line of algebra before you choose an answer or type an SPR entry.

Example 2: Solving by graphing both sides

Problem: 1.4(x + 5) = 0.6x + 13. What is the solution to the given equation?

Thinking: Graph y = 1.4(x + 5) and y = 0.6x + 13. The lines cross at the point (7.5, 17.5). The solution to the equation is the x-coordinate, 7.5.

Check by algebra: Distribute to get $1.4x + 7 = 0.6x + 13$. Then $0.8x = 6$, so $x = 7.5$.

Trap: Entering 17.5, the y-coordinate. It is the value of each side when $x = 7.5$; only the x-coordinate of the intersection solves the equation.

Example 3: Counting solutions without solving

Problem: How many solutions does the system $y = (x - 3)^2 - 1$ and $y = -2x + 6$ have?

Thinking: Graph both. The line crosses the parabola twice, once near $x = 0.6$ and once near $x = 3.4$. The system has exactly two solutions.

Check by algebra: Set the expressions equal: $x^2 - 6x + 8 = -2x + 6$, so $x^2 - 4x + 2 = 0$. The discriminant is $16 - 8 = 8$, which is positive, so there are two real solutions. The question never needed their exact values.

Trap: If the window is too small, one intersection can sit off the screen. When a count matters, zoom out once before you answer.

Example 4 (harder-module): A slider finds the constant

Problem: In the system $y = x^2$ and $y = 2x + c$, c is a constant. For what value of c does the system have exactly one solution?

Thinking with a slider: Graph $y = x^2$ and $y = 2x + c$ with a slider for c. As c decreases, the line slides down until it just touches the parabola. That happens at $c = -1$, where the single point is (1, 1).

Check by algebra: Set $x^2 = 2x + c$, or $x^2 - 2x - c = 0$. Exactly one solution means the discriminant is 0: $4 + 4c = 0$, so $c = -1$.

Why both methods matter: The slider shows what is happening and gets you close, but sliders move in steps and may stop at a value such as −0.98. The discriminant gives the exact answer. On harder questions, use the graph to understand the situation and algebra to finish.

When not to use the calculator

The calculator is a tool, not a rule. Skip it in these situations:

- **One or two steps of algebra finish the job.** Typing $3x + 5 = 20$ into a calculator takes longer than solving it.
- **The question asks about structure.** "Which expression is equivalent…" with letters such as a and b, or "What does the 4 represent in this model?" are about meaning, not computation.
- **The answer must be exact and the graph gives a decimal.** You still need the algebra to turn 0.4166… into 5/12.
- **A long expression invites typing errors.** One missing parenthesis can produce a convincing but wrong graph.

A useful rule: if you can write the first line of algebra right away, write it. If you would have to guess at a method, or if the algebra would take more than a few lines, graph.

TIP Use the calculator to *check* as often as to *solve*. After solving by hand, graphing or substituting your answer takes a few seconds and catches sign errors and other slips.

The reference sheet

The reference sheet is not a full formula list. Here is what it contains:

Topic	What the sheet gives
Circle	$A = \pi r^2$; $C = 2\pi r$
Rectangle	$A = \ell w$
Triangle	$A = (1/2)bh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Special right triangles	30°-60°-90° with sides x, $x\sqrt{3}$ and 2x; 45°-45°-90° with sides s, s and $s\sqrt{2}$
Rectangular solid	$V = \ell wh$
Right circular cylinder	$V = \pi r^2 h$
Sphere	$V = (4/3)\pi r^3$
Right circular cone	$V = (1/3)\pi r^2 h$
Pyramid	$V = (1/3)\ell wh$
Angles and arcs	A circle has 360 degrees of arc, or 2π radians. The angles of a triangle add up to 180 degrees.

Just as useful is knowing what the sheet does **not** give you. You must know these from memory: slope and the forms of a line, distance and midpoint, the circle equation, the quadratic formula and discriminant, the vertex formula, SOH-CAH-TOA and $\sin x° = \cos(90° - x°)$, similar-triangle proportions, arc length and sector area, degree-radian conversion, exponent rules, percent change, exponential models, mean and median, and probability. Appendix D collects them.

Know the sheet well enough that you rarely need to open it. It pays off most on volume questions, where the cone, sphere and pyramid formulas are easy to mix up, and on special right triangles.

Example 5: Working backward from a volume

Problem: A sphere has a volume of 36π cubic centimeters. What is the radius, in centimeters, of the sphere?

Thinking: The sheet gives $V = (4/3)\pi r^3$. Set $(4/3)\pi r^3 = 36\pi$. Divide both sides by π to get $(4/3)r^3 = 36$, then multiply by 3/4 to get $r^3 = 27$. So $r = 3$.

Traps: Using the cone or cylinder formula by mistake gives a different value, so match the formula to the shape named in the question. Stopping at $r^3 = 27$ and answering 27 solves for the wrong quantity.

Common traps

Trap	How to spot it
Typing a mixed number such as 2 1/4	Your answer has a whole number and a fraction side by side. Convert to 9/4 or 2.25.

Trap	How to spot it
A short repeating decimal such as .67	Your decimal has fewer characters than the space allows. Use the fraction or fill the space.
Entering the y-coordinate when the question asks for x	You graphed both sides of an equation. Only the x-coordinate is the solution.
Missing an intersection	A window can hide solutions. Adjust it and confirm the count algebraically.
Trusting a slider's decimal	The slider stopped at a value such as 2.98. Confirm the exact value with algebra.
Wrong angle mode	A trigonometry result looks strange. Check whether the calculator is in degree or radian mode.
Typing units or symbols	Your entry has %, $, a comma or a word. Remove it.

Check yourself

1. How many characters may a negative SPR answer use, and how many may a positive one use?
2. How would you enter an answer of 7/6? Name two acceptable forms besides the fraction.
3. You graph both sides of an equation and the graphs meet at (4, −9). What is the solution?
4. Which volume formulas appear on the reference sheet, and which line and circle facts do not?
5. Name two situations in which algebra is faster than the calculator.
6. Answers: (1) Six for a negative answer, including the minus sign; five for a positive answer. (2) 1.166 or 1.167, since 7/6 = 1.1666…; never 1.17. (3) x = 4. (4) Rectangular solid, cylinder, sphere, cone and pyramid; slope, line forms, distance, midpoint and the circle equation are not on it. (5) Examples: a one-step or two-step equation; a question about the meaning of a number in a model or about the structure of an expression.

Practice Set

10 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C1501

$3(2x - 1) = 2x + 6$

What value of x is the solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

2 M-C1502

The monthly price of a streaming plan increased from $16.00 to $18.00. The price increased by p%. What is the value of p?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

3 M-C1503

A right circular cone has a radius of 3 inches and a height of 10 inches. What is the volume, in cubic inches, of the cone?

A 10π **B** 30π **C** 90π **D** 100π

---

4 M-C1504

$y = (3/4)x - 2$

$5x - 2y = 18$

Which ordered pair (x, y) is the solution to the given system of equations?

A (1, 4) **B** (2, −4) **C** (4, 1) **D** (6, 5/2)

---

5 M-C1505

A bike-share service charges a fixed starting fee plus a constant rate per minute. The total cost of a 20-minute ride is $7.00, and the total cost of a 45-minute ride is $12.50. What is the total cost, in dollars, of a 32-minute ride?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

6 M-C1506

A student recorded the heights, in centimeters, of 10 bean seedlings two weeks after planting: 8.5, 12.0, 10.4, 7.9, 11.6, 9.2, 13.1, 10.0, 8.8, 11.5. What is the median height, in centimeters, of the 10 seedlings?

A 10.0 **B** 10.2 **C** 10.3 **D** 10.4

---

7 M-C1507

$y = -x^2 + 4x + 1$

$y = 2x + 3$

How many solutions does the given system of equations have?

A Zero **B** Exactly one **C** Exactly two **D** Infinitely many

---

8 M-C1508

In right triangle ABC, angle C is a right angle, the measure of angle A is 60°, and AB = 14. The area of triangle ABC is $k\sqrt{3}$ square units, where k is a constant. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

9 M-C1509

The function f is defined by $f(x) = (3x - 1)(x + 2)$. The graph of $y = f(x)$ in the xy-plane has its vertex at (h, k). What is the value of h?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

10 M-C1510

Which expression is equivalent to $(2x + 1)^2 - (x - 3)^2$?

A $3x^2 - 8$ **B** $3x^2 - 2x + 10$ **C** $(x - 2)(3x - 2)$ **D** $(x + 4)(3x - 2)$

Practice Set Explanations

1. Correct answer: 9/4 or 2.25 M-C1501

Linear equations in one variable · Algebra · Easier

Distribute to get $6x - 3 = 2x + 6$. Subtract $2x$ and add 3 to get $4x = 9$, so $x = 9/4$. Enter 9/4 or 2.25; both fit in the five-character space. Do not enter the mixed number 2 1/4. A common wrong answer is 7/4, which comes from distributing as $6x - 1$ instead of $6x - 3$.

1. Distribute the 3: $6x - 3 = 2x + 6$.
2. Subtract $2x$ from both sides: $4x - 3 = 6$.
3. Add 3 to both sides: $4x = 9$.
4. Divide by 4: $x = 9/4$, which can be entered as 9/4 or 2.25.

Calculator option: Graph $y = 3(2x - 1)$ and $y = 2x + 6$. They intersect at $x = 2.25$, which you can enter as 2.25 or as 9/4.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

2. Correct answer: 12.5 or 25/2 M-C1502

Percentages · Problem-Solving and Data Analysis · Easier

Percent increase is the change divided by the original amount, times 100. The change is $18 - 16 = 2$ dollars, and $2/16 = 0.125$, so the increase is 12.5% and $p = 12.5$. Type 12.5 with no percent sign. Entering 0.125 gives the decimal rate, not p, and about 11.1 comes from dividing by the new price, 18, instead of the original price.

1. Find the change in price: $18.00 - 16.00 = 2.00$ dollars.
2. Divide by the original price: $2.00/16.00 = 0.125$.
3. Convert to a percent: $0.125 \times 100 = 12.5$, so the increase is 12.5%.
4. Enter 12.5 (or 25/2) with no percent sign.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

3. Correct answer: B M-C1503

Area and volume · Geometry and Trigonometry · Easier

B *30π*

The reference sheet gives the volume of a right circular cone as $V = (1/3)\pi r^2 h$. Substituting $r = 3$ and $h = 10$ gives $V = (1/3)\pi(9)(10) = 30\pi$ cubic inches. The work here is choosing the right formula from the sheet and putting each measurement in the right place.

1. Choose the formula from the reference sheet: $V = (1/3)\pi r^2 h$, where r is the radius and h is the height.
2. Substitute $r = 3$ and $h = 10$: $V = (1/3)\pi(3^2)(10)$.
3. Simplify: $(1/3)(9)(10) = 30$, so $V = 30\pi$ cubic inches.

A This computes $(1/3)\pi(3)(10)$: the radius was not squared before multiplying by the height.

C This is $\pi r^2 h$, the volume of a cylinder with the same radius and height; the cone formula includes a factor of 1/3.

D This swaps the radius and the height, computing $(1/3)\pi(10^2)(3) = 100\pi$ instead of using $r = 3$ and $h = 10$.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

4. Correct answer: C M-C1504

Systems of two linear equations in two variables · Algebra · Medium

C *(4, 1)*

The solution is the point that lies on both lines, which is where their graphs intersect. Substituting $y = (3/4)x - 2$ into the second equation gives $5x - 2((3/4)x - 2) = 18$, so $(7/2)x + 4 = 18$ and $x = 4$. Then $y = (3/4)(4) - 2 = 1$, and (4, 1) checks in both equations.

1. Substitute $y = (3/4)x - 2$ into $5x - 2y = 18$: $5x - 2((3/4)x - 2) = 18$.
2. Distribute: $5x - (3/2)x + 4 = 18$, so $(7/2)x = 14$.
3. Solve: $x = 14 \times (2/7) = 4$.
4. Find y: $y = (3/4)(4) - 2 = 1$, so the solution is (4, 1).

A This reverses the coordinates of the solution. The point (1, 4) fails both equations; for example, $(3/4)(1) - 2$ is not 4.

B This point satisfies only $5x - 2y = 18$. Checking one equation is not enough: $(3/4)(2) - 2 = -1/2$, not -4.

D This point satisfies only the first equation. Checking the second equation shows $5(6) - 2(5/2) = 25$, not 18.

Calculator option: Graph both equations exactly as given and select the intersection point; the calculator shows (4, 1). An equation in standard form graphs just as well as one solved for y.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

5. Correct answer: 9.64 M-C1505

Linear functions · Algebra · Medium

The rate is the slope: $(12.50 - 7.00)/(45 - 20) = 0.22$ dollar per minute. The fee is $7.00 - 0.22(20) = 2.60$ dollars, so the cost of an m-minute ride is $0.22m + 2.60$, and a 32-minute ride costs 9.64 dollars. Enter 9.64 with no dollar sign; the fraction 241/25 is too long to fit. Answering 7.04 leaves out the starting fee.

1. Let C(m) be the cost, in dollars, of an m-minute ride.
2. Slope: $(12.50 - 7.00)/(45 - 20) = 5.50/25 = 0.22$ dollar per minute.
3. Fee: $7.00 - 0.22(20) = 2.60$, so $C(m) = 0.22m + 2.60$.
4. Evaluate: $C(32) = 0.22(32) + 2.60 = 7.04 + 2.60 = 9.64$.

Calculator option: Enter the points (20, 7) and (45, 12.5) in a table and fit a linear model with the regression (~)

feature. It reports a slope of 0.22 and an intercept of 2.6; evaluate the model at 32.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

6. Correct answer: B M-C1506

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Medium

B *10.2*

Sort the heights: 7.9, 8.5, 8.8, 9.2, 10.0, 10.4, 11.5, 11.6, 12.0, 13.1. With 10 values, the median is the mean of the 5th and 6th values: (10.0 + 10.4)/2 = 10.2 centimeters. A calculator median function returns the same result from the unsorted list.

1. Order the 10 heights from least to greatest: 7.9, 8.5, 8.8, 9.2, 10.0, 10.4, 11.5, 11.6, 12.0, 13.1.
2. With an even count, the median is the mean of the two middle values, the 5th and 6th.
3. Compute (10.0 + 10.4)/2 = 10.2 centimeters.

A 10.0 is only the 5th value of the sorted list. With an even number of values, the median is the mean of the two middle values.

C 10.3 is the mean, 103.0/10, not the median. The question asks for the middle of the sorted list.

D This averages the 5th and 6th values of the unsorted list, 11.6 and 9.2. The values must be sorted before the middle is found.

Calculator option: Type the 10 heights as a list inside the median function; the calculator returns 10.2 without any sorting by hand. The mean function on the same list returns 10.3, a reminder that the two measures differ.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

7. Correct answer: A M-C1507

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

A *Zero*

Setting the expressions equal gives $-x^2 + 4x + 1 = 2x + 3$, or $x^2 - 2x + 2 = 0$. Its discriminant is $(-2)^2 - 4(1)(2) = -4$, which is negative, so there is no real solution. On a graph, the line stays above the parabola, whose highest point is (2, 5), and the two graphs never meet.

1. Set the right sides equal: $-x^2 + 4x + 1 = 2x + 3$.
2. Rearrange: $x^2 - 2x + 2 = 0$.
3. Compute the discriminant $b^2 - 4ac = (-2)^2 - 4(1)(2) = -4$.
4. A negative discriminant means no real solution, so the system has zero solutions.

B Exactly one solution would require the line to touch the parabola at a single point, but the vertical gap between the graphs is never less than 1.

C Two solutions come from a sign slip in the discriminant, such as computing 4 + 8 = 12 instead of 4 − 8 = −4.

D Infinitely many solutions occur only when both equations describe the same graph, and a line can never coincide with a parabola.

Calculator option: Graph both equations and look for intersection points. None appear in any window, and zooming in near x = 1 shows the line staying at least 1 unit above the parabola.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

8. Correct answer: 49/2 or 24.5 M-C1508

Right triangles and trigonometry · Geometry and Trigonometry · Medium

AB is the hypotenuse, and angle B is 30°. The reference sheet shows a 30°-60°-90° triangle with sides x, $x\sqrt{3}$ and 2x, so 2x = 14 and x = 7. The legs are AC = 7 (opposite the 30° angle) and BC = $7\sqrt{3}$. The area is $(1/2)(7)(7\sqrt{3}) = (49/2)\sqrt{3}$, so k = 49/2, or 24.5. Answering 49 leaves out the factor 1/2.

1. Angle B = 180° − 90° − 60° = 30°, so ABC is a 30°-60°-90° triangle with hypotenuse AB = 14.
2. From the reference sheet, the hypotenuse is 2x, so x = 7: the short leg AC = 7 and the long leg BC = $7\sqrt{3}$.
3. Area = (1/2)(AC)(BC) = $(1/2)(7)(7\sqrt{3}) = (49/2)\sqrt{3}$.
4. So k = 49/2, entered as 49/2 or 24.5.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

9. Correct answer: -5/6 or -.8333 or -0.833 M-C1509

Nonlinear functions · Advanced Math · Harder

The vertex lies halfway between the zeros, x = 1/3 and x = −2, so h = (1/3 + (−2))/2 = −5/6. Expanding to $3x^2 + 5x - 2$ and using x = −b/(2a) = −5/6 confirms it. Enter −5/6, −.8333 or −0.833; −.83 is too short. A sign slip gives 5/6, and using 1 instead of 1/3 as a zero gives −1/2.

1. Find the zeros: 3x − 1 = 0 gives x = 1/3, and x + 2 = 0 gives x = −2.
2. The axis of symmetry is halfway between the zeros: h = (1/3 + (−2))/2.
3. Simplify: 1/3 − 2 = −5/3, and (−5/3)/2 = −5/6.
4. Check: $f(x) = 3x^2 + 5x - 2$, so −b/(2a) = −5/6. Enter −5/6, −.8333 or −0.833.

Calculator option: Graph y = (3x − 1)(x + 2) and select the vertex; the calculator shows about (−0.833, −4.083). Enter −0.833 or −.8333, or recognize −0.8333… as −5/6.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

10. Correct answer: D M-C1510

Equivalent expressions · Advanced Math · Harder

D *(x + 4)(3x − 2)*

The expression is a difference of squares, $a^2 - b^2 = (a - b)(a + b)$, with $a = 2x + 1$ and $b = x - 3$. Then $a - b = 2x + 1 - x + 3 = x + 4$ and $a + b = 3x - 2$, giving $(x + 4)(3x - 2)$. Expanding both the original and this product gives $3x^2 + 10x - 8$, which confirms the match.

1. Let $a = 2x + 1$ and $b = x - 3$, so the expression is $a^2 - b^2 = (a - b)(a + b)$.
2. Compute $a - b = (2x + 1) - (x - 3) = x + 4$.
3. Compute $a + b = (2x + 1) + (x - 3) = 3x - 2$.
4. So the expression equals $(x + 4)(3x - 2)$, which expands to $3x^2 + 10x - 8$.

A This squares each term separately, treating $(2x + 1)^2$ as $4x^2 + 1$ and $(x - 3)^2$ as $x^2 + 9$, which drops both middle terms.

B This expands both squares correctly but subtracts only the first term of $x^2 - 6x + 9$; the minus sign applies to every term.

C This forgets to distribute the minus sign in $a - b$, writing $2x + 1 - x - 3 = x - 2$ instead of $x + 4$.

Calculator option: Graph $y = (2x + 1)^2 - (x - 3)^2$, then graph each choice; only choice D lies exactly on the original. As a quick check, $x = 1$ gives 5 for the original and for D, but −5, 11 and −1 for A, B and C.

Review: Chapter 15, The Math Toolkit: Calculator, Reference Sheet and Student-Produced Responses (page 228)

CHAPTER 16

Linear Equations in One Variable

A linear equation in one variable has a single unknown, usually x, and no power of it higher than 1: no x^2, no $\sqrt{x}$ and no x in a denominator. An equation such as $5(x - 2) = 3x + 4$ is the most basic algebra on the test. It appears as a stand-alone question, inside word problems, and as the last step of systems, geometry and data questions.

This chapter builds a fast, reliable method, then covers the variations that make these equations hard: equations with no solution or infinitely many solutions, unknown constants, and questions that ask for an expression instead of x.

What the question looks like

Most questions in this skill take one of five forms:

- **Solve.** An equation is given, often with parentheses, fractions or decimals, and you find x, either from choices or as a student-produced response.
- **Solve for an expression.** For example, "If $3x + 2 = 11$, what is the value of $6x + 4$?"
- **Count solutions.** "How many solutions does the given equation have?" The choices are usually zero, exactly one, exactly two and infinitely many.
- **Find a constant.** "In the given equation, k is a constant. The equation has no solution. What is the value of k?"
- **Model a context.** A short situation with a fixed amount and a rate. You write the equation, solve it or both.

How to recognize it

The equation has one variable, and every term that contains it is a number times the variable, or will be once you distribute and clear fractions. If you see x^2, a variable in an exponent or a variable under a radical, you are in Chapter 21 instead. If you see two variables, such as x and y, you are in Chapter 17 or 18.

Letters such as a, b and k are constants (also called parameters) that stand for fixed numbers. They make a question harder without changing the skill.

In a word problem, look for a starting amount plus or minus a rate times an unknown, such as a fee plus a monthly charge or a balance minus spending per week.

The method

1. **Clear fractions and decimals.** Multiply every term on both sides by the least common denominator, or by 10 or 100 for decimals.
2. **Distribute** any number in front of parentheses to every term inside, including its sign.
3. **Combine like terms** on each side separately.
4. **Collect** the variable terms on one side and the constants on the other. Moving the smaller variable term avoids negative coefficients.
5. **Divide** both sides by the coefficient of the variable.

6. **Check and reread.** Substitute your answer into the original equation, then reread the question. Does it ask for x, or for something else?

Step 6 is the one students skip, yet substituting takes about 15 seconds and catches most slips.

Clearing fractions and decimals

The least common denominator (LCD) is the smallest number that every denominator divides into evenly. Multiply every term by it, including any constant that has no denominator, and the fractions disappear. With decimals, multiply by 10 for one decimal place or by 100 for two.

How many solutions?

After the first three steps, any linear equation can be written as $ax + b = cx + d$. Compare the sides:

After simplifying	Number of solutions	Example
Different x-coefficients ($a \neq c$)	Exactly one	$5x + 2 = 3x + 8$
Same x-coefficients, different constants ($a = c$, $b \neq d$)	None	$4x + 1 = 4x - 7$
Same x-coefficients and same constants ($a = c$, $b = d$)	Infinitely many	$2x + 6 = 2x + 6$

If you keep solving in the last two rows, the x-terms cancel. A false statement left behind, such as $1 = -7$, means no solution. A true statement, such as $6 = 6$, means infinitely many. A linear equation can never have exactly two solutions.

To find a constant that gives no solution, set the x-coefficients equal and check that the constants differ. For infinitely many solutions, set the x-coefficients equal and the constants equal.

Solving for an expression

When a question asks for an expression instead of x, look for a multiple. If $3x + 2 = 11$, then $6x + 4 = 2(3x + 2) = 22$, and you never need x. When the same expression appears more than once, as in $5(x - 1) = (x - 1) + 12$, treat it as a single unknown.

Building an equation from words

Name the unknown first, with its units: "let m be the number of months." Then translate one phrase at a time. A one-time amount is a constant. An amount "per" or "each" multiplies the unknown. "Total," "is" or "costs" becomes the equals sign. Before solving, test your equation with an easy value, such as $m = 1$, to confirm that it describes the situation.

Worked examples

Example 1: Fractions on both sides

Problem: $(x - 4)/3 + 2 = (x + 1)/2$. What is the solution to the given equation?

Thinking: The LCD of 3 and 2 is 6. Multiply every term by 6: $2(x - 4) + 12 = 3(x + 1)$. Distribute: $2x - 8 + 12 = 3x + 3$, so $2x + 4 = 3x + 3$. Subtract $2x$ and 3 from both sides: $x = 1$.

Check: The left side is $(1 - 4)/3 + 2 = -1 + 2 = 1$, and the right side is $(1 + 1)/2 = 1$.

Trap: Forgetting to multiply the lone 2 by 6 gives $2x - 8 + 2 = 3x + 3$, which leads to $x = -9$. Every term gets multiplied, not just the fractions.

Example 2: Decimals and variables on both sides

Problem: $0.6x - 1.5 = 0.25x + 2$. What is the solution to the given equation?

Thinking: Multiply every term by 100: $60x - 150 = 25x + 200$. Subtract $25x$: $35x - 150 = 200$. Add 150: $35x = 350$, so $x = 10$.

Check: $0.6(10) - 1.5 = 4.5$ and $0.25(10) + 2 = 4.5$.

Trap: Multiplying by 10 instead of 100 leaves the decimal $2.5x$ on the right. Multiply by enough to clear the longest decimal.

Example 3: Counting solutions

Problem: $5(x + 2) - 3x = 2(x + 4) + 2$. How many solutions does the given equation have?

Thinking: The left side is $5x + 10 - 3x = 2x + 10$. The right side is $2x + 8 + 2 = 2x + 10$. The two sides are identical, so the equation is true for every x: infinitely many solutions.

Variation: With + 1 instead of the final + 2, the right side becomes $2x + 9$: same x-coefficient, different constant, so no solution.

Trap: Seeing the x-terms cancel and answering "zero" by reflex. Cancellation only rules out "exactly one"; the leftover statement decides between none and infinitely many.

Example 4: Solving for an expression

Problem: If $8 - 3x = 2$, what is the value of $9x - 24$?

Thinking: Notice that $9x - 24 = -3(8 - 3x)$. Since $8 - 3x = 2$, the expression equals $-3(2) = -6$.

Check by solving: $8 - 3x = 2$ gives $x = 2$, and $9(2) - 24 = -6$.

Trap: Answering 2, which is x, or 6, which drops the negative sign. Name what the question asks for before you finish.

Example 5: An equation from a context

Problem: A water tank holds 450 liters and drains at 18 liters per minute. A second tank holds 120 liters and fills at 15 liters per minute. After how many minutes will the two tanks hold the same amount of water?

Thinking: Let t be the number of minutes. The first tank holds $450 - 18t$ liters and the second holds $120 + 15t$ liters. Set them equal: $450 - 18t = 120 + 15t$. Add $18t$ and subtract 120: $330 = 33t$, so $t = 10$.

Check: After 10 minutes, the tanks hold $450 - 180 = 270$ liters and $120 + 150 = 270$ liters.

Trap: Answering 270, the shared amount of water, instead of the time. On a graph, the intersection is (10, 270), and the question asks for the first coordinate.

Example 6 (harder-module): Two constants and no solution

Problem: In the equation $(2x + a)/3 = (bx - 4)/6$, a and b are constants. The equation has no solution. What must be true about a and b?

Thinking: Multiply both sides by 6: $2(2x + a) = bx - 4$, so $4x + 2a = bx - 4$. No solution requires equal x-coefficients and different constants. So $b = 4$, and $2a \neq -4$, which means $a \neq -2$.

Conclusion: $b = 4$, and a can be any number except -2. With $a = -2$, the sides would be identical, giving infinitely many solutions instead.

Trap: Setting the constants equal ($2a = -4$) is the condition for infinitely many solutions. Write down which condition you need before you match anything.

Common traps

Trap	How to spot it
Distributing to only the first term	A number in front of parentheses touched only one term inside. Recheck each product.
Dropping a negative sign	A minus sign stands in front of parentheses, as in $-2(x - 5)$. The second term becomes $+10$.
Clearing only the fractions	A constant without a denominator was not multiplied by the LCD.
Moving a term with the wrong sign	A term changed sides but kept its sign. Add or subtract the same amount on both sides.
Answering x when the question asks for an expression	The last line of the question names something other than x.
Choosing "zero" whenever x cancels	The leftover statement is true, which means infinitely many solutions.
Mixing up the conditions for none and infinitely many	You matched the constants when the question said "no solution."
Reporting a shared total instead of the unknown	In a context, the answer is the quantity you named as the variable, in its units.

Harder-module variations

- **Constants that must meet a condition.** Expect "no solution" and "infinitely many solutions" with one or two constants, as in Example 6. Write the equation as $ax + b = cx + d$ first, then use the table.
- **A given solution.** "If $x = 4$ is a solution, what is the value of a?" Substitute the number for x, and the equation becomes a linear equation in a.
- **Heavier fractions.** Three fraction terms, or a subtraction in front of a fraction, as in $(3x)/4 - (x - 2)/6 = 5$. The LCD method still works; slow down when you distribute the negative sign.
- **A percent inside a context.** A total that includes a 15% tip is 1.15 times the base amount. Divide the total by 1.15; subtracting 15% of the total does not undo the tip.
- **A hidden multiple.** Given $2x - 3 = 7$, find $6 - 4x$. Spot the factor first: $6 - 4x = -2(2x - 3) = -14$.

TIP On a harder question with constants, a slider graph can show what is happening, but the exact value comes from matching coefficients. Use the graph to check the algebra.

Check yourself

1. What are the six steps of the method, in order?
2. After simplifying, an equation reads $7x - 3 = 7x + 4$. How many solutions does it have, and why?

3. What values of a and b give $ax + 5 = 3x + b$ infinitely many solutions?
4. If $4x - 6 = 18$, what is the value of $2x - 3$, and what is the fastest way to see it?
5. A club charges a $30 fee plus $15 per class. Write an equation for a member who paid $165 in total for c classes.
6. Answers: (1) Clear fractions and decimals, distribute, combine like terms, collect, divide, then check and reread. (2) None: the x-coefficients match but the constants differ, so the equation reduces to $-3 = 4$, which is false. (3) $a = 3$ and $b = 5$. (4) 9, because $2x - 3$ is half of $4x - 6$. (5) $30 + 15c = 165$, which gives $c = 9$.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/linear-equations-one-variable

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C1601

$3(x + 4) = 27$

What is the solution to the given equation?

A 5 **B** 23/3 **C** 9 **D** 13

2 M-C1602

$7x - 4 = 3x + 18$

What value of x is the solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

3 M-C1603

If $4x + 6 = 26$, what is the value of $2x + 3$?

A 5 **B** 10 **C** 13 **D** 26

4 M-C1604

A gym charges a one-time sign-up fee of $45 plus $28 for each month of membership. A customer has paid a total of $269 for the sign-up fee and m months of membership. Which equation represents this situation?

A $45m + 28 = 269$ **B** $28m + 45 = 269$ **C** $28m - 45 = 269$ **D** $28(m + 45) = 269$

5 M-C1605

$(2x - 1)/3 = (x + 4)/2$

What is the solution to the given equation?

A 11/4 **B** 6 **C** 13 **D** 14

6 M-C1606

$(3/4)x - 2 = (1/6)x + 5$

What value of x is the solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

7 M-C1607

$4(2x - 3) = 3(x + 6) - 5$

What is the solution to the given equation?

A 1/5 **B** 13/5 **C** 16/5 **D** 5

8 M-C1608

$0.4(x + 15) = 0.25x + 9$

What is the solution to the given equation?

A 20 **B** 56 **C** 60 **D** 100

9 M-C1609

$3(2x - 4) + x = 7x - 12$

How many solutions does the given equation have?

A Zero **B** Exactly one **C** Exactly two **D** Infinitely many

10 M-C1610

Phone plan P costs $25 per month plus $6 for each gigabyte of data used that month. Phone plan Q costs $53 per month plus $2.50 for each gigabyte of data used that month. For what number of gigabytes of data used in a month do the two plans cost the same amount?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

11 M-C1611

$3(2x + 5) = (2x + 5) + 24$

Based on the given equation, what is the value of $2x + 5$?

A 7/2 **B** 12 **C** 24 **D** 36

12 M-C1612

$(ax)/3 - 4 = (x + a)/2$

In the given equation, a is a constant. If $x = 6$ is a solution to the equation, what is the value of a?

A −2/3 **B** 22/9 **C** 14/3 **D** 7

13 M-C1613

$k(3x - 2) = 6x + 5$

In the given equation, k is a constant. The equation has no solution. What is the value of k?

A −5/2 **B** 1/2 **C** 2 **D** 6

14 M-C1614

$4(ax + 3) - 2x = 10x + b$

In the given equation, a and b are constants. The equation has infinitely many solutions. Which choice gives the values of a and b?

A $a = 3$ and $b = 12$ **B** $a = 3$ and $b = 3$ **C** $a = 2$ and $b = 12$ **D** $a = 5/2$ and $b = 12$

15 M-C1615

A taxi ride costs a base fare of $4.00 plus $2.50 per mile. A rider paid the fare plus a tip equal to 20% of the fare, for a total of $45.60. How many miles long was the ride?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

Practice Set Explanations

1. Correct answer: A M-C1601

Linear equations in one variable · Algebra · Easier

A *5*

Divide both sides by 3 to get x + 4 = 9, then subtract 4 to get x = 5. Distributing first works too: 3x + 12 = 27, so 3x = 15 and x = 5. Check by substituting: 3(5 + 4) = 3(9) = 27.

1. Divide both sides by 3: x + 4 = 9.
2. Subtract 4 from both sides: x = 5.
3. Check: 3(5 + 4) = 27.

B This distributes the 3 only to x, writing 3x + 4 = 27, which gives 3x = 23 and x = 23/3.

C 9 is the value of x + 4, not x. One more step, subtracting 4 from both sides, is still needed.

D This adds 12 instead of subtracting it after distributing: 3x = 27 + 12 = 39, so x = 13, which does not check.

Review: Chapter 16, Linear Equations in One Variable (page 239)

2. Correct answer: 11/2 or 5.5 M-C1602

Linear equations in one variable · Algebra · Easier

Subtract 3x from both sides to get 4x − 4 = 18, then add 4 to get 4x = 22, so x = 22/4 = 11/2. Enter 11/2 or 5.5, not the mixed number 5 1/2. A common wrong answer is 7/2, which comes from subtracting 4 instead of adding it, so that 4x = 14.

1. Subtract 3x from both sides: 4x − 4 = 18.
2. Add 4 to both sides: 4x = 22.
3. Divide by 4: x = 22/4 = 11/2, or 5.5.

Calculator option: Graph y = 7x − 4 and y = 3x + 18; the lines intersect at x = 5.5.

Review: Chapter 16, Linear Equations in One Variable (page 239)

3. Correct answer: C M-C1603

Linear equations in one variable · Algebra · Easier

C *13*

Notice that 4x + 6 = 2(2x + 3). Since 2(2x + 3) = 26, dividing by 2 gives 2x + 3 = 13. Solving for x first also works (4x = 20, so x = 5, and 2(5) + 3 = 13), but the structure shortcut takes a single step.

1. Factor the left side: 4x + 6 = 2(2x + 3).
2. So 2(2x + 3) = 26.
3. Divide both sides by 2: 2x + 3 = 13.

A 5 is the value of x. The question asks for the value of the expression 2x + 3, not for x.

B 10 is the value of 2x. The constant 3 still needs to be added to reach the value of 2x + 3.

D 26 is the value of 4x + 6, which is twice the requested expression 2x + 3.

Review: Chapter 16, Linear Equations in One Variable (page 239)

4. Correct answer: B M-C1604

Linear equations in one variable · Algebra · Easier

B *28m + 45 = 269*

The monthly charge depends on m, so it is 28m dollars. The sign-up fee is paid once, so it is a constant 45 added to the total. That gives 28m + 45 = 269. Solving shows 28m = 224 and m = 8 months, a whole number, which supports the model.

1. Let m be the number of months of membership.
2. Monthly charges: 28m dollars. One-time fee: 45 dollars.
3. Total paid: 28m + 45 = 269.

A This swaps the roles of the two amounts, charging $45 per month and treating $28 as the one-time fee.

C The sign-up fee adds to the amount paid. Subtracting it would mean the customer got $45 back instead of paying it.

D This adds 45 to the number of months, as if the customer paid for 45 extra months instead of a single $45 fee.

Review: Chapter 16, Linear Equations in One Variable (page 239)

5. Correct answer: D M-C1605

Linear equations in one variable · Algebra · Medium

D *14*

Multiply both sides by 6, the least common denominator, to clear the fractions: 2(2x − 1) = 3(x + 4). Distribute to get 4x − 2 = 3x + 12, so x = 14. Check: (2(14) − 1)/3 = 27/3 = 9 and (14 + 4)/2 = 9.

1. Multiply both sides by 6: 6 × (2x − 1)/3 = 2(2x − 1) and 6 × (x + 4)/2 = 3(x + 4).
2. Distribute: 4x − 2 = 3x + 12.
3. Subtract 3x and add 2 on both sides: x = 14.
4. Check: both sides equal 9 when x = 14.

A This multiplies each numerator by its own denominator, 3(2x − 1) = 2(x + 4), instead of by the other side's denominator.

B This distributes the 3 only to x on the right side, writing 3x + 4 instead of 3x + 12.

C This distributes the 2 only to 2x on the left side, writing 4x − 1 instead of 4x − 2.

Calculator option: Graph y = (2x − 1)/3 and y = (x + 4)/2; they intersect at (14, 9). Widen the viewing window if the intersection is off the screen.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: 12 M-C1606

Linear equations in one variable · Algebra · Medium

Multiply every term by 12, the least common denominator: 9x − 24 = 2x + 60. Subtract 2x and add 24 to get 7x = 84, so x = 12. Check: (3/4)(12) − 2 = 7 and (1/6)(12) + 5 = 7. A common wrong answer is 1, which comes from multiplying only the fractions by 12 and leaving −2 and 5 unchanged.

1. Multiply every term on both sides by 12: 9x − 24 = 2x + 60.
2. Subtract 2x from both sides: 7x − 24 = 60.
3. Add 24 to both sides: 7x = 84, so x = 12.

Calculator option: Graph y = (3/4)x − 2 and y = (1/6)x + 5 and read the intersection, (12, 7). Enter only the x-coordinate.

Review: Chapter 16, Linear Equations in One Variable (page 239)

7. Correct answer: D M-C1607

Linear equations in one variable · Algebra · Medium

D *5*

Distribute on both sides: 8x − 12 = 3x + 18 − 5, which simplifies to 8x − 12 = 3x + 13. Subtract 3x to get 5x − 12 = 13, add 12 to get 5x = 25, and divide to get x = 5. Check: 4(7) = 28 and 3(11) − 5 = 28.

1. Distribute: 8x − 12 = 3x + 18 − 5.
2. Combine constants on the right: 8x − 12 = 3x + 13.
3. Subtract 3x and add 12 on both sides: 5x = 25.
4. Divide by 5: x = 5.

A This subtracts 12 instead of adding it when moving the constant, which gives 5x = 13 − 12 = 1.

B This distributes the 3 only to x on the right, writing 3x + 6 − 5 instead of 3x + 18 − 5, so 5x = 13.

C This distributes the 4 only to 2x on the left, writing 8x − 3 instead of 8x − 12, so 5x = 16.

Review: Chapter 16, Linear Equations in One Variable (page 239)

8. Correct answer: A M-C1608

Linear equations in one variable · Algebra · Medium

A *20*

Distribute: 0.4x + 6 = 0.25x + 9. Subtract 0.25x and 6 from both sides to get 0.15x = 3, so x = 3/0.15 = 20. Multiplying every term by 100 first, 40x + 600 = 25x + 900, avoids decimals and gives the same answer. Check: 0.4(35) = 14 and 0.25(20) + 9 = 14.

1. Distribute 0.4: 0.4x + 6 = 0.25x + 9.
2. Subtract 0.25x from both sides: 0.15x + 6 = 9.
3. Subtract 6: 0.15x = 3.
4. Divide: x = 3/0.15 = 20.

B This computes 0.4 × 15 as 0.6 instead of 6, a decimal-place slip that leads to 0.15x = 8.4.

C This drops the product 0.4 × 15 entirely, solving 0.4x = 0.25x + 9, so 0.15x = 9 and x = 60.

D This adds 6 to 9 instead of subtracting it when moving the constant, so 0.15x = 15 and x = 100.

Calculator option: Graph y = 0.4(x + 15) and y = 0.25x + 9; they cross at (20, 14). Graphing sidesteps decimal-place slips like the one behind choice B.

Review: Chapter 16, Linear Equations in One Variable (page 239)

9. Correct answer: D M-C1609

Linear equations in one variable · Algebra · Medium

D *Infinitely many*

Simplify the left side: 6x − 12 + x = 7x − 12. Both sides are now identical, so every value of x makes the equation true. Subtracting 7x from both sides leaves −12 = −12, a true statement with no variable, which signals infinitely many solutions.

1. Distribute on the left: 6x − 12 + x.
2. Combine like terms: 7x − 12 = 7x − 12.
3. Subtract 7x from both sides: −12 = −12, which is always true.
4. Every real number x is a solution, so there are infinitely many.

A This assumes that x-terms canceling means no solution. The leftover statement, −12 = −12, is true, so every value of x works.

B This assumes every linear equation has exactly one solution. That requires different x-coefficients, but both sides simplify to 7x − 12.

C This treats the equation like a quadratic. A linear equation in one variable has zero, one or infinitely many solutions, never exactly two.

Calculator option: Graph y = 3(2x − 4) + x and y = 7x − 12. The two graphs are the same line, so they share every point.

Review: Chapter 16, Linear Equations in One Variable (page 239)

10. Correct answer: 8 M-C1610

Linear equations in one variable · Algebra · Medium

Let g be the number of gigabytes. Set the two monthly costs equal: 25 + 6g = 53 + 2.5g. Subtract 2.5g and 25 from both sides to get 3.5g = 28, so g = 8. Check: both plans cost $73 at 8 gigabytes. Entering 73 is a common error: it is the shared cost, not the number of gigabytes the question asks for.

1. Let g be the number of gigabytes used in a month.
2. Plan P costs 25 + 6g dollars; plan Q costs 53 + 2.5g dollars.
3. Set them equal: 25 + 6g = 53 + 2.5g, so 3.5g = 28.
4. Divide: g = 8. Both plans then cost 25 + 48 = 73 dollars.

Calculator option: Graph y = 25 + 6x and y = 53 + 2.5x. They intersect at (8, 73); the x-coordinate answers the question.

Review: Chapter 16, Linear Equations in One Variable (page 239)

11. Correct answer: B M-C1611

Linear equations in one variable · Algebra · Medium

B *12*

Treat 2x + 5 as a single quantity u. The equation becomes 3u = u + 24, so 2u = 24 and u = 12. Therefore 2x + 5 = 12. Solving for x first (x = 7/2) and then substituting also works, but it takes longer and gives more chances for a slip.

1. Let u = 2x + 5, so the equation is 3u = u + 24.
2. Subtract u from both sides: 2u = 24.
3. Divide by 2: u = 12, so 2x + 5 = 12.

A 7/2 is the value of x, not of 2x + 5. The question asks for the value of the expression.

C 24 is the value of 2u, which is 2(2x + 5). One more division by 2 is needed.

D 36 is the value of each side of the equation, such as 3(2x + 5), not the value of 2x + 5 itself.

Review: Chapter 16, Linear Equations in One Variable (page 239)

12. Correct answer: C M-C1612

Linear equations in one variable · Algebra · Harder

C *14/3*

Substitute x = 6: (6a)/3 − 4 = (6 + a)/2, so 2a − 4 = (6 + a)/2. Multiply both sides by 2: 4a − 8 = 6 + a. Then 3a = 14 and a = 14/3. Check: 2(14/3) − 4 = 16/3, and (6 + 14/3)/2 = (32/3)/2 = 16/3.

1. Substitute x = 6: (6a)/3 − 4 = (6 + a)/2.
2. Simplify the left side: 2a − 4 = (6 + a)/2.
3. Multiply both sides by 2: 4a − 8 = 6 + a.
4. Subtract a and add 8: 3a = 14, so a = 14/3.

A This changes −4 to +4 while clearing the fraction, giving 4a + 8 = 6 + a and 3a = −2.

B This multiplies both fractions by 6 but not the constant −4, giving 12a − 4 = 18 + 3a and 9a = 22.

D This divides only the 6 by 2 on the right side, treating (6 + a)/2 as 3 + a, which gives 2a − 4 = 3 + a.

Review: Chapter 16, Linear Equations in One Variable (page 239)

13. Correct answer: C M-C1613

Linear equations in one variable · Algebra · Harder

C *2*

Distribute: 3kx − 2k = 6x + 5. A linear equation has no solution when the x-coefficients match but the constants differ. Setting 3k = 6 gives k = 2. Then the constants are −2k = −4 and 5, which differ, so the equation becomes 6x − 4 = 6x + 5, which is never true.

1. Distribute k: 3kx − 2k = 6x + 5.
2. For no solution, the x-coefficients must be equal: 3k = 6, so k = 2.
3. Check the constants: −2(2) = −4, which is not 5.
4. The equation becomes 6x − 4 = 6x + 5, which is never true, so k = 2.

A This makes the constants equal (−2k = 5) instead of the x-coefficients. With k = −5/2, the equation has exactly one solution, x = 0.

B This inverts the ratio, solving 6k = 3 instead of 3k = 6. With k = 1/2, the x-coefficients are 3/2 and 6, so there is one solution.

D This sets k equal to 6 without accounting for the 3 inside the parentheses; then 18x − 12 = 6x + 5 has one solution.

Calculator option: Graph y = k(3x − 2) and y = 6x + 5 with a slider for k. At k = 2 the lines are parallel and never meet; the algebra confirms that the value is exact.

Review: Chapter 16, Linear Equations in One Variable (page 239)

14. Correct answer: A M-C1614

Linear equations in one variable · Algebra · Harder

A *a = 3 and b = 12*

Distribute and collect the x-terms: 4ax + 12 − 2x = (4a − 2)x + 12. For infinitely many solutions, the two sides must be identical, so the x-coefficients match, 4a − 2 = 10, and the constants match, 12 = b. The first equation gives 4a = 12, so a = 3 and b = 12.

1. Distribute: 4ax + 12 − 2x = 10x + b.
2. Group the x-terms: (4a − 2)x + 12 = 10x + b.
3. Match x-coefficients: 4a − 2 = 10, so a = 3.
4. Match constants: b = 12.

B This distributes the 4 only to ax, so the constant on the left is 3 instead of 4 × 3 = 12.

C This treats −2x as +2x when matching coefficients, solving 4a + 2 = 10 to get a = 2.

D This ignores the −2x term and solves 4a = 10. The left side's x-coefficient is 4a − 2, not 4a.

Review: Chapter 16, Linear Equations in One Variable (page 239)

15. Correct answer: 13.6 or 68/5 M-C1615

Linear equations in one variable · Algebra · Harder

Let m be the number of miles. The fare is 4 + 2.5m dollars, and the total with a 20% tip is 1.2(4 + 2.5m) = 45.60. Divide by 1.2: 4 + 2.5m = 38, so 2.5m = 34 and m = 13.6. Ignoring the tip gives 16.64, and subtracting 20% of $45.60 gives about 12.99; neither undoes the tip correctly.

1. Let m be the length of the ride in miles. The fare is 4 + 2.5m dollars.
2. A 20% tip makes the total 120% of the fare: 1.2(4 + 2.5m) = 45.60.
3. Divide both sides by 1.2: 4 + 2.5m = 38.
4. Subtract 4 and divide by 2.5: m = 34/2.5 = 13.6.

Calculator option: Graph y = 1.2(4 + 2.5x) and the horizontal line y = 45.6; they intersect at x = 13.6.

Review: Chapter 16, Linear Equations in One Variable (page 239)

CHAPTER 17

Linear Equations in Two Variables and Linear Functions

A linear function changes by the same amount every time its input increases by 1. Its graph is a straight line, and that line can be described by an equation in two variables, such as $y = 2x + 5$ or $3x + 4y = 20$. The test splits this material into two skills, linear functions and linear equations in two variables, but they rest on the same few ideas: slope, intercepts and the forms of a line. This chapter teaches those ideas, then shows how the test asks about them.

What the question looks like

- **Find the equation.** From two points, a table, a graph or a description, choose or build the equation of a line.
- **Interpret a number.** "Which statement is the best interpretation of 1.5 in this context?" The number is usually a slope or an intercept.
- **Use function notation.** Find $f(4)$, or find the input that makes $f(x)$ equal a given value.
- **Parallel and perpendicular lines.** Find a line, a slope or a constant that makes two lines parallel or perpendicular.
- **Intercepts.** Find where a line crosses an axis, or say what that crossing means in context.
- **Two-item combinations.** An equation such as $12a + 8b = 240$ describes all the ways to combine two items that cost \$12 and \$8 for a total of exactly \$240.

How to recognize it

The variables appear only to the first power, with no products of variables, variable denominators, radicals or variable exponents. A function such as $f(x) = 4 - 2x$ is linear because its input appears only to the first power. A table is linear when equal steps in x always produce equal steps in y. A graph is linear when it is a straight line.

A question that asks for a point satisfying two equations is a system (Chapter 18). A question about one line belongs here.

The method

Slope three ways

Slope is the change in y divided by the change in x, often called "rise over run": $m = (y_2 - y_1)/(x_2 - x_1)$. Subtract in the same order on top and bottom. If the x-coordinates match, the line is vertical and its slope is undefined.

- **From two points:** substitute into the formula.
- **From a table:** pick any two rows and divide the change in y by the change in x. The steps in x may not be equal, so always divide by the change between the rows you chose.
- **From a graph:** find two points where the line crosses grid intersections, then count the rise and the run between them. A line that falls from left to right has a negative slope.

For example, this table shows a linear relationship. From x = 2 to x = 5, y rises 6 while x rises 3, so the slope is 2. From x = 5 to x = 11, y rises 12 while x rises 6: the slope is again 2. Working back from (2, 9), the y-intercept is 9 − 2(2) = 5, so y = 2x + 5.

x	2	5	11
y	9	15	27

The three forms of a line

Form	Equation	Best for
Slope-intercept	$y = mx + b$	Reading the slope m and the y-intercept b directly
Point-slope	$y - y_1 = m(x - x_1)$	Writing a line from its slope and any point (x_1, y_1)
Standard	$Ax + By = C$	Contexts with two items; finding both intercepts quickly

In standard form, A and B cannot both be zero. The slope is $-A/B$ and the y-intercept is C/B when $B \neq 0$; the x-intercept is C/A when $A \neq 0$. A vertical line, $x = c$, has undefined slope and cannot use slope-intercept or point-slope form.

Function notation

In $f(x) = 3x - 7$, the letter f names the function, x is the input and f(x) is the output. "Find f(4)" means substitute 4 for x. "Find x when f(x) = 4" means set the output equal to 4 and solve. The point (4, f(4)) lies on the graph of $y = f(x)$.

Meaning in context

- The **slope** is a rate: units of the output per one unit of the input, such as dollars per month.
- The **y-intercept** (or starting value) is the output when the input is 0.
- The **x-intercept** is the input when the output reaches 0, such as the day a balance runs out.

To interpret a number, write the units next to each variable before you read the choices, decide whether the number is a rate or a value, and then say it as a sentence. Eliminate any choice whose units do not match; many wrong interpretations fail this test.

Parallel and perpendicular lines

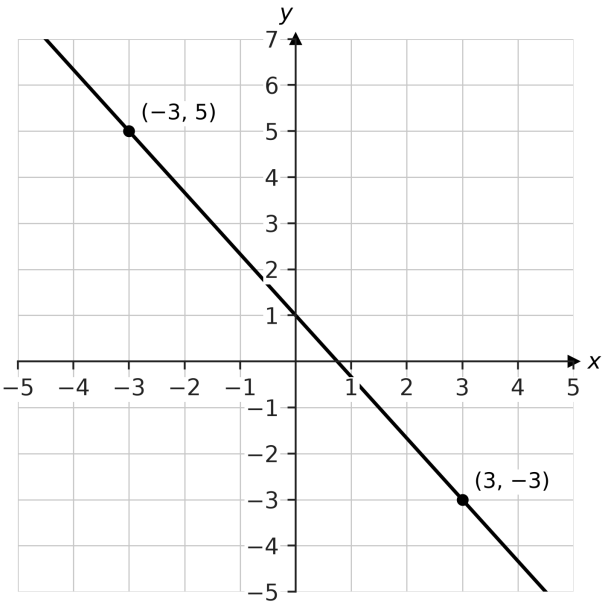
Nonvertical parallel lines have equal slopes and different y-intercepts; equal slopes and intercepts give the same line. Distinct vertical lines are parallel too. For two nonvertical perpendicular lines, the slopes multiply to −1: 2/3 pairs with −3/2. A horizontal line and a vertical line are also perpendicular.

Two-item contexts: ax + by = c

When x and y count two kinds of items, a and b are their prices or sizes and c is the total. The x-intercept, c/a, is the number of x-items when y = 0, and the y-intercept is c/b. The slope, −a/b, is the trade-off: each additional x-item means a/b fewer y-items.

Worked examples

Example 1: Equation from a graph



Problem: The graph shows the line y = f(x). Which equation defines f?

Thinking: Use the labeled points (−3, 5) and (3, −3). From left to right, the line falls 8 units while it runs 6 units, so the slope is −8/6 = −4/3. The line crosses the y-axis at (0, 1), so f(x) = (−4/3)x + 1.

Check: f(3) = −4 + 1 = −3, which matches the labeled point.

Trap: Counting the run as 3, from the y-axis to x = 3, instead of 6, from x = −3 to x = 3. Count between the same two points for both rise and run.

Example 2: Two points, three forms

Problem: A line passes through (−1, 8) and (3, −4). Write its equation in slope-intercept and standard form.

Thinking: The slope is (−4 − 8)/(3 − (−1)) = −12/4 = −3. Point-slope form with (−1, 8) gives y − 8 = −3(x + 1). Distribute: y − 8 = −3x − 3, so y = −3x + 5. Add 3x to both sides for standard form: 3x + y = 5.

Check: (3, −4) gives 3(3) + (−4) = 5.

Trap: Writing y − 8 = −3(x − 1). The point's x-coordinate is −1, so x − (−1) becomes x + 1.

Example 3: Slope and intercept in context

Problem: The cost C, in dollars, to rent a kayak for h hours is C = 18h + 25. What do 18 and 25 mean?

Thinking: The variable h is in hours and C is in dollars. The coefficient 18 multiplies h, so it is a rate: the cost rises $18 for each additional hour. The constant 25 is the value of C when h = 0, a fixed $25 charge paid no matter how long the rental lasts.

Trap: Calling 25 "the cost of the first hour." The first hour costs 18 + 25 = $43; the 25 is a fee charged even before the clock starts.

Example 4: Function notation in both directions

Problem: The function f is defined by f(x) = (2/3)x − 4. Find f(9), and find the value of a for which f(a) = 10.

Thinking: For f(9), substitute 9 for x: (2/3)(9) − 4 = 6 − 4 = 2. For f(a) = 10, set the output equal to 10: (2/3)a − 4 = 10, so (2/3)a = 14 and a = 21.

Trap: Computing f(10) when the question gives f(a) = 10. The 10 is an output, not an input.

Example 5: A perpendicular line

Problem: Line p passes through (2, −1) and is perpendicular to the line 2x − 5y = 10. Which equation defines line p?

Thinking: Solve 2x − 5y = 10 for y: y = (2/5)x − 2, with slope 2/5. The perpendicular slope is −5/2. Point-slope form gives y + 1 = (−5/2)(x − 2), so y = (−5/2)x + 5 − 1 = (−5/2)x + 4.

Check: (2/5)(−5/2) = −1, and (2, −1) satisfies y = (−5/2)(2) + 4 = −1.

Trap: Reading the slope of 2x − 5y = 10 as 2 or as −2/5. Solve for y, or use −A/B = −2/(−5) = 2/5.

Example 6: Two items and a total

Problem: A museum sells adult tickets for $14 and student tickets for $8. One day's ticket sales totaled $560, so 14a + 8s = 560. What do the intercepts and the slope of the graph (with a on the horizontal axis) mean?

Thinking: The a-intercept, 560/14 = 40, means 40 adult tickets and no student tickets. The s-intercept, 560/8 = 70, means 70 student tickets and no adult tickets. Solving for s gives s = 70 − 1.75a, so the slope is −1.75: each additional adult ticket replaces 1.75 student tickets in the same total, because 14/8 = 1.75. Actual sales require whole tickets: four additional adult tickets replace seven student tickets.

Trap: Taking the slope as 14 − 8 = 6. The slope is a ratio of the coefficients, not a difference.

Example 7 (harder-module): A constant that makes lines perpendicular

Problem: In the xy-plane, the line 2x + ky = 5 is perpendicular to the line y = (4/3)x − 1, where k is a constant. What is the value of k?

Thinking: The second line has slope 4/3, so a perpendicular line has slope −3/4. The slope of 2x + ky = 5 is −2/k. Set −2/k = −3/4. Cross-multiply: 3k = 8, so k = 8/3.

Check: With k = 8/3, the slope is −2 ÷ (8/3) = −3/4, and (4/3)(−3/4) = −1.

Trap: Setting −2/k equal to 4/3 finds the value that makes the lines parallel, k = −3/2. Decide which slope you need before you write the equation.

Common traps

Trap	How to spot it
Rise and run upside down	You divided the change in x by the change in y. Slope is rise over run.
Using a table's first row as the y-intercept	The first row's input is not 0. Work back to x = 0.
Sign errors in point-slope form	The point has a negative coordinate. Write x − (−3) as x + 3.
Misreading slope from standard form	The slope of Ax + By = C is −A/B, not A or A/B.

Trap	How to spot it
Calling the constant "the first unit's cost"	The constant is the value when the input is 0.
Confusing f(a) with a	The question gives an output; set f(x) equal to it and solve.
Opposite or reciprocal, but not both	A perpendicular slope changes sign and flips. Check that the product is −1.
Treating a trade-off as a price difference	In ax + by = c, the slope is −a/b, a ratio.

Harder-module variations

- **Unknown constants in two equations.** Find k so that two lines are parallel, perpendicular or the same line. Write each slope in terms of k, then set up the right condition.
- **Point-slope models in context.** In S(t) = 200(t − 2) + 900, the 900 is the value at t = 2, not at t = 0. Substitute t = 0 to find the starting value.
- **Functions described by a rule instead of a formula.** "f increases by 10 every time x increases by 4" means the slope is 10/4 = 2.5.
- **Matching a required form.** "Written as 2x + by = c, what is c?" Find the line, then scale it so that the x-coefficient matches the given one.
- **Interpreting intercepts that are not starting values.** An x-intercept can be the time when a quantity runs out, and a vertical intercept can be the most of one item a budget allows.

Check yourself

1. How do you find the slope of a line from a table whose x-values do not increase by equal steps?
2. What are the slope and y-intercept of 4x − 2y = 12?
3. If f(x) = 5 − 2x, what is f(−3), and for what x is f(x) = −3?
4. A line has slope 3/5. What is the slope of a line parallel to it, and of a line perpendicular to it?
5. In C = 0.12m + 9, where C is a monthly bill in dollars and m is minutes used, what does 0.12 mean?
6. Answers: (1) Divide the change in y by the change in x between the same two rows. (2) Solving for y gives y = 2x − 6, so the slope is 2 and the y-intercept is −6. (3) f(−3) = 11, and f(x) = −3 when x = 4. (4) Parallel: 3/5. Perpendicular: −5/3. (5) The bill increases by $0.12 for each additional minute.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

Linear Functions
skillbyskillprep.com/skills/linear-functions

Linear Equations in Two Variables
skillbyskillprep.com/skills/linear-equations-two-variables

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C1701

The function f is defined by f(x) = 7 − 3x. What is the value of f(−2)?

A −8 **B** 1 **C** 3 **D** 13

---

2 M-C1702

A student measured the height of a sunflower seedling every day. The equation h = 1.8t + 6 models the height h, in centimeters, of the seedling t days after the student began measuring. What is the best interpretation of 6 in this context?

A The seedling grew 6 centimeters each day.

B The seedling grew 1.8 centimeters every 6 days.

C The seedling stopped growing after 6 days.

D The seedling was 6 centimeters tall when the student began measuring.

---

3 M-C1703

The graph of the linear function g in the xy-plane passes through the points (2, 5) and (6, 13). What is the slope of the graph of g?

A 1/2 **B** 2 **C** 4 **D** 8

---

4 M-C1704

3x + 5y = 45

The graph of the given equation in the xy-plane crosses the y-axis at the point (0, b). What is the value of b?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

5 M-C1705

Bean plant height (hypothetical data)

Days after sprouting, d	Height, h(d) (centimeters)
3	9.2
7	15.6
11	22.0
15	28.4

The table shows the height h(d), in centimeters, of a bean plant d days after it sprouted, for four values of d. The height is a linear function of d. Which equation defines h?

A h(d) = 1.6d + 9.2 **B** h(d) = 4.4d + 1.6 **C** h(d) = 1.6d + 4.4 **D** h(d) = 6.4d + 9.2

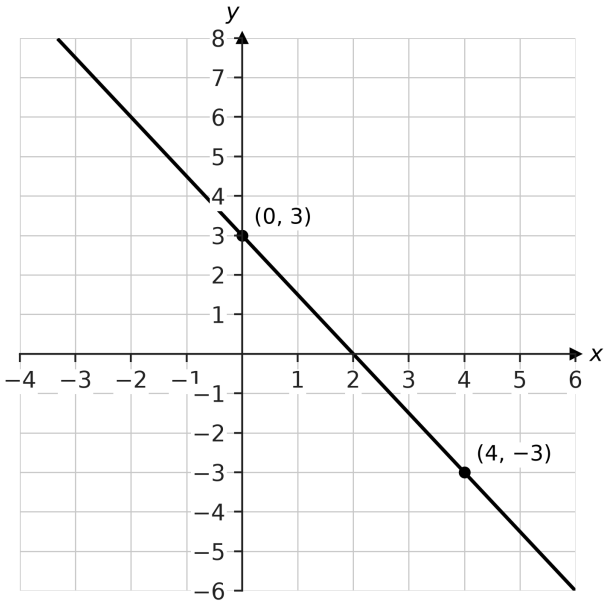
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6 M-C1706

A prepaid phone plan starts with a balance of $60. The function B(d) = 60 − 1.5d gives the balance, in dollars, remaining after d days of use. The graph of y = B(d) crosses the horizontal axis at the point (40, 0). Which statement is the best interpretation of this point in this context?

A After 40 days of use, the balance is $0.
B After 60 days of use, the balance is $0.
C The plan starts with a balance of $40.
D The balance decreases by $40 each day.

7 M-C1707



The graph of y = f(x), where f is a linear function, is shown. What is the value of f(10)?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

8 M-C1708

In the xy-plane, line k has a slope of −2/3 and passes through the point (6, −1). Which equation defines line k?

A 2x + 3y = −3
B 2x + 3y = 9
C 2x − 3y = 15
D 3x + 2y = 16

9 M-C1709

A gym sells day passes for $12 each and class passes for $20 each. One day, the gym sold d day passes and c class passes for a total of $1,040, so 12d + 20c = 1,040. If the gym had sold 10 more day passes while keeping the same total of $1,040, how many fewer class passes would it have sold?

Student-produced response. Write your answer on the answer sheet or below.

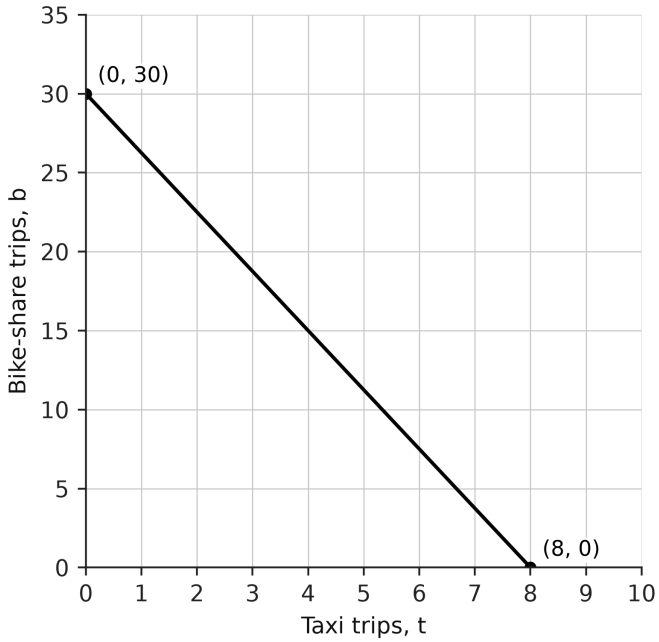
Answer: _____

10 M-C1710

In the xy-plane, line m is perpendicular to the line with equation 3x + 4y = 12 and passes through the point (3, 1). What is the y-coordinate of the y-intercept of line m?

A −3
B −5/4
C 13/4
D 5

11 M-C1711



A commuter spends exactly $60 each month on taxi trips and bike-share trips. Each taxi trip costs $7.50, and each bike-share trip costs $2.00. The graph of $7.5t + 2b = 60$, where t is the number of taxi trips and b is the number of bike-share trips, is shown. Which statement is the best interpretation of the slope of the graph in this context?

A Each taxi trip costs $3.75 more than each bike-share trip.

B For each additional bike-share trip, the commuter can take 3.75 fewer taxi trips.

C In the linear model, each additional taxi trip corresponds to 3.75 fewer bike-share trips.

D If the commuter takes no taxi trips, the commuter can take 3.75 bike-share trips.

12 M-C1712

The linear function f satisfies $f(0) = -5$ and $f(x + 3) = f(x) + 12$ for all values of x. What is the value of $f(5)$?

A 7 **B** 10 **C** 15 **D** 55

13 M-C1713

A streaming service launched a new plan. The function $S(t) = 350(t - 3) + 4{,}800$ models the number of subscribers to the plan t months after launch, for $0 \leq t \leq 12$. Which statement is the best interpretation of 4,800 in this context?

A The plan had 4,800 subscribers 3 months after launch.

B The plan had 4,800 subscribers when it launched.

C The plan gained 4,800 subscribers each month.

D The plan gained 4,800 subscribers during its first 3 months.

14 M-C1714

In the xy-plane, line q passes through the points (−3, 7) and (5, 1). The equation of line q can be written in the form 3x + by = c, where b and c are constants. What is the value of c?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

15 M-C1715

$(k - 1)x + 3y = 7$

$4x - 6y = 5$

In the given equations, k is a constant. The graphs of the equations in the xy-plane are parallel lines. What is the value of k?

A −2 **B** −1 **C** 3 **D** 11/2

Practice Set Explanations

1. Correct answer: D M-C1701

Linear functions · Algebra · Easier

D *13*

Substitute −2 for x: f(−2) = 7 − 3(−2) = 7 + 6 = 13. The product 3(−2) is −6, and subtracting −6 is the same as adding 6. Writing the input in parentheses keeps its negative sign attached during the substitution.

1. Replace x with −2: f(−2) = 7 − 3(−2).
2. Multiply: −3(−2) = 6.
3. Add: 7 + 6 = 13.

A This subtracts first, computing (7 − 3)(−2) = −8. Multiplication must be done before subtraction.

B This treats −3(−2) as −6, a sign error. The product of two negative numbers is positive, so the result is 7 + 6.

C This solves f(x) = −2, which gives x = 3. The question asks for the output when the input is −2.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

2. Correct answer: D M-C1702

Linear functions · Algebra · Easier

D *The seedling was 6 centimeters tall when the student began measuring.*

In h = 1.8t + 6, the constant 6 is the value of h when t = 0. Since t counts days after measuring began, t = 0 is the first measurement, so the seedling was 6 centimeters tall then. The coefficient 1.8 is the growth rate, in centimeters per day.

1. The model has the form h = (rate)t + (starting value).
2. When t = 0, h = 1.8(0) + 6 = 6.
3. So 6 is the height, in centimeters, when the student began measuring.

A The daily growth is the coefficient of t, 1.8 centimeters per day. The 6 is the starting height, not a rate.

B This mixes up the two numbers. The model says the seedling grows 1.8 centimeters every 1 day, not every 6 days.

C A linear model with a positive slope keeps increasing; nothing in it shows growth stopping. The 6 is the height at t = 0.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

3. Correct answer: B M-C1703

Linear functions · Algebra · Easier

B *2*

Slope is the change in y divided by the change in x: (13 − 5)/(6 − 2) = 8/4 = 2. The y-values rise 2 units for every 1-unit increase in x. Subtract the coordinates in the same order in the numerator and the denominator.

1. Change in y: 13 − 5 = 8.
2. Change in x: 6 − 2 = 4.
3. Slope = 8/4 = 2.

A This divides the change in x by the change in y, 4/8. Slope is rise over run, not run over rise.

C 4 is the change in x, the run, by itself. It belongs in the denominator of the slope.

D 8 is the change in y, the rise, by itself. It still needs to be divided by the run, 4.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

4. Correct answer: 9 M-C1704

Linear equations in two variables · Algebra · Easier

Every point on the y-axis has an x-coordinate of 0. Substituting x = 0 gives 5y = 45, so y = 9 and the y-intercept is (0, 9). A common wrong answer is 15, the x-intercept, which comes from setting y = 0 instead of x = 0.

1. On the y-axis, x = 0.
2. Substitute: 3(0) + 5y = 45, so 5y = 45.
3. Divide by 5: y = 9, so b = 9.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

5. Correct answer: C M-C1705

Linear functions · Algebra · Medium

C *h(d) = 1.6d + 4.4*

From one row to the next, d increases by 4 and h(d) increases by 6.4, so the slope is 6.4/4 = 1.6 centimeters per day. Use a row to find the intercept: 9.2 = 1.6(3) + b, so b = 4.4. Check another row: 1.6(15) + 4.4 = 28.4.

1. Slope: (15.6 − 9.2)/(7 − 3) = 6.4/4 = 1.6.
2. Write h(d) = 1.6d + b and use the point (3, 9.2): 9.2 = 4.8 + b.
3. Solve: b = 4.4, so h(d) = 1.6d + 4.4.
4. Check with (15, 28.4): 1.6(15) + 4.4 = 28.4.

A This uses the first height in the table, 9.2, as the starting value, but that height is for d = 3, not d = 0.

B This swaps the slope and the intercept. At d = 3, it gives 4.4(3) + 1.6 = 14.8, not 9.2.

D This treats each row as one day, starting at 9.2 and adding 6.4 per row. The first row is d = 3, and each row spans 4 days.

Calculator option: Enter the table and fit a linear regression model; the calculator reports a slope of 1.6 and an intercept of 4.4, matching choice C.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

6. Correct answer: A M-C1706

Linear functions · Algebra · Medium

A *After 40 days of use, the balance is $0.*

The point (40, 0) means d = 40 and B(d) = 0. The input d counts days of use and the output is the remaining balance in dollars, so after 40 days the balance has dropped to $0. Check: 60 − 1.5(40) = 60 − 60 = 0.

1. In the point (40, 0), d = 40 days and B(40) = 0 dollars.
2. Verify: B(40) = 60 − 1.5(40) = 0.
3. So the balance is used up after 40 days of use.

B This uses the starting balance, 60, as the number of days. The function gives B(60) = 60 − 90 = −30, not 0.

C The starting balance is B(0), which is $60. The 40 in the point is a number of days, not dollars.

D The balance falls by $1.50 per day, the coefficient of d. The 40 is a number of days, not a daily amount.

Calculator option: Graph y = 60 − 1.5x to see the line cross the horizontal axis at (40, 0). The graph confirms the point, but the interpretation still depends on what each axis measures.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

7. Correct answer: -12 M-C1707

Linear functions · Algebra · Medium

The labeled points (0, 3) and (4, −3) give a slope of (−3 − 3)/(4 − 0) = −6/4 = −1.5, and (0, 3) is the y-intercept. So f(x) = −1.5x + 3 and f(10) = −15 + 3 = −12. Enter −12 with the minus sign. Answering 18 comes from using a slope of +1.5 instead of −1.5.

1. Use the points (0, 3) and (4, −3): slope = (−3 − 3)/(4 − 0) = −6/4 = −3/2.
2. The line crosses the y-axis at (0, 3), so the y-intercept is 3.
3. So f(x) = (−3/2)x + 3.
4. Evaluate: f(10) = (−3/2)(10) + 3 = −15 + 3 = −12.

Calculator option: Graph y = −1.5x + 3 and check that it passes through both labeled points, then evaluate it at x = 10. The point x = 10 is off the printed grid, so an equation is needed.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

8. Correct answer: B M-C1708

Linear equations in two variables · Algebra · Medium

B $2x + 3y = 9$

Point-slope form gives y − (−1) = (−2/3)(x − 6), so y + 1 = (−2/3)x + 4 and y = (−2/3)x + 3. Multiply by 3: 3y = −2x + 9, or 2x + 3y = 9. Check: 2(6) + 3(−1) = 9, and solving 2x + 3y = 9 for y shows a slope of −2/3.

1. Point-slope form: y + 1 = (−2/3)(x − 6).
2. Distribute: y + 1 = (−2/3)x + 4, so y = (−2/3)x + 3.
3. Multiply every term by 3: 3y = −2x + 9.
4. Rearrange: 2x + 3y = 9.

A This uses the point's y-coordinate, −1, as the y-intercept: y = (−2/3)x − 1. The point (6, −1) does not satisfy this equation.

C This line passes through (6, −1), but solving for y gives y = (2/3)x − 5, so its slope is positive 2/3.

D This line passes through (6, −1), but its slope is −3/2 because the coefficients of x and y were swapped.

Calculator option: Graph y = (−2/3)(x − 6) − 1, then graph each choice. Only 2x + 3y = 9 lies exactly on top of it.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: 6 M-C1709

Linear equations in two variables · Algebra · Medium

Ten more day passes add 10 × 12 = 120 dollars. To keep the total at $1,040, class-pass sales must fall by $120, which is 120/20 = 6 class passes. Equivalently, the slope of c as a function of d is −12/20 = −3/5, and (−3/5)(10) = −6. Answering 50/3 inverts the ratio of the two prices.

1. Let d be the number of day passes and c the number of class passes, with 12d + 20c = 1,040.
2. Ten extra day passes add 12 × 10 = 120 dollars.
3. Class-pass sales must drop by 120 dollars to keep the total fixed.
4. Each class pass costs $20, so the drop is 120/20 = 6 class passes.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

10. Correct answer: A M-C1710

Linear equations in two variables · Algebra · Medium

A -3

Rewrite 3x + 4y = 12 as y = (−3/4)x + 3, so its slope is −3/4. A perpendicular line has the negative reciprocal slope, 4/3. Using (3, 1): 1 = (4/3)(3) + b = 4 + b, so b = −3. Check: the slopes multiply to (−3/4)(4/3) = −1.

1. Solve the given equation for y: y = (−3/4)x + 3, so its slope is −3/4.
2. The perpendicular slope is the negative reciprocal: 4/3.
3. Write y = (4/3)x + b and substitute (3, 1): 1 = 4 + b.
4. Solve: b = −3.

B This uses a slope of 3/4, the opposite of the original slope, but forgets to take the reciprocal.

C This uses the original slope, −3/4, which produces a line parallel to the given line, not perpendicular to it.

D This uses −4/3, the reciprocal of the original slope without changing its sign. Perpendicular slopes have opposite signs.

Calculator option: Graph 3x + 4y = 12 and y = (4/3)(x − 3) + 1. The second line crosses the y-axis at (0, −3) and meets the first line at a right angle.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

11. Correct answer: C M-C1711

Linear equations in two variables · Algebra · Medium

C *In the linear model, each additional taxi trip corresponds to 3.75 fewer bike-share trips.*

With b on the vertical axis, b = 30 − 3.75t, so the slope is −3.75 bike-share trips per taxi trip. A taxi trip uses $7.50, the cost of 7.50/2.00 = 3.75 bike-share trips. This is the model's rate of exchange; actual combinations require whole trips. For example, four more taxi trips replace fifteen bike-share trips.

1. Let t be taxi trips (horizontal axis) and b be bike-share trips (vertical axis).
2. Solve 7.5t + 2b = 60 for b: b = 30 − 3.75t.
3. The slope is −3.75, so b decreases by 3.75 when t increases by 1.
4. In the linear model, each additional taxi trip corresponds to 3.75 fewer bike-share trips.

A The slope comes from a ratio of the prices, 7.50/2.00, not a difference. A taxi trip costs $5.50 more, not $3.75 more.

B This reverses the roles. One more bike-share trip uses $2.00, which would buy only about 0.27 of a taxi trip.

D With no taxi trips, the commuter can take 60/2 = 30 bike-share trips. That is the vertical intercept, not the slope.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

12. Correct answer: C M-C1712

Linear functions · Algebra · Harder

C *15*

Increasing the input by 3 increases the output by 12, so the slope of f is 12/3 = 4. Since f(0) = −5, f(x) = 4x − 5, and f(5) = 4(5) − 5 = 15. Check the condition: f(x + 3) = 4x + 7, which equals (4x − 5) + 12.

1. The condition says f increases by 12 whenever x increases by 3.
2. Slope = 12/3 = 4.
3. With f(0) = −5, the function is f(x) = 4x − 5.
4. Evaluate: f(5) = 20 − 5 = 15.

A 7 is f(3) = f(0) + 12, the value after one jump of 3 units. The input 5 is not a multiple of 3, so the slope is needed.

B This uses 3, the change in the input, as the slope. Slope is the change in output for a 1-unit change in input.

D This uses 12 as the slope, but 12 is the change in output over 3 units of input, not over 1 unit.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

13. Correct answer: A M-C1713

Linear functions · Algebra · Harder

A *The plan had 4,800 subscribers 3 months after launch.*

When t = 3, the factor t − 3 equals 0, so S(3) = 350(0) + 4,800 = 4,800. The model is in point-slope form: its graph passes through (3, 4,800) with slope 350. So 4,800 is the number of subscribers 3 months after launch, and the plan gains 350 subscribers per month.

1. Notice that S(t) = 350(t − 3) + 4,800 is in point-slope form.
2. Substitute t = 3: S(3) = 350(0) + 4,800 = 4,800.
3. So the graph passes through (3, 4,800): 4,800 subscribers 3 months after launch.
4. For comparison, S(0) = 3,750 subscribers at launch.

B The number at launch is S(0) = 350(−3) + 4,800 = 3,750. The value 4,800 occurs when t = 3, not when t = 0.

C The monthly gain is the slope, 350 subscribers per month. The 4,800 is a subscriber count at one time, not a rate.

D During the first 3 months, the plan gained 350 × 3 = 1,050 subscribers, growing from 3,750 to 4,800.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

14. Correct answer: 19 M-C1714

Linear equations in two variables · Algebra · Harder

The slope of q is (1 − 7)/(5 − (−3)) = −6/8 = −3/4. Solving 3x + by = c for y shows that the slope is −3/b, so b = 4. Substituting (5, 1) into 3x + 4y = c gives c = 15 + 4 = 19, and (−3, 7) checks: −9 + 28 = 19. Entering 4 gives b, not c, and 19/4 is the y-intercept.

1. Slope of q: (1 − 7)/(5 − (−3)) = −6/8 = −3/4.
2. Solving 3x + by = c for y gives slope −3/b, so −3/b = −3/4 and b = 4.
3. Substitute (5, 1) into 3x + 4y = c: c = 15 + 4 = 19.
4. Check with (−3, 7): 3(−3) + 4(7) = −9 + 28 = 19.

Calculator option: After finding b = 4, graph 3x + 4y = c with a slider for c and plot the two given points; the line passes through both only when c = 19.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

15. Correct answer: B M-C1715

Linear equations in two variables · Algebra · Harder

B *−1*

Parallel lines have equal slopes. The second line is y = (2/3)x − 5/6, with slope 2/3. The first line has slope −(k − 1)/3. Setting −(k − 1)/3 = 2/3 gives k − 1 = −2, so k = −1. The y-intercepts, 7/3 and −5/6, differ, so the lines are distinct and parallel.

1. Solve 4x − 6y = 5 for y: y = (2/3)x − 5/6, so its slope is 2/3.
2. Solve the first equation for y: y = (−(k − 1)/3)x + 7/3.
3. Set the slopes equal: −(k − 1)/3 = 2/3, so k − 1 = −2.
4. Solve: k = −1. The y-intercepts differ, so the lines are parallel.

A This sets −k/3 = 2/3, using k as the x-coefficient and forgetting that the coefficient is k − 1.

C This sets (k − 1)/3 = 2/3, dropping the negative sign that appears when the first equation is solved for y.

D This gives the first line a slope of −3/2, the negative reciprocal of 2/3, which makes the lines perpendicular, not parallel.

Calculator option: Graph both equations with a slider for k. The lines look parallel only at k = −1; the algebra confirms that the value is exact.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

CHAPTER 18

Systems of Two Linear Equations

A system of two linear equations is a pair of linear equations in the same two variables. A solution to the system is an ordered pair (x, y) that makes both equations true at the same time; on a graph, it is the point where the two lines meet. Systems questions can appear in either Math module, as stacked equations, word problems or graphs, in multiple-choice or student-produced response (SPR) form.

This chapter gives you two algebra methods, a quick test for the number of solutions and a routine for turning a story into a system, with the graphing calculator as your check.

What the question looks like

Systems questions come in four families.

Family	Typical stem	What you produce
Solve	What is the solution (x, y) to the given system of equations?	x, y, or an expression such as x + y
Number of solutions	How many solutions does the given system have? The given system has no solution. What is the value of k?	zero, one or infinitely many, or a constant
Context	Which system of equations represents this situation? How many adult tickets were sold?	two equations, or one unknown amount
Graph	The graphs of the equations in a system are shown. What is the solution?	the intersection point, or what it means

The equations are usually printed one above the other and called "the given system." Context questions often print no equations at all: you build them from two facts in the story.

How to recognize it

Look for any of these signals:

- Two equations that use the same two variables, printed together.
- The phrases "solution (x, y)," "intersection," "no solution" or "infinitely many solutions."
- A story with two unknown amounts and two separate facts about them, usually a total count and a total value (dollars, liters, points).
- Two lines drawn in the same xy-plane.

The method

1. **Name the target.** Is it x, y, an expression, a constant or a count of solutions? Many questions want something other than x.

2. **Line up the equations.** Put the x-terms, y-terms and constants in matching columns. Standard form, ax + by = c, makes the columns easy to compare.
3. **Choose a method.** Use substitution when a variable is already alone or has a coefficient of 1 or −1. Use elimination when two coefficients already match or can be matched with one multiplication. Graph when the numbers are messy and you need only the intersection.
4. **Solve for one variable,** then find the other only if the question needs it.
5. **Check the pair in both equations** or in the story. Wrong choices often fit only one equation.

Substitution

Solve one equation for one variable, then replace that variable in the other equation with the expression you found. The result is a single equation in one variable. Put the substituted expression in parentheses so that you distribute correctly.

Elimination

Add or subtract the two equations so that one variable cancels. If no coefficients match, first multiply one or both equations by a number. Multiply every term, including the constant on the right side; skipping the constant is the most common slip.

How many solutions?

Two different lines meet once or never. Equations describing the same line share every point. For nonvertical lines, compare slopes and y-intercepts:

Slopes	y-intercepts	Graph	Number of solutions
Different	Any	Lines cross once	Exactly one
Same	Different	Parallel lines	Zero (no solution)
Same	Same	One line, drawn twice	Infinitely many

A vertical line meets every nonvertical line once; two vertical lines are either distinct (no solution) or identical (infinitely many). In standard form, match a variable's nonzero coefficients by multiplying one or both equations. If the other coefficients then match too, the lines are parallel or identical: identical when the constants also match (infinitely many solutions), parallel when they differ (no solution). If the other coefficients do not match, there is exactly one solution.

NOTE A system of two linear equations never has exactly two solutions. Two lines that share two points are the same line.

Using the graphing calculator

Graph both equations and select the intersection to see its coordinates. Use coefficients to confirm parallel or identical lines; a graph outside the window or very close to another can be misleading. A slider can find an unknown constant that makes the lines parallel or identical, but the algebra above is usually faster.

Worked examples

Worked example 1: Substitution

Problem. Solve the system $y = 2x - 5$ and $3x + 4y = 13$. What is the solution (x, y)?

Think. The first equation already gives y, so substitute $2x - 5$ for y in the second equation.

1. $3x + 4(2x - 5) = 13$
2. $3x + 8x - 20 = 13$
3. $11x = 33$, so $x = 3$.
4. $y = 2(3) - 5 = 1$.

Answer. (3, 1). Check: $3(3) + 4(1) = 13$, and $1 = 2(3) - 5$.

Why the traps fail. (1, 3) reverses the coordinates. If you multiply only the $2x$ by 4 and leave -5 alone, you get $11x - 5 = 13$ and $x = 18/11$, a value that fails the check.

Worked example 2: Elimination aimed at the target

Problem. $3x + 2y = 4$ and $5x + 3y = 7$. What is the value of y?

Think. The target is y, so eliminate x. Multiply the first equation by 5 and the second by 3 so that both x-coefficients are 15.

1. $15x + 10y = 20$
2. $15x + 9y = 21$
3. Subtract the second equation from the first: $y = -1$.

Answer. $y = -1$. Check: with $y = -1$, the first equation gives $3x - 2 = 4$, so $x = 2$, and $5(2) + 3(-1) = 7$.

Why the traps fail. If you multiply the first equation by 5 but leave its constant as 4, the subtraction gives $y = -17$, which fails the check. And $x = 2$ is the other coordinate, not the target.

Worked example 3: The constant that gives no solution

Problem. In the system $kx + 6y = 9$ and $2x + 3y = 5$, k is a constant. For what value of k does the system have no solution?

Think. No solution means parallel lines: matching x- and y-coefficients but different constants. Multiply the second equation by 2 so that its y-coefficient matches the first: $4x + 6y = 10$. Now compare $kx + 6y = 9$ with $4x + 6y = 10$. If $k = 4$, the coefficients match while the constants, 9 and 10, differ.

Answer. $k = 4$. Check with slopes: both lines then have slope $-2/3$, and their y-intercepts ($3/2$ and $5/3$) differ.

Why the traps fail. $k = 2$ copies the x-coefficient without scaling; with $k = 2$ the slopes differ and the lines cross once. If the first equation had been $kx + 6y = 10$, then $k = 4$ would make the equations identical, giving infinitely many solutions instead of none.

Worked example 4: Writing a system from a story

Problem. A science museum sold 420 tickets in one day. Adult tickets cost \$15 and student tickets cost \$9. Ticket sales totaled \$4,980. How many adult tickets were sold?

Think. There are two unknowns (adult tickets a and student tickets s) and two facts (a count and a money total). A small table turns each fact into an equation.

Quantity	Adult	Student	Total
Number of tickets	a	s	420
Dollars	15a	9s	4,980

Each row is an equation: a + s = 420 and 15a + 9s = 4,980. Substitute s = 420 − a:

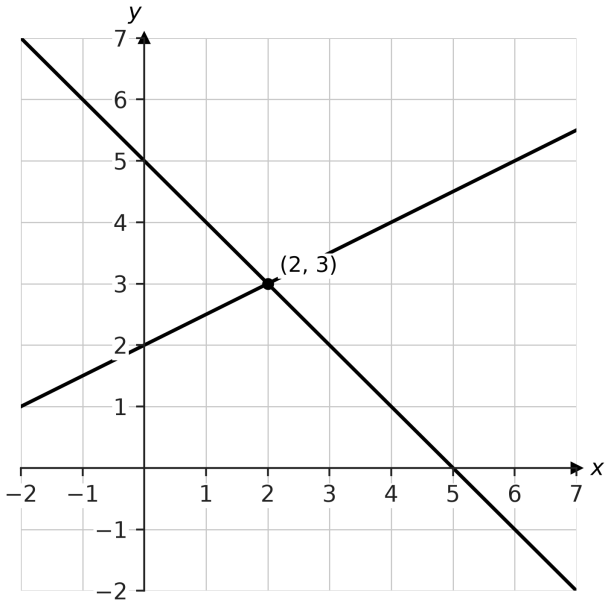
1. 15a + 9(420 − a) = 4,980
2. 15a + 3,780 − 9a = 4,980
3. 6a = 1,200, so a = 200.

Answer. 200 adult tickets (and 220 student tickets). Check: 15(200) + 9(220) = 3,000 + 1,980 = 4,980.

Why the traps fail. 220 is the number of student tickets. Swapping the prices (writing 9a + 15s) also produces 220, so if you get 220, check which price belongs with which variable.

Worked example 5: Reading the solution from a graph

Problem. The graphs of y = −x + 5 (the falling line) and y = (1/2)x + 2 (the rising line) are shown. What is the solution to the system?



Think. The solution is the point on both lines, which is where they cross. Read the x-coordinate along the horizontal axis first, then the y-coordinate: the lines meet at (2, 3). Confirm with algebra: −x + 5 = (1/2)x + 2 gives 3 = (3/2)x, so x = 2 and y = −2 + 5 = 3.

Answer. (2, 3).

Why the traps fail. (0, 5) and (0, 2) are the y-intercepts; each lies on only one line. (3, 2) reverses the coordinates. If a crossing point is unlabeled, confirm your reading with the equations.

Worked example 6 (harder module): Solving for an expression

Problem. 5x + 2y = 26 and 2x + 5y = 16. What is the value of x + y?

Think. The coefficients are mirror images, so adding the equations gives equal coefficients on x and y: 7x + 7y = 42. Divide by 7: x + y = 6.

Answer. 6. Solving for x = 14/3 and y = 4/3 also works, but it is slower and invites arithmetic slips.

Why the traps fail. 42 is 7(x + y), not x + y. Subtracting the equations gives 3x − 3y = 10, which leads to x − y = 10/3, the wrong expression.

Common traps

Trap	How to spot it
Answering with x when the question asks for y, x + y or a constant	Reread the final sentence of the stem before you choose.
Losing a sign when subtracting equations	Subtract every column, constants included, and write the subtraction out.
Scaling only part of an equation	After multiplying, every term in that equation, constant included, should have changed.
Mixing up "no solution" and "infinitely many"	No solution: same slope, different intercepts. Infinitely many: the very same line.
Pairing a price or rate with the wrong variable	Build a count row and a value row; each price sits under its own item.
Reading an intercept instead of the intersection	The solution is where the lines cross each other, not where either crosses an axis.
A pair that fits only one equation	Check your answer in both equations before you move on.

Harder-module variations

- **Two unknown constants.** For infinitely many solutions, every coefficient and the constant must follow one ratio. If 2x + by = 10 and ax + 6y = 30 describe the same line, the constants show the ratio, 30/10 = 3. So a = 3(2) = 6, and 6 = 3b gives b = 2.
- **Mixtures and percents.** Write one equation for the total amount and one for the pure substance. Mixing a 10% salt solution with a 40% salt solution to make 60 liters of a 20% solution gives x + y = 60 and 0.10x + 0.40y = 0.20(60) = 12, so x = 40 liters of the 10% solution. As a check, 20% is closer to 10% than to 40%, so most of the mixture should be the 10% solution.
- **Equations in unfamiliar forms.** Rewrite before comparing. For x/2 + y/3 = 4, multiply by 6 to get 3x + 2y = 24.
- **A known solution and an unknown constant.** If a stem tells you that (2, −1) is a solution, substitute x = 2 and y = −1 into each equation, then solve for any constant that remains.
- **Interpreting the solution.** In a context, the intersection is the moment or amount at which two quantities are equal: two plans cost the same, or two tanks hold the same volume. The x-value tells you when, and the y-value tells you how much.

Check yourself

1. When is substitution usually faster than elimination?
2. To eliminate y from 3x − 2y = 8 and 5x + 4y = 6, what should you do first?
3. Two lines have the same slope and different y-intercepts. How many solutions does the system have?

4. A story gives the total number of items sold and the total money collected. Which two equations do you write?
5. If $2x + y = 9$ and $x + 2y = 6$, what is $x + y$?
6. **Answers:** (1) When one variable is already alone or has a coefficient of 1 or −1. (2) Multiply the first equation by 2, every term included, then add the equations. (3) Zero; the lines are parallel. (4) A count equation (the numbers of items add to the total count) and a value equation (price times quantity for each item, added, equals the money total). (5) Adding gives $3x + 3y = 15$, so $x + y = 5$.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/systems-of-linear-equations

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C1801

$y = 3x - 2$
$2x + y = 13$
What is the solution (x, y) to the given system of equations?

A (2, 4) **B** (4, 5) **C** (3, 7) **D** (7, 3)

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2 M-C1802

A community theater sold 180 tickets to a play. Adult tickets cost $8 each and student tickets cost $5 each, and ticket sales totaled $1,140. Which system of equations can be used to find a, the number of adult tickets sold, and s, the number of student tickets sold?

A $a + s = 180$
$8a + 5s = 1{,}140$

B $a + s = 1{,}140$
$8a + 5s = 180$

C $a + s = 180$
$5a + 8s = 1{,}140$

D $a - s = 180$
$8a + 5s = 1{,}140$

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3 M-C1803

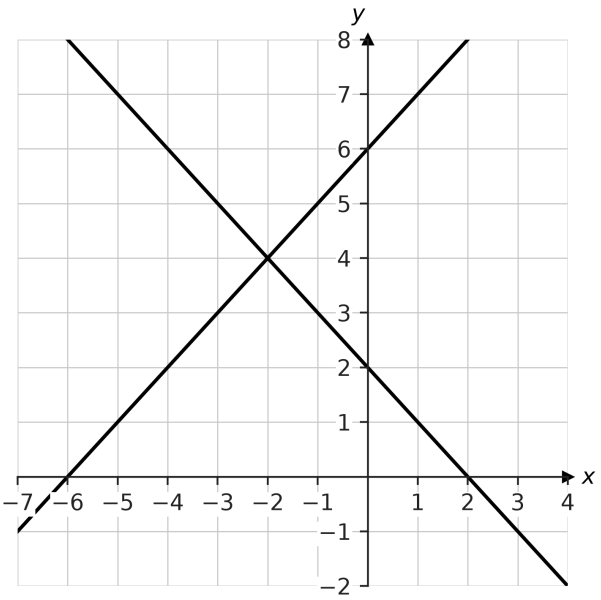
$4x + y = 23$
$2x - y = 7$
The solution to the given system of equations is (x, y). What is the value of x?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

4 M-C1804



The graphs of the two linear equations in a system are shown in the xy-plane. What is the solution (x, y) to the system?

A (0, 2) **B** (0, 6) **C** (−2, 4) **D** (4, −2)

5 M-C1805

$3x + 2y = 7$

$5x - 4y = 41$

If (x, y) is the solution to the given system of equations, what is the value of y?

A −8 **B** −4 **C** 4 **D** 5

6 M-C1806

$2y = 6x - 8$

$3x - y = 4$

How many solutions does the given system of equations have?

A Zero **B** Exactly one **C** Exactly two **D** Infinitely many

7 M-C1807

At a basketball game, a concession stand sold hot dogs for $4 each and pretzels for $3 each. The stand sold 150 of these items for a total of $530. How many hot dogs did the stand sell?

A 70 **B** 75 **C** 80 **D** 320

8 M-C1808

$6x - ky = 5$

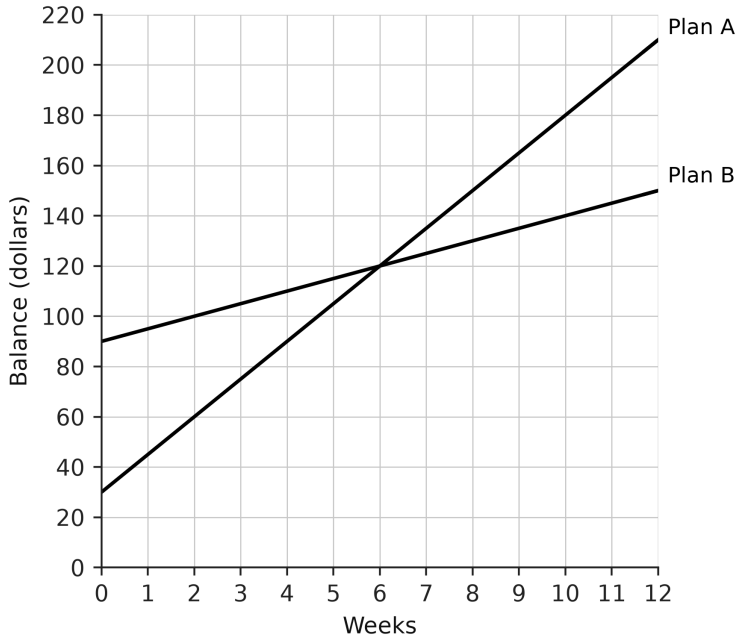
$2x - 3y = 9$

In the given system of equations, k is a constant. For what value of k does the system have no solution?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

9 M-C1809



The graph shows the balances, in dollars, of two savings plans x weeks after the plans begin. What is the best interpretation of the point where the two graphs intersect?

A After 6 weeks, the combined balance of the two plans is $120.

B After 6 weeks, the two plans have the same balance, $120.

C Each plan increases by $120 every 6 weeks.

D After 120 weeks, the two plans have the same balance, $6.

10 M-C1810

A lab technician mixes a 15% acid solution with a 35% acid solution to make 40 liters of a solution that is 30% acid. How many liters of the 15% acid solution does the technician use?

A 10 **B** 12 **C** 20 **D** 30

11 M-C1811

$6x - 4y = c$

$y = 1.5x - 2$

In the given system of equations, c is a constant. For what value of c does the system have infinitely many solutions?

A −8 **B** −2 **C** 2 **D** 8

12 M-C1812

$3x + 7y = 47$

$7x + 3y = 33$

If (x, y) is the solution to the given system of equations, what is the value of $x - y$?

A $-7/2$ **B** $7/2$ **C** 8 **D** 14

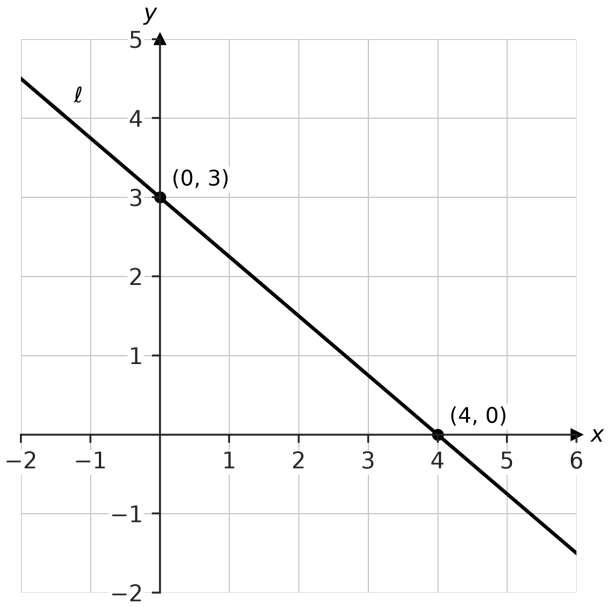
13 M-C1813

A bakery sells muffins in small boxes of 6 and large boxes of 12. All small boxes have the same price, and all large boxes have the same price. An order of 3 small boxes and 2 large boxes costs $54, and an order of 5 small boxes and 1 large box costs $48. What is the price, in dollars, of one muffin when it is bought in a large box?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

14 M-C1814



Line ℓ is shown in the xy-plane. A system of equations consists of the equation of line ℓ and the equation $y = kx + 5$, where k is a constant. If the system has no solution, what is the value of k?

A $-4/3$ **B** $-3/4$ **C** $3/4$ **D** $4/3$

15 M-C1815

$ax + 6y = 8$

$3x + by = 12$

In the given system of equations, a and b are constants. If the system has infinitely many solutions, what is the value of $a + b$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

Practice Set Explanations

1. Correct answer: C M-C1801

Systems of two linear equations in two variables · Algebra · Easier

C *(3, 7)*

Substitute 3x − 2 for y in the second equation: 2x + (3x − 2) = 13, so 5x − 2 = 13, 5x = 15 and x = 3. Then y = 3(3) − 2 = 7. Check the pair in both equations: 7 = 9 − 2, and 2(3) + 7 = 13.

1. Substitute y = 3x − 2 into 2x + y = 13: 2x + 3x − 2 = 13.
2. Combine and solve: 5x = 15, so x = 3.
3. Find y: y = 3(3) − 2 = 7.
4. Check (3, 7) in both equations.

A (2, 4) satisfies y = 3x − 2 but not the second equation, because 2(2) + 4 = 8, not 13. A solution must work in both equations.

B (4, 5) satisfies 2x + y = 13 but not y = 3x − 2, because 3(4) − 2 = 10, not 5.

D This reverses the coordinates. A solution is written (x, y), and here x = 3 and y = 7.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

2. Correct answer: A M-C1802

Systems of two linear equations in two variables · Algebra · Easier

A *a + s = 180*
8a + 5s = 1,140

Each fact in the story becomes one equation. The tickets add to 180: a + s = 180. The money adds to $1,140, with $8 for each adult ticket and $5 for each student ticket: 8a + 5s = 1,140. (Solving gives a = 80 and s = 100, and 640 + 500 = 1,140.)

1. Let a = the number of adult tickets and s = the number of student tickets.
2. Count equation: a + s = 180.
3. Money equation: 8a + 5s = 1,140.

B This swaps the totals. The 180 counts tickets, so it belongs with a + s; the 1,140 is dollars, so it belongs with 8a + 5s.

C This attaches each price to the wrong variable. Adult tickets cost $8, so the number of adult tickets, a, must be multiplied by 8.

D The two kinds of tickets add to 180. The equation a − s = 180 would mean 180 more adult tickets than student tickets were sold.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

3. Correct answer: 5 M-C1803

Systems of two linear equations in two variables · Algebra · Easier

The y-terms are opposites, so adding the equations eliminates y: 6x = 30, and x = 5. Then y = 23 − 4(5) = 3, and the check 2(5) − 3 = 7 works. A common wrong answer is 3, which is the value of y; another is 30, the value of 6x before dividing by 6.

1. Add the equations: (4x + 2x) + (y − y) = 23 + 7, so 6x = 30.
2. Divide by 6: x = 5.
3. Check: y = 23 − 20 = 3, and 2(5) − 3 = 7.

Calculator option: Graph both equations and select the intersection point, (5, 3); its x-coordinate is 5.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

4. Correct answer: C M-C1804

Systems of two linear equations in two variables · Algebra · Easier

C *(−2, 4)*

The solution to a system is the point that lies on both graphs, which is where the lines intersect. The lines cross 2 units left of the y-axis and 4 units above the x-axis, at (−2, 4). Check with the equations of the lines, y = x + 6 and y = −x + 2: −2 + 6 = 4 and 2 + 2 = 4.

1. Locate the point where the two lines cross.
2. Read the coordinates: x = −2 and y = 4.
3. Check with the lines y = x + 6 and y = −x + 2, read from their intercepts.

A (0, 2) is where the falling line crosses the y-axis. It lies on only one line, so it does not satisfy both equations.

B (0, 6) is the y-intercept of the rising line. A solution must lie on both lines, not on one of them.

D This reverses the coordinates of the intersection. The crossing point is 2 units left of the y-axis, so x = −2 and y = 4.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

5. Correct answer: B M-C1805

Systems of two linear equations in two variables · Algebra · Medium

B *−4*

Use elimination. Multiply the first equation by 2 so the y-coefficients are opposites: 6x + 4y = 14. Add this to the second equation: 11x = 55, so x = 5. Substitute into the first equation: 15 + 2y = 7, so 2y = −8 and y = −4. Check: 5(5) − 4(−4) = 25 + 16 = 41.

1. Multiply 3x + 2y = 7 by 2: 6x + 4y = 14.
2. Add to 5x − 4y = 41: 11x = 55, so x = 5.
3. Substitute: 3(5) + 2y = 7, so 2y = −8.
4. Divide by 2: y = −4.

A This is the value of 2y. After reaching 2y = −8, divide both sides by 2 to get y = −4.

C This drops the negative sign. From 15 + 2y = 7, subtracting 15 gives 2y = −8, which is negative.

D This is the value of x, not y. The question asks for the second coordinate of the solution.

Calculator option: Graph both equations and select the intersection; it is (5, −4).

Review: Chapter 18, Systems of Two Linear Equations (page 264)

6. Correct answer: D M-C1806

Systems of two linear equations in two variables · Algebra · Medium

D *Infinitely many*

Rewrite both equations in slope-intercept form. Dividing the first by 2 gives y = 3x − 4. Solving the second for y gives y = 3x − 4 as well. Both equations describe the same line, so every point on that line satisfies both: the system has infinitely many solutions.

1. Divide 2y = 6x − 8 by 2: y = 3x − 4.
2. Solve 3x − y = 4 for y: y = 3x − 4.
3. Same slope and same y-intercept means the same line, so there are infinitely many solutions.

A Zero solutions requires the same slope with different y-intercepts. These lines share the slope 3 and also the y-intercept −4.

B Exactly one solution requires different slopes. Both equations have slope 3, so the lines cannot cross at a single point.

C Two lines that share two points are the same line, so a system of two linear equations never has exactly two solutions.

Calculator option: Graph both equations: only one line appears, because the second graph lies exactly on top of the first.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

7. Correct answer: C M-C1807

Systems of two linear equations in two variables · Algebra · Medium

C *80*

Let h be the number of hot dogs and p the number of pretzels. Then h + p = 150 and 4h + 3p = 530. Substitute p = 150 − h: 4h + 3(150 − h) = 530, so h + 450 = 530 and h = 80. Check: 80 hot dogs and 70 pretzels bring in $320 + $210 = $530.

1. Let h = hot dogs and p = pretzels: h + p = 150 and 4h + 3p = 530.
2. Substitute p = 150 − h: 4h + 450 − 3h = 530.
3. Solve: h = 80.
4. Check: 4(80) + 3(70) = 320 + 210 = 530.

A 70 is the number of pretzels. The same value appears if the $4 price is attached to pretzels and the $3 price to hot dogs.

B This assumes equal numbers of each item. Selling 75 hot dogs and 75 pretzels would bring in $525, not $530.

D $320 is the money taken in from hot dogs (80 × $4), not the number of hot dogs sold.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

8. Correct answer: 9 M-C1808

Systems of two linear equations in two variables · Algebra · Medium

A linear system has no solution when its lines are parallel: the x- and y-coefficients are in the same ratio, but the constants are not. Multiply the second equation by 3 to match the x-coefficients: 6x − 9y = 27. The first equation matches this when k = 9, and because 5 ≠ 27 the lines are parallel and distinct. Entering 3 copies the unscaled coefficient; −9 mishandles the sign in −ky.

1. Multiply 2x − 3y = 9 by 3: 6x − 9y = 27.
2. The x-coefficients now match, so parallel lines need −k = −9.
3. So k = 9; the constants 5 and 27 differ, so the lines never meet.

Calculator option: Put k on a slider and graph both equations; the lines are parallel only when k = 9.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

9. Correct answer: B M-C1809

Systems of two linear equations in two variables · Algebra · Medium

B *After 6 weeks, the two plans have the same balance, $120.*

A point on both graphs gives an x-value and a y-value that fit both plans at once. The graphs meet at (6, 120), so at 6 weeks each plan has a balance of $120. The models confirm it: Plan A is y = 30 + 15x and Plan B is y = 90 + 5x, and both equal 120 when x = 6.

1. The intersection is the one point that lies on both graphs.
2. Read it from the grid: x = 6 weeks and y = 120 dollars.
3. Interpret: after 6 weeks, the two plans have the same balance, $120.

A Each graph shows one plan's own balance. At the intersection each plan holds $120, so together they hold $240.

C The intersection compares the balances at one moment; it does not describe growth. Plan A grows $15 per week and Plan B grows $5 per week.

D This swaps the coordinates. The horizontal axis shows weeks and the vertical axis shows dollars, so the point means 6 weeks and $120.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

10. Correct answer: A M-C1810

Systems of two linear equations in two variables · Algebra · Medium

A *10*

Let x be the liters of 15% solution and y the liters of 35% solution. Volume: x + y = 40. Acid: 0.15x + 0.35y = 0.30(40) = 12. Substitute y = 40 − x: 0.15x + 14 − 0.35x = 12, so −0.20x = −2 and x = 10. Check: 0.15(10) + 0.35(30) = 1.5 + 10.5 = 12 liters of acid.

1. Let x = liters of 15% solution and y = liters of 35% solution.
2. Volume equation: x + y = 40. Acid equation: 0.15x + 0.35y = 0.30(40) = 12.
3. Substitute y = 40 − x: −0.20x + 14 = 12, so x = 10.

B 12 liters is the amount of pure acid in the final mixture, 0.30 × 40, not the volume of the 15% solution.

C This splits the 40 liters evenly. Equal amounts of the two solutions would make a 25% solution, not a 30% solution.

D 30 liters is the volume of the 35% solution. The question asks for the volume of the 15% solution.

Calculator option: Graph x + y = 40 and 0.15x + 0.35y = 12; the intersection is (10, 30).

Review: Chapter 18, Systems of Two Linear Equations (page 264)

11. Correct answer: D M-C1811

Systems of two linear equations in two variables · Algebra · Medium

D *8*

Infinitely many solutions means both equations describe the same line. Rewrite y = 1.5x − 2 in the form of the first equation: subtracting y and adding 2 to both sides gives 1.5x − y = 2. Multiply by 4: 6x − 4y = 8. The x- and y-coefficients now match the first equation, so c must equal 8.

1. Rewrite y = 1.5x − 2 as 1.5x − y = 2.
2. Multiply every term by 4: 6x − 4y = 8.
3. Matching the first equation, 6x − 4y = c, requires c = 8.

A This comes from a sign error in rearranging, writing 1.5x − y = −2. Moving −2 to the other side makes it +2.

B This is the y-intercept of the second line. The equations are in different forms, so the constant must be found by rewriting and scaling.

C This rewrites the second equation correctly as 1.5x − y = 2 but forgets to multiply by 4 to match the coefficients 6 and −4.

Calculator option: Graph y = 1.5x − 2 and 6x − 4y = c with c on a slider; the two graphs coincide only when c = 8.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

12. Correct answer: A M-C1812

Systems of two linear equations in two variables · Algebra · Harder

A *−7/2*

Subtract the first equation from the second: (7x + 3y) − (3x + 7y) = 33 − 47, so 4x − 4y = −14. Divide by 4: x − y = −14/4 = −7/2. Solving for each variable gives x = 9/4 and y = 23/4, which confirms 9/4 − 23/4 = −14/4, but the direct route avoids those fractions.

1. Subtract the first equation from the second: 4x − 4y = −14.
2. Divide both sides by 4: x − y = −14/4.
3. Simplify: x − y = −7/2.

B This is y − x. Subtracting the second equation from the first gives 4y − 4x = 14, so y − x = 7/2 and x − y = −7/2.

C This is x + y, found by adding the equations (10x + 10y = 80). The question asks for x − y.

D This is 4y − 4x, from subtracting the second equation from the first. Dividing by −4 is still needed to isolate x − y.

Calculator option: Graph both equations; the intersection is (2.25, 5.75), and 2.25 − 5.75 = −3.5.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

13. Correct answer: 1.5 or 3/2 M-C1813

Systems of two linear equations in two variables · Algebra · Harder

Let s and b be the prices of a small box and a large box. Then 3s + 2b = 54 and 5s + b = 48. From the second equation, b = 48 − 5s; substituting gives 3s + 96 − 10s = 54, so s = 6 and b = 18. A large box holds 12 muffins, so each costs 18/12 = $1.50. Entering 18 gives the box price; 1 is the per-muffin price in a small box.

1. Let s = the price of a small box and b = the price of a large box: 3s + 2b = 54 and 5s + b = 48.
2. Substitute b = 48 − 5s: 3s + 2(48 − 5s) = 54, so −7s = −42 and s = 6.
3. Find b: b = 48 − 5(6) = 18.
4. Divide by 12 muffins: 18/12 = 1.5 dollars per muffin.

Calculator option: Graph 3x + 2y = 54 and 5x + y = 48; the intersection (6, 18) gives the box prices, and 18 ÷ 12 = 1.5.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

14. Correct answer: B M-C1814

Systems of two linear equations in two variables · Algebra · Harder

B *−3/4*

A system of two linear equations has no solution when the lines are parallel and distinct, so the second line must have the same slope as ℓ. Line ℓ passes through (0, 3) and (4, 0), so its slope is (0 − 3)/(4 − 0) = −3/4. With k = −3/4, the line y = −(3/4)x + 5 has y-intercept 5 instead of 3, so the lines never meet.

1. No solution means parallel lines with different y-intercepts.
2. Slope of ℓ from (0, 3) and (4, 0): $(0 - 3)/(4 - 0) = -3/4$.
3. So $k = -3/4$; the y-intercepts 5 and 3 differ, so the lines are distinct.

A This divides the change in x by the change in y. Slope is the change in y divided by the change in x: $-3/4$.

C This drops the sign. Line ℓ falls from left to right, so its slope is negative.

D This is the negative reciprocal of the slope, which would make the lines perpendicular and give exactly one solution.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

15. Correct answer: 11 M-C1815

Systems of two linear equations in two variables · Algebra · Harder

Infinitely many solutions means one equation is a constant multiple of the other, so every coefficient follows the same ratio as the constants: $8/12 = 2/3$. Then $a = (2/3)(3) = 2$, and $6 = (2/3)b$ gives $b = 9$. Both equations reduce to $x + 3y = 4$, so $a + b = 11$. Entering 9 comes from copying the coefficients without scaling ($a = 3$ and $b = 6$); 8.5 comes from using the ratio upside down.

1. Same line means the first equation is 2/3 of the second, because $8 = (2/3)(12)$.
2. x-coefficients: $a = (2/3)(3) = 2$.
3. y-coefficients: $6 = (2/3)b$, so $b = 9$.
4. Add: $a + b = 2 + 9 = 11$.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

CHAPTER 19

Linear Inequalities

A linear inequality compares two linear expressions with one of the symbols <, >, ⩽ or ⩾. Its solution is usually a whole range of values rather than a single number. The digital SAT asks you to solve one-variable inequalities, to write inequalities from real limits such as budgets and minimums, and to decide which points satisfy a two-variable inequality or a system of inequalities. These questions belong to the Algebra domain and can appear in either Math module.

What the question looks like

Family	Typical stem	What you produce
Solve	Which inequality represents all solutions to the given inequality?	a simplified inequality such as $x \geqslant -3$
Model	Which inequality represents this situation?	an inequality or a system built from the story
Test points	Which point (x, y) is a solution to the given system of inequalities?	the one point that satisfies every inequality
Interpret	What is the maximum number of items that can be bought?	a whole number at the edge of the solution set
Read a graph	Which system of inequalities is represented by the graph?	inequalities that match the boundary lines and the solution region

How to recognize it

The symbols are the obvious signal. In word problems, the signal is a phrase that sets a limit. Translate it before you do anything else.

Phrase in the story	Symbol	Example
at least, no less than, a minimum of	⩾	at least 12 people: $p \geqslant 12$
at most, no more than, a maximum of, cannot exceed	⩽	at most \$40: $c \leqslant 40$
more than, greater than, exceeds	>	more than 3 hours: $h > 3$
fewer than, less than, below	<	fewer than 30 seats: $s < 30$

TRAP "No more than 50" sets an upper limit ($\leqslant 50$), and "no fewer than 50" sets a lower limit ($\geqslant 50$). Read the whole phrase, not just the word "more" or "fewer."

The method

1. **Translate** any words into a symbol, using the table above.

2. **Solve as you would an equation.** Undo addition and subtraction first, then multiplication and division.
3. **Flip the symbol when you multiply or divide both sides by a negative number.** Adding or subtracting, even a negative number, never flips it.
4. **Answer the question asked.** For a whole-number answer, round in the direction that keeps the inequality true, not to the nearest whole number.
5. **Check with a test value** from your solution set. For two-variable problems, test points in every inequality.

Why the symbol flips

Multiplying by a negative number reverses order on the number line. For example, $2 < 5$, but multiplying both sides by -1 gives $-2 > -5$. Dividing both sides by a negative number works the same way: $<$ becomes $>$, and $\leqslant$ becomes $\geqslant$.

Two-variable inequalities and their graphs

The graph of an inequality such as $y < 2x + 1$ is a boundary line plus all the points on one side of it.

- The boundary is the line $y = 2x + 1$.
- The line is **solid** for $\leqslant$ or $\geqslant$, because points on the line are solutions, and **dashed** for $<$ or $>$, because they are not.
- When the inequality is solved for y, "$y >$" or "$y \geqslant$" means the region above the line, and "$y <$" or "$y \leqslant$" means the region below it.
- If y is not isolated, test a point that is not on the line, such as (0, 0). If it makes the inequality true, the region containing that point is the solution set.

A system of inequalities is solved by the region where the individual regions overlap. A point is a solution to the system only if it makes **every** inequality true.

Using the graphing calculator

Type each inequality directly; the calculator draws solid or dashed boundaries and shades each region, and the overlap is the solution set of a system. You can also type a point to see where it falls. A point that sits exactly on a dashed boundary is not a solution even though it appears to touch the shading, so confirm borderline points by substituting them.

Worked examples

Worked example 1: Distributing and flipping

Problem. Which inequality represents all solutions to $2(x - 4) > 5x + 7$?

Think. Distribute, collect the x-terms on one side, then divide.

1. $2x - 8 > 5x + 7$
2. Subtract $5x$ from both sides: $-3x - 8 > 7$.
3. Add 8 to both sides: $-3x > 15$.
4. Divide by -3 and flip the symbol: $x < -5$.

Answer. $x < -5$. Check with $x = -6$: $2(-10) = -20$ and $5(-6) + 7 = -23$, and $-20 > -23$ is true.

Why the traps fail. $x > -5$ forgets the flip; testing $x = 0$ gives $-8 > 7$, which is false. Writing $2(x - 4)$ as $2x - 4$ leads to $-3x > 11$, a different boundary.

Worked example 2: A budget, so round down

Problem. A club has $250 to spend on a party. Renting a room costs $75, and each pizza costs $12. What is the greatest number of pizzas the club can buy?

Think. The total cost cannot exceed the budget: $75 + 12p \leqslant 250$. Subtract 75 to get $12p \leqslant 175$, so $p \leqslant 175/12 \approx 14.6$.

Answer. 14 pizzas. Fourteen pizzas bring the total to 75 + 168 = $243; fifteen would bring it to $255, over the budget.

Why the traps fail. Rounding 14.6 to the nearest whole number gives 15, which breaks the budget. Ignoring the room fee gives $250 \div 12 \approx 20.8$, which answers a different question.

Worked example 3: A goal, so round up

Problem. A fundraiser must raise at least $900. It has already raised $260 and sells wristbands for $6 each. What is the minimum number of wristbands it must sell?

Think. $260 + 6w \geqslant 900$, so $6w \geqslant 640$ and $w \geqslant 640/6 \approx 106.7$.

Answer. 107. Selling 106 wristbands brings the total to only $896, just short of the goal.

Why the traps fail. 106 rounds down and falls short of the goal. With a minimum ("at least"), every value below the boundary fails, so round up.

Worked example 4: Two limits in one story

Problem. A gardener buys x bags of soil at $8 each and y bags of mulch at $5 each. She can spend at most $120 and needs at least 4 bags of soil. Which system describes her choices, and can she buy 6 bags of soil and 14 bags of mulch?

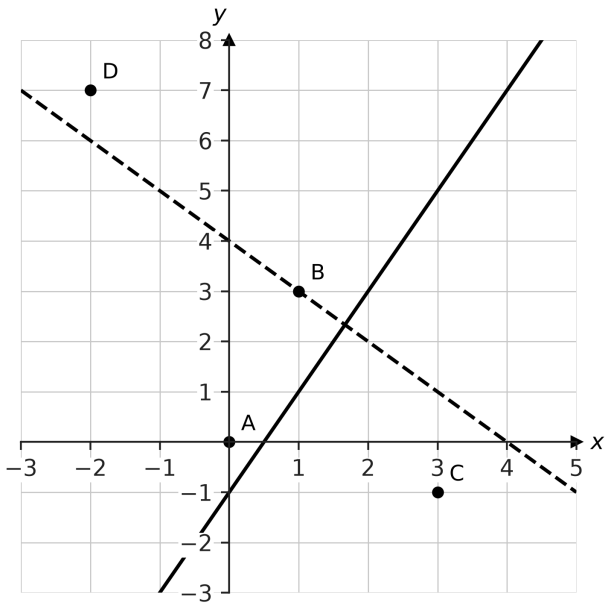
Think. Money: $8x + 5y \leqslant 120$. Soil minimum: $x \geqslant 4$. Test (6, 14): $8(6) + 5(14) = 48 + 70 = 118$, and $118 \leqslant 120$ is true; $6 \geqslant 4$ is also true.

Answer. The system is $8x + 5y \leqslant 120$ and $x \geqslant 4$, with x and y nonnegative whole numbers. The pair (6, 14) satisfies these conditions, so yes.

Why the traps fail. The pair (3, 18) passes the money test (24 + 90 = 114) but fails $x \geqslant 4$. Checking only one inequality is the most common point-testing mistake.

Worked example 5: Testing points against a graph

Problem. The graph shows the boundary lines for the system $y \geqslant 2x - 1$ and $y < -x + 4$. The solid line is $y = 2x - 1$ and the dashed line is $y = -x + 4$. Which of the labeled points A(0, 0), B(1, 3), C(3, −1) and D(−2, 7) is a solution to the system?



Think. The solid boundary counts; the dashed boundary does not. The solution region lies on or above the solid line and below the dashed line. Test each point in both inequalities.

Point	$y \geq 2x - 1$	$y < -x + 4$	Solution?
A(0, 0)	$0 \geqslant -1$, true	$0 < 4$, true	Yes
B(1, 3)	$3 \geqslant 1$, true	$3 < 3$, false	No (on the dashed line)
C(3, −1)	$-1 \geqslant 5$, false	$-1 < 1$, true	No
D(−2, 7)	$7 \geqslant -5$, true	$7 < 6$, false	No

Answer. A(0, 0).

Why the traps fail. B seems to touch the region, but a point on a dashed boundary is excluded. C and D each pass one inequality and fail the other.

Worked example 6 (harder module): Parallel boundaries

Problem. In the system $y > 3x + 2$ and $y < 3x + k$, k is a constant. For which values of k does the system have no solution?

Think. Both boundary lines have slope 3, so they are parallel. The first inequality describes the points above $y = 3x + 2$; the second describes the points below $y = 3x + k$. A strip of solutions between the lines exists only if the second line sits above the first, which means $k > 2$. If $k = 2$, both strict inequalities exclude the one shared line, so no point works.

Answer. The system has no solution when $k \leqslant 2$.

Why the traps fail. Answering $k < 2$ alone forgets that $k = 2$ also leaves no points: no y-value can be both greater than and less than $3x + 2$. Testing one value of k, such as $k = 0$, shows one case but not the whole condition.

Common traps

Trap	How to spot it
Forgetting to flip after multiplying or dividing by a negative	Circle any negative coefficient before you divide, then test a value.
Flipping when you only added or subtracted	Only multiplying or dividing by a negative flips the symbol.
Reversing "at least" and "at most"	"At least" is a floor (⩾); "at most" is a ceiling (⩽).
Rounding a whole-number answer to the nearest integer	For ⩽ or ⩾, round toward allowed integers. A strict < or > excludes an integer boundary: $x < 9$ has greatest integer 8. Check the neighbor.
Counting a point on a dashed line	Substitute: if the two sides are equal and the symbol is < or >, the point is excluded.
Testing only one inequality of a system	A solution must make every inequality true.
Choosing the wrong side when y has a negative coefficient	Solve for y (flipping if you divide by a negative) or test (0, 0).

Harder-module variations

- **Expressions instead of x.** If $3x - 5 \geqslant 7$, then $x \geqslant 4$, so $6x + 1 \geqslant 25$, and the least possible value of $6x + 1$ is 25. Solve for x first, then build the expression.
- **Compound inequalities.** For $-3 < 2x + 1 \leqslant 9$, apply each step to all three parts: $-4 < 2x \leqslant 8$, so $-2 < x \leqslant 4$.
- **Comparing two plans.** To find when one plan costs less than another, put the two cost expressions in an inequality and solve. The equality point is where the costs match; the inequality tells you which side of that point you want.
- **Standard-form boundaries.** For $3x + 2y > 6$, solve for y: $y > -(3/2)x + 3$, so the region lies above a dashed line. Testing (0, 0) gives $0 > 6$, which is false, confirming that the region is on the side away from the origin.
- **Context limits.** Counts of people or items are whole numbers and cannot be negative, so a story can rule out values that the algebra alone allows.

Check yourself

1. What happens to the inequality symbol when you divide both sides by −4? What happens when you subtract 4?
2. Translate "a maximum of 30 students" and "no fewer than 8 chaperones" into inequalities.
3. Is the boundary line solid or dashed for $y > x - 2$, and which side of it holds the solutions?
4. How do you decide whether (2, 5) is a solution to a system of two inequalities?
5. The algebra gives $n \leqslant 23.6$, where n counts tickets. What is the greatest possible value of n?
6. **Answers:** (1) Dividing by −4 flips it; subtracting 4 does not. (2) $s \leqslant 30$ and $c \geqslant 8$. (3) Dashed, with the solutions above the line. (4) Substitute it into both inequalities; it is a solution only if both are true. (5) 23, because 24 would break the limit.

Practice Set

12 questions, arranged from easier to harder. Answer all of them before reading the explanations.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/linear-inequalities

1 M-C1901

Which inequality represents all solutions to $4x - 7 > 9$?

A $x > 1/2$ **B** $x > 4$ **C** $x < 4$ **D** $x > 16$

2 M-C1902

A student has \$120 in a savings account and plans to deposit \$25 into the account at the end of each week, with no withdrawals. Which inequality can be used to find the numbers of weeks, w, after which the account will hold at least \$500?

A $25w + 120 \leqslant 500$ **B** $120w + 25 \geqslant 500$ **C** $25w \geqslant 500$ **D** $120 + 25w \geqslant 500$

3 M-C1903

Which inequality represents all solutions to $-4x + 3 \leqslant 15$?

A $x \leqslant -3$ **B** $x \geqslant -9/2$ **C** $x \geqslant -3$ **D** $x \geqslant 3$

4 M-C1904

At a soccer tournament, a concession stand sells drinks for \$2 each and sandwiches for \$5 each. The stand's goal is to take in at least \$600. Which inequality represents the numbers of drinks, d, and sandwiches, s, that the stand must sell to meet its goal?

A $2d + 5s \geqslant 600$ **B** $2d + 5s \leqslant 600$ **C** $5d + 2s \geqslant 600$ **D** $d + s \geqslant 600$

5 M-C1905

$x/3 - 2 < (x - 4)/5$

What is the greatest integer value of x that satisfies the given inequality?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

6 M-C1906

$y < -2x + 5$

$y \geqslant x - 4$

Which point (x, y) is a solution to the given system of inequalities?

A (2, 1) **B** (4, 0) **C** (−1, −6) **D** (2, −1)

7 M-C1907

A bakery's oven can bake at most 240 items in a day. To fill its standing orders, the bakery must bake at least 60 loaves of bread each day. If m is the number of muffins and b is the number of loaves the bakery bakes in one day, which system of inequalities represents these conditions?

A $m + b \geqslant 240$
$b \geqslant 60$

B $m + b \leqslant 240$
$b \geqslant 60$

C $m + b \leqslant 240$
$b \leqslant 60$

D $m + b \leqslant 240$
$m \geqslant 60$

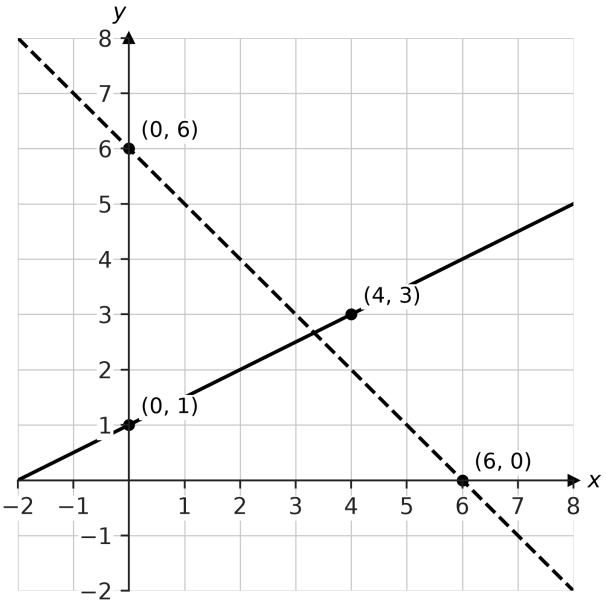
8 M-C1908

An amateur film festival must take in at least $1,525 in ticket sales to cover the cost of a screening. Student tickets cost $6 and adult tickets cost $10. The festival has already sold 140 student tickets and will sell no more student tickets. What is the minimum number of adult tickets the festival must sell to cover the cost?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

9 M-C1909



The graph shows the two boundary lines of a system of linear inequalities. The solutions to the system are all the points that lie on or above the solid line and below the dashed line. Which system of inequalities has this solution set?

A $y \geqslant (1/2)x + 1$
$y < -x + 6$

B $y > (1/2)x + 1$
$y \leqslant -x + 6$

C $y \leqslant (1/2)x + 1$
$y < -x + 6$

D $y \geqslant 2x + 1$
$y < -x + 6$

10 M-C1910

$y \leqslant 4x - 5$

$y > -x + 2$

The point (3, k) is a solution to the given system of inequalities, where k is a constant. Which of the following could be the value of k?

A −3 **B** −1 **C** 7 **D** 8

11 M-C1911

A lab technician has 40 milliliters of a solution that is 30% acid. The technician will add w milliliters of pure water to dilute it, and the diluted solution must be at most 12% acid. What is the minimum number of milliliters of water the technician must add?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

12 M-C1912

Kai has $420 in a savings account and withdraws $35 at the end of each week. Lena has $60 in a savings account and deposits $25 at the end of each week. Neither makes any other changes. What is the greatest whole number of weeks after which Kai's balance is still greater than Lena's balance?

A 5 **B** 6 **C** 7 **D** 12

Practice Set Explanations

1. Correct answer: B M-C1901

Linear inequalities in one or two variables · Algebra · Easier

B $x > 4$

Solve as you would an equation. Add 7 to both sides: $4x > 16$. Divide both sides by 4: $x > 4$. Dividing by a positive number does not change the direction of the inequality. Check with $x = 5$: $4(5) - 7 = 13$, and $13 > 9$ is true.

1. Add 7 to both sides: $4x > 16$.
2. Divide both sides by 4, a positive number, so the symbol stays the same: $x > 4$.

A This subtracts 7 from 9 instead of adding 7 to both sides, giving $4x > 2$. To undo the -7, add 7.

C The symbol flips only when you multiply or divide by a negative number. Dividing by 4 keeps the direction.

D This stops at $4x > 16$. Dividing both sides by 4 gives the solution for x itself.

Review: Chapter 19, Linear Inequalities (page 278)

2. Correct answer: D M-C1902

Linear inequalities in one or two variables · Algebra · Easier

D $120 + 25w \geqslant 500$

After w weeks, the balance is the starting \$120 plus \$25 for each week, or $120 + 25w$ dollars. "At least \$500" means greater than or equal to 500, so the inequality is $120 + 25w \geqslant 500$. Solving gives $w \geqslant 15.2$, so the account first reaches the goal after 16 weeks.

1. Let w = the number of weeks. The balance after w weeks is $120 + 25w$ dollars.
2. "At least \$500" translates to $\geqslant 500$.
3. Inequality: $120 + 25w \geqslant 500$.

A "At least" means greater than or equal to. The symbol $\leqslant$ would describe a balance of at most \$500.

B This swaps the roles of the numbers. The \$25 is added each week, so it multiplies w; the \$120 is a one-time starting amount.

C This leaves out the \$120 already in the account, so it describes the deposits alone reaching \$500.

Review: Chapter 19, Linear Inequalities (page 278)

3. Correct answer: C M-C1903

Linear inequalities in one or two variables · Algebra · Easier

C $x \geqslant -3$

Subtract 3 from both sides: $-4x \leqslant 12$. Divide both sides by -4; dividing by a negative number reverses the inequality, so $x \geqslant -3$. Check with $x = 0$, which should work: $-4(0) + 3 = 3$, and $3 \leqslant 15$ is true. A value outside the solution set, such as $x = -5$, gives 23, which is not $\leqslant 15$.

1. Subtract 3 from both sides: $-4x \leqslant 12$.
2. Divide by -4 and reverse the symbol: $x \geqslant -3$.
3. Check $x = 0$: $3 \leqslant 15$ is true, and $0 \geqslant -3$.

A This divides by -4 without reversing the inequality symbol. Dividing both sides by a negative number always flips the direction.

B This adds 3 to both sides instead of subtracting it, giving $-4x \leqslant 18$. To undo + 3, subtract 3.

D This loses the negative sign: $12 \div (-4)$ is -3, not 3. The test value $x = 0$ satisfies the inequality but is not in $x \geqslant 3$.

Review: Chapter 19, Linear Inequalities (page 278)

4. Correct answer: A M-C1904

Linear inequalities in one or two variables · Algebra · Easier

A $2d + 5s \geqslant 600$

Selling d drinks at \$2 each brings in $2d$ dollars, and selling s sandwiches at \$5 each brings in $5s$ dollars. The total, $2d + 5s$, must be at least 600, which means greater than or equal to 600: $2d + 5s \geqslant 600$.

1. Money from drinks: $2d$ dollars. Money from sandwiches: $5s$ dollars.
2. Total money: $2d + 5s$.
3. "At least \$600": $2d + 5s \geqslant 600$.

B The symbol $\leqslant$ means at most \$600. The goal is to take in at least \$600, which requires $\geqslant$.

C This attaches each price to the wrong item. Drinks cost \$2, so the number of drinks, d, must be multiplied by 2.

D This counts items instead of dollars. Selling 600 items is not the same as taking in \$600.

Review: Chapter 19, Linear Inequalities (page 278)

5. Correct answer: 8 M-C1905

Linear inequalities in one or two variables · Algebra · Medium

Multiply every term by 15, the least common denominator: $5x - 30 < 3(x - 4)$, so $5x - 30 < 3x - 12$. Subtract $3x$ and add 30: $2x < 18$, so $x < 9$. Because 9 itself is excluded by the strict symbol $<$, the greatest integer solution is 8. Entering 9 ignores the strict symbol; forgetting to multiply the -2 by 15 leads to $x < -5$ and an answer of -6.

1. Multiply every term by 15: $5x - 30 < 3x - 12$.
2. Subtract $3x$ and add 30: $2x < 18$.
3. Divide by 2: $x < 9$.
4. The greatest integer less than 9 is 8.

Calculator option: Graph $y = x/3 - 2$ and $y = (x - 4)/5$; they cross at $x = 9$, and the first graph is below the second only to the left of that point.

Review: Chapter 19, Linear Inequalities (page 278)

6. Correct answer: D M-C1906

Linear inequalities in one or two variables · Algebra · Medium

D *(2, −1)*

Test each point in both inequalities. For (2, −1), the first inequality gives $-1 < -2(2) + 5 = 1$, which is true, and the second gives $-1 \geqslant 2 - 4 = -2$, which is also true. A point is a solution to a system only when it makes every inequality true, and (2, −1) is the only choice that does.

1. Substitute each point into $y < -2x + 5$ and $y \geqslant x - 4$.
2. (2, −1): $-1 < 1$ is true and $-1 \geqslant -2$ is true.
3. Every other choice makes at least one inequality false.

A For (2, 1), the first inequality becomes $1 < 1$, which is false. The point lies on the boundary $y = -2x + 5$, and $<$ excludes the boundary.

B For (4, 0), the first inequality becomes $0 < -3$, which is false, even though the second inequality, $0 \geqslant 0$, is true.

C For (−1, −6), the second inequality becomes $-6 \geqslant -5$, which is false. The point satisfies the first inequality only.

Calculator option: Graph both inequalities; the solution set is where the shaded regions overlap. Substitute to confirm, because a point on a dashed boundary can look as if it is inside.

Review: Chapter 19, Linear Inequalities (page 278)

7. Correct answer: B M-C1907

Linear inequalities in one or two variables · Algebra · Medium

B $m + b \leqslant 240$
$b \geqslant 60$

The oven limit applies to all items together, so muffins plus loaves can be no more than 240: $m + b \leqslant 240$. The standing orders require at least 60 loaves: $b \geqslant 60$. Both conditions must hold at the same time, so together they form the system.

1. Let m = the number of muffins and b = the number of loaves.
2. Oven capacity, at most 240 items: $m + b \leqslant 240$.
3. Loaf minimum, at least 60: $b \geqslant 60$.

A "At most 240" means the total cannot exceed 240, which is $m + b \leqslant 240$, not $m + b \geqslant 240$.

C "At least 60 loaves" sets a minimum, so $b \geqslant 60$. The inequality $b \leqslant 60$ would cap the loaves at 60.

D The minimum of 60 applies to loaves of bread, b, not to muffins, m.

Review: Chapter 19, Linear Inequalities (page 278)

8. Correct answer: 69 M-C1908

Linear inequalities in one or two variables · Algebra · Medium

The student tickets bring in 6(140) = $840. If a is the number of adult tickets, then $840 + 10a \geqslant 1{,}525$, so $10a \geqslant 685$ and $a \geqslant 68.5$. Ticket counts are whole numbers, and 68 adult tickets would fall short ($1,520 in all), so the minimum is 69. Entering 68 rounds the wrong way; 153 ignores the student tickets already sold.

1. Money from student tickets: $6 \times 140 = 840$ dollars.
2. Let a = the number of adult tickets: $840 + 10a \geqslant 1{,}525$.
3. Solve: $10a \geqslant 685$, so $a \geqslant 68.5$.
4. Round up to a whole number: 69, since 68 tickets give only $1,520.

Review: Chapter 19, Linear Inequalities (page 278)

9. Correct answer: A M-C1909

Linear inequalities in one or two variables · Algebra · Medium

A $y \geqslant (1/2)x + 1$
$y < -x + 6$

The solid line passes through (0, 1) and (4, 3), so its slope is 2/4 = 1/2 and its equation is $y = (1/2)x + 1$. A solid line means points on it are included, so "on or above" is $y \geqslant (1/2)x + 1$. The dashed line passes through (0, 6) and (6, 0), so it is $y = -x + 6$; dashed means the line is excluded, so "below" is $y < -x + 6$.

1. Solid line through (0, 1) and (4, 3): slope 2/4 = 1/2, so $y = (1/2)x + 1$.
2. Dashed line through (0, 6) and (6, 0): slope −1, so $y = -x + 6$.
3. On or above the solid line: $y \geqslant (1/2)x + 1$. Below the dashed line: $y < -x + 6$.

B This swaps the symbols. The solid boundary needs $\geqslant$, because points on it count, and the dashed boundary needs $<$, because points on it do not.

C The symbol $\leqslant$ describes points on or below the solid line. The solution set lies on or above it.

D The solid line rises 2 units over a run of 4 units, so its slope is 1/2, not 2.

Calculator option: Type both inequalities into the graphing calculator; it draws the same solid and dashed boundaries and shades their overlap, which should match the description.

Review: Chapter 19, Linear Inequalities (page 278)

10. Correct answer: C M-C1910

Linear inequalities in one or two variables · Algebra · Harder

C *7*

Substitute $x = 3$ into each inequality. The first gives $k \leqslant 4(3) - 5 = 7$. The second gives $k > -3 + 2 = -1$. So k must satisfy $-1 < k \leqslant 7$. Among the choices, only 7 works: it is

allowed because the first inequality includes equality, while −1 is excluded by the strict symbol >.

1. Substitute x = 3: k ⩽ 7 and k > −1.
2. Combine: −1 < k ⩽ 7.
3. Of the choices, only 7 lies in this interval.

A With k = −3, the second inequality fails: −3 > −1 is false.

B With k = −1, the second inequality becomes −1 > −1, which is false. The strict symbol > excludes the boundary value.

D With k = 8, the first inequality becomes 8 ⩽ 7, which is false.

Calculator option: Graph both inequalities and the vertical line x = 3; on that line, the overlap runs from just above y = −1 up to and including y = 7.

Review: Chapter 19, Linear Inequalities (page 278)

11. Correct answer: 60 M-C1911

Linear inequalities in one or two variables · Algebra · Harder

Adding water does not change the amount of acid: 0.30(40) = 12 milliliters. The diluted volume is 40 + w milliliters, and its acid can be at most 12% of that volume, so 12 ⩽ 0.12(40 + w). Dividing by 0.12 gives 100 ⩽ 40 + w, so w ⩾ 60. Entering 100 gives the total volume rather than the water added.

1. Acid in the original solution: 0.30 × 40 = 12 milliliters.
2. Let w = milliliters of water added; the new volume is 40 + w milliliters.
3. At most 12% acid: 12 ⩽ 0.12(40 + w).
4. Divide by 0.12: 100 ⩽ 40 + w, so w ⩾ 60.

Calculator option: Graph y = 12/(40 + x) and y = 0.12; the first graph reaches 0.12 at x = 60 and stays below it for larger x.

Review: Chapter 19, Linear Inequalities (page 278)

12. Correct answer: A M-C1912

Linear inequalities in one or two variables · Algebra · Harder

A *5*

After w weeks, Kai has 420 − 35w dollars and Lena has 60 + 25w dollars. Kai's balance is greater when 420 − 35w > 60 + 25w, which gives 360 > 60w, or w < 6. The greatest whole number less than 6 is 5: after 5 weeks, Kai has $245 and Lena has $185.

1. Let w = the number of weeks. Kai: 420 − 35w. Lena: 60 + 25w.
2. Kai greater: 420 − 35w > 60 + 25w, so 360 > 60w and w < 6.
3. The greatest whole number less than 6 is 5.

B After 6 weeks both balances equal $210. Kai's balance is not greater, so 6 fails the strict inequality.

C This adds 60 to 420 instead of subtracting it when collecting the constants, giving 480 > 60w and w < 8. The correct constant is 420 − 60 = 360.

D 12 is the number of weeks until Kai's account is empty (420 ÷ 35); it ignores Lena's growing balance.

Calculator option: Graph y = 420 − 35x and y = 60 + 25x; they cross at x = 6, and Kai's line is above Lena's only to the left of that point.

Review: Chapter 19, Linear Inequalities (page 278)

CHAPTER 20

Equivalent Expressions

Two expressions are equivalent when they have the same value for every value of the variable for which both are defined. For example, $2(x + 3)$ and $2x + 6$ are equivalent. Equivalent-expression questions belong to the Advanced Math domain. They ask you to expand, factor, simplify exponents and radicals, combine rational expressions, match coefficients and divide polynomials. You rarely solve for x; you rewrite.

What the question looks like

Family	Typical stem
Expand or factor	Which expression is equivalent to $(2x - 3)(x + 4)$?
Exponents and radicals	For $x > 0$, which expression is equivalent to $x^{3/2}$?
Rational expressions	Which expression is equivalent to $1/x + 1/(x + 2)$?
Matching coefficients	If $(ax + 1)(x - 3) = 2x^2 + bx - 3$ for all values of x, what is the value of b?
Division and remainders	Which expression is equivalent to $(x^2 + 4x + 7)/(x + 1)$?

How to recognize it

- The stem says "equivalent," "can be rewritten in the form" or "for all values of x."
- There is no equation to solve, or the equation is an identity: both sides must match for every x.
- The answer choices are expressions that look different but might be the same.
- A condition such as "for $x > 0$" or "$x \neq 3$" appears. It keeps radicals and denominators defined; it is not a signal to solve.

The method

1. **Compare the forms.** Look at the choices first. Factored choices mean you factor; expanded choices mean you expand; exponent choices mean you convert radicals to exponents.
2. **Rewrite carefully,** one step at a time. Distribute every term, combine like terms and track every sign.
3. **Match your result to a choice.** If nothing matches exactly, reorder the terms or factor out a constant before deciding.
4. **For identities, match coefficients.** Expand the side with unknowns, then set the x^2 coefficients equal, the x coefficients equal and the constants equal.
5. **Check with a test value.** Substitute a number such as $x = 2$ or $x = 3$ into the original and into your choice. Avoid 0 and 1, which make many wrong choices look right.

CHECK A test value can only rule choices out. If two choices both match the original at $x = 2$, test them again at $x = 3$; equivalent expressions agree at every allowed value.

Expanding and factoring

Pattern	Expanded form
$(a + b)^2$	$a^2 + 2ab + b^2$
$(a - b)^2$	$a^2 - 2ab + b^2$
$(a + b)(a - b)$	$a^2 - b^2$
$(x + p)(x + q)$	$x^2 + (p + q)x + pq$

To factor, work in this order: take out the greatest common factor; look for a difference of squares or a perfect square; then factor any remaining trinomial $x^2 + bx + c$ by finding two numbers whose product is c and whose sum is b.

When the leading coefficient is not 1, split the middle term. For $2x^2 + 7x + 3$, multiply the leading coefficient by the constant ($2 \times 3 = 6$), find two numbers with product 6 and sum 7 (6 and 1), and rewrite: $2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$.

Exponents and radicals

The rules below hold for positive bases x and y. With zero or negative bases, check that each expression is defined and that the rule applies; for example, $\sqrt{(x^2)} = |x|$ for real x, not always x. Denominators must be nonzero.

Rule	Example
$x^a \cdot x^b = x^{a+b}$	$x^2 \cdot x^5 = x^7$
$x^a / x^b = x^{a-b}$	$x^6 / x^2 = x^4$
$(x^a)^b = x^{ab}$	$(x^3)^4 = x^{12}$
$(xy)^a = x^a y^a$	$(2x)^3 = 8x^3$
$x^{-a} = 1/x^a$	$x^{-2} = 1/x^2$
$x^{m/n}$ is the nth root of x^m	$x^{2/3} = \sqrt[3]{(x^2)}$, and $x^{1/2} = \sqrt{x}$

In a rational exponent, the denominator is the root and the numerator is the power. Converting every radical to an exponent first turns most radical questions into exponent arithmetic.

Rational expressions

A rational expression is a fraction with a polynomial in the numerator and the denominator. To simplify, factor completely and cancel common **factors**, never individual terms: in $(x + 3)/(x + 5)$, nothing cancels. To add or subtract, rewrite both fractions over a common denominator, then combine the numerators, putting parentheses around any numerator you subtract.

Cancelling can hide a restriction. The expression $(x^2 - 4)/(x - 2)$ equals $x + 2$ for every x except $x = 2$, where the original denominator is zero. That is why stems add conditions such as "for $x > 2$" or "$x \neq 2$."

Polynomial division

Dividing p(x) by $x + 3$ produces a quotient and a remainder, written as quotient + remainder/$(x + 3)$. One quick method is to rewrite the numerator as a multiple of the divisor plus a leftover amount. The **remainder theorem** gives the remainder directly: dividing p(x) by $x - a$ leaves the remainder p(a). If $p(a) = 0$, then $x - a$ is a factor of p(x).

Using the graphing calculator

Graph the original expression and a choice to find mismatches. Matching graphs or table values suggest a match but do not prove one; simplify and check domain restrictions. A slider can estimate an unknown constant, then matching coefficients gives its exact value.

Worked examples

Worked example 1: Expanding a product

Problem. Which expression is equivalent to $(3x - 2)(2x + 5)$?

Think. Multiply each term in the first factor by each term in the second: $3x(2x) = 6x^2$, $3x(5) = 15x$, $-2(2x) = -4x$ and $-2(5) = -10$. Combine the middle terms: $15x - 4x = 11x$.

Answer. $6x^2 + 11x - 10$. Test with $x = 2$: $(4)(9) = 36$, and $24 + 22 - 10 = 36$.

Why the traps fail. $6x^2 - 10$ drops the middle terms. $6x^2 + 19x - 10$ adds $15x$ and $4x$ instead of $15x$ and $-4x$. $6x^2 + 11x + 10$ loses the sign of $-2(5)$.

Worked example 2: Factoring completely

Problem. Factor $3x^2 - 30x + 75$ completely.

Think. All three terms share the factor 3: $3(x^2 - 10x + 25)$. The trinomial is a perfect square, because $25 = 5^2$ and $10x = 2(5)(x)$.

Answer. $3(x - 5)^2$.

Why the traps fail. $(3x - 5)^2$ expands to $9x^2 - 30x + 25$, so it mishandles the common factor. $3(x - 5)(x + 5)$ is a difference of squares, which has no middle term at all.

Worked example 3: Rational exponents

Problem. For $x > 0$, which expression is equivalent to $(16x^8)^{1/4} \cdot x^{-1/2}$?

Think. Apply the exponent $1/4$ to each factor: $16^{1/4} = 2$, because $2^4 = 16$, and $(x^8)^{1/4} = x^2$. Then multiply by $x^{-1/2}$ by adding exponents: $x^2 \cdot x^{-1/2} = x^{3/2}$.

Answer. $2x^{3/2}$, which can also be written $2x\sqrt{x}$.

Why the traps fail. $4x^{3/2}$ takes $16^{1/4}$ to be 4, but 4 is $16^{1/2}$. $2x^{5/2}$ treats $x^{-1/2}$ as $x^{1/2}$. $2x^{-1}$ multiplies the exponents 2 and $-1/2$ instead of adding them.

Worked example 4: Subtracting rational expressions

Problem. Which expression is equivalent to $3/(x - 2) - 1/(x + 1)$?

Think. The common denominator is $(x - 2)(x + 1)$. Multiply each numerator by the factor its denominator lacks, and keep the second numerator in parentheses: $3(x + 1) - (x - 2) = 3x + 3 - x + 2 = 2x + 5$.

Answer. $(2x + 5)/((x - 2)(x + 1))$.

Why the traps fail. Writing $-(x - 2)$ as $-x - 2$ gives a numerator of $2x + 1$. Subtracting straight across, $(3 - 1)/((x - 2) - (x + 1))$, gives $-2/3$, which is not equivalent: at $x = 3$ the original equals 2.75.

Worked example 5: Matching coefficients

Problem. In the equation $(ax - 2)(3x + 5) = 12x^2 + bx - 10$, a and b are constants, and the equation is true for all values of x. What is the value of b?

Think. Expand the left side: $3ax^2 + 5ax - 6x - 10$. Because the equation holds for all x, matching coefficients are equal.

- x^2 terms: $3a = 12$, so $a = 4$.
- x terms: $b = 5a - 6 = 20 - 6 = 14$.
- Constants: $-10 = -10$, a useful check.

Answer. b = 14.

Why the traps fail. b = 4 reports a instead of b. b = 26 comes from writing $-2(3x)$ as $+6x$. b = 20 forgets the $-6x$ term entirely.

Worked example 6 (harder module): Division with a remainder

Problem. The expression $(2x^2 + 9x + 1)/(x + 4)$ can be written as $2x + 1 + k/(x + 4)$, where k is a constant. What is the value of k?

Think. Multiply the divisor by the quotient: $(x + 4)(2x + 1) = 2x^2 + 9x + 4$. The numerator, $2x^2 + 9x + 1$, is 3 less than that. So $2x^2 + 9x + 1 = (x + 4)(2x + 1) - 3$, and dividing by $x + 4$ gives $2x + 1 - 3/(x + 4)$.

Answer. $k = -3$. The remainder theorem agrees: with $p(x) = 2x^2 + 9x + 1$, $p(-4) = 32 - 36 + 1 = -3$.

Why the traps fail. $k = 1$ assumes the original constant is the remainder. $k = 3$ subtracts in the wrong order ($4 - 1$ instead of $1 - 4$). Evaluating $p(4) = 69$ uses the wrong sign; the divisor $x + 4$ equals zero at $x = -4$.

Common traps

Trap	How to spot it
Dropping the middle term when squaring a binomial	$(a - b)^2$ always has a $-2ab$ term.
Losing a sign when distributing a negative	Put the subtracted expression in parentheses and distribute the minus sign to every term.
Adding exponents when you should multiply, or the reverse	Same base multiplied: add. Power of a power: multiply.
Flipping a rational exponent	In $x^{m/n}$, the bottom number n is the root.
Cancelling terms instead of factors	Factor first; cancel only factors that multiply the entire numerator and denominator.
Adding fractions straight across	Rewrite with a common denominator before combining.
Using the wrong sign in the remainder theorem	A divisor of $x + 3$ means evaluate at $x = -3$.

Harder-module variations

- **Completing the square to reveal a form.** To write $x^2 + 6x + 2$ as $(x + h)^2 + k$, take half of 6, which is 3: $(x + 3)^2 = x^2 + 6x + 9$, so subtract 7. The result, $(x + 3)^2 - 7$, shows $h = 3$ and $k = -7$.

- **Sum of coefficients.** For $ax^2 + bx + c$, the sum $a + b + c$ is the polynomial's value at $x = 1$, a shortcut when a question asks for a sum of coefficients.
- **Grouped squares.** Expressions such as $(x + 1)^2 - 9$ or $(x^2 - 2x + 1) - y^2$ are differences of squares in disguise. Name the two squared quantities, then use $A^2 - B^2 = (A - B)(A + B)$.
- **Unknown constants with division.** If $p(x)$ contains a constant k and the remainder for a divisor is given, evaluate p at the value that makes the divisor zero, set the result equal to the remainder and solve for k.
- **Different bases.** Rewrite powers with a common base before combining them: $8^x = (2^3)^x = 2^{3x}$ and $4 = 2^2$, so $8^x \cdot 4 = 2^{3x + 2}$.
- **Expressions that simplify to a constant.** Some rational expressions cancel completely and leave a number. If a question promises a constant and your result still contains x, look for a sign slip.

Check yourself

1. What should you look for first when you factor any polynomial?
2. Rewrite $\sqrt{(x^5)}$ with a rational exponent.
3. Why can't you cancel the x's in $(x + 4)/(x + 9)$?
4. What is the remainder when $x^3 - 2x + 5$ is divided by $x - 1$?
5. If $(2x + p)(x - 4) = 2x^2 + qx - 12$ for all values of x, what are p and q?
6. **Answers:** (1) A greatest common factor. (2) $x^{5/2}$. (3) The x's are terms in sums, not factors of the whole numerator and denominator. (4) $1 - 2 + 5 = 4$. (5) The constants give $-4p = -12$, so $p = 3$, and $q = p - 8 = -5$.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/equivalent-expressions

Practice Set

18 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2001

Which expression is equivalent to $(x + 6)(x - 2)$?

A $x^2 - 12$ **B** $x^2 - 4x - 12$ **C** $x^2 + 4x - 12$ **D** $x^2 + 8x - 12$

2 M-C2002

Which expression is equivalent to $(3x^4)^2$?

A $9x^8$ **B** $6x^8$ **C** $9x^6$ **D** $3x^8$

3 M-C2003

The expression $5(x + 3) - 2(x - 4)$ is equivalent to $ax + b$, where a and b are constants. What is the value of b?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

4 M-C2004

Which expression is equivalent to $9x^2 - 49$?

A $(3x - 7)^2$ **B** $(3x + 7)(3x - 7)$ **C** $(9x - 7)(x + 7)$ **D** $(9x + 7)(9x - 7)$

5 M-C2005

What is the remainder when $2x^3 - 5x^2 + x + 7$ is divided by $x - 2$?

A -31 **B** 0 **C** 5 **D** 7

6 M-C2006

Which expression is equivalent to $2x^3 - 8x^2 - 24x$?

A $2x(x - 6)(x + 2)$ **B** $2(x - 6)(x + 2)$ **C** $2x(x + 6)(x - 2)$ **D** $2x(x - 3)(x + 4)$

7 M-C2007

For $x > 0$, which expression is equivalent to $\sqrt[3]{(x^5)} \cdot \sqrt{x}$?

A $x^{5/6}$ **B** $x^{11/10}$ **C** $x^{6/5}$ **D** $x^{13/6}$

8 M-C2008

Which expression is equivalent to $(x^2 - 9)/(x^2 + 5x + 6)$, for $x > 0$?

A $(x - 3)/(x + 2)$ **B** $-9/(5x + 6)$ **C** $(x + 3)/(x + 2)$ **D** $(x + 3)/(x - 2)$

9 M-C2009

For $x > 0$, the expression $x^2\sqrt{(x^3)}$ is equivalent to x^k, where k is a constant. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

10 M-C2010

$(ax + 4)(2x - 3) = 10x^2 + bx - 12$

In the given equation, a and b are constants, and the equation is true for all values of x. What is the value of b?

A −23 **B** −7 **C** 5 **D** 23

11 M-C2011

Which expression is equivalent to $(x^2 + 7x + 14)/(x + 3)$, for $x > 0$?

A $x + 4 + 14/(x + 3)$ **B** $x + 4 - 2/(x + 3)$ **C** $x + 10 + 44/(x + 3)$ **D** $x + 4 + 2/(x + 3)$

12 M-C2012

Which expression is equivalent to $2/(x - 1) + 3/(x + 4)$, for $x > 1$?

A $5/(2x + 3)$

B $(5x + 11)/((x - 1)(x + 4))$

C $(5x + 5)/((x - 1)(x + 4))$

D $(5x + 5)/(x^2 - 4)$

13 M-C2013

Which expression is equivalent to $x^2 - 8x + 10$?

A $(x - 4)^2 - 6$ **B** $(x - 4)^2 + 10$ **C** $(x + 4)^2 - 6$ **D** $(x - 8)^2 - 54$

14 M-C2014

The expression $(2x - 5)^2 - (x + 1)(x - 1)$ can be written in the form $ax^2 + bx + c$, where a, b and c are constants. What is the value of $a + b + c$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

15 M-C2015

For positive values of a and b, which expression is equivalent to $(a^3b^{-2})^{1/2}/(a^{-1/2}b)$?

A a^2 **B** a/b^2 **C** a^2/b^3 **D** a^2/b^2

16 M-C2016

Which expression is equivalent to $(x^2 - 6x + 9) - (y^2 - 4y + 4)$?

A $(x - y - 5)(x + y - 5)$ **B** $(x - y - 1)(x + y - 5)$ **C** $(x - y - 5)(x + y - 1)$ **D** $(x - y - 1)^2$

17 M-C2017

For $x > 2$, the expression $(5x^2 + 10x)/(x^2 - 4) - 10/(x - 2)$ is equivalent to a constant k. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

18 M-C2018

$p(x) = 2x^3 + kx^2 - 5x + 4$

In the polynomial p, k is a constant. When p(x) is divided by $x + 2$, the remainder is 10. What is the value of k?

A 0 **B** 1/2 **C** 3 **D** 8

Practice Set Explanations

1. Correct answer: C M-C2001

Equivalent expressions · Advanced Math · Easier

C $x^2 + 4x - 12$

Multiply each term of the first factor by each term of the second: $x \cdot x = x^2$, $x(-2) = -2x$, $6 \cdot x = 6x$ and $6(-2) = -12$. Combine the middle terms: $-2x + 6x = 4x$. The product is $x^2 + 4x - 12$. Check with $x = 3$: $(9)(1) = 9$, and $9 + 12 - 12 = 9$.

1. Distribute: $x(x - 2) + 6(x - 2) = x^2 - 2x + 6x - 12$.
2. Combine like terms: $x^2 + 4x - 12$.

A This multiplies only the first terms and the last terms. It drops the middle terms, $-2x$ and $6x$, which combine to $4x$.

B This reverses the sign of the middle term. The middle terms are $6x$ and $-2x$, which combine to $+4x$.

D This adds 6 and 2 to get the middle coefficient. The constants are 6 and -2, so the middle coefficient is $6 + (-2) = 4$.

Review: Chapter 20, Equivalent Expressions (page 289)

2. Correct answer: A M-C2002

Equivalent expressions · Advanced Math · Easier

A $9x^8$

A power of a product raises every factor to that power: $(3x^4)^2 = 3^2 \cdot (x^4)^2$. Then $3^2 = 9$, and a power of a power multiplies the exponents: $(x^4)^2 = x^{4 \cdot 2} = x^8$. The result is $9x^8$.

1. Square each factor: $3^2 \cdot (x^4)^2$.
2. Compute: $3^2 = 9$ and $(x^4)^2 = x^8$, so the result is $9x^8$.

B This multiplies the coefficient 3 by 2 instead of squaring it. Squaring means $3 \times 3 = 9$.

C This adds the exponents, $4 + 2 = 6$. For a power of a power, multiply the exponents: $4 \times 2 = 8$.

D This applies the outer exponent to x^4 only. The coefficient 3 is inside the parentheses, so it is squared too.

Review: Chapter 20, Equivalent Expressions (page 289)

3. Correct answer: 23 M-C2003

Equivalent expressions · Advanced Math · Easier

Distribute both constants, keeping the minus sign with the 2: $5(x + 3) = 5x + 15$ and $-2(x - 4) = -2x + 8$. Combine like terms: $(5x - 2x) + (15 + 8) = 3x + 23$, so $b = 23$. A common wrong answer is 7, which comes from writing $-2(x - 4)$ as $-2x - 8$; another is 3, which is a, not b.

1. Distribute: $5x + 15 - 2x + 8$.
2. Combine like terms: $3x + 23$.
3. Match with $ax + b$: $b = 23$.

Review: Chapter 20, Equivalent Expressions (page 289)

4. Correct answer: B M-C2004

Equivalent expressions · Advanced Math · Easier

B $(3x + 7)(3x - 7)$

The expression is a difference of two squares: $9x^2 = (3x)^2$ and $49 = 7^2$. The pattern $a^2 - b^2 = (a + b)(a - b)$ gives $(3x + 7)(3x - 7)$. Check by expanding: $9x^2 - 21x + 21x - 49 = 9x^2 - 49$.

1. Write each term as a square: $(3x)^2 - 7^2$.
2. Apply $a^2 - b^2 = (a + b)(a - b)$: $(3x + 7)(3x - 7)$.

A $(3x - 7)^2$ expands to $9x^2 - 42x + 49$. A squared binomial has a middle term and a positive constant.

C This expands to $9x^2 + 56x - 49$. The middle terms cancel only when the factors are a sum and a difference of the same two terms.

D This uses $9x$ as the square root of $9x^2$. But $(9x)^2 = 81x^2$, so this product is $81x^2 - 49$.

Calculator option: Graph $y = 9x^2 - 49$ and a choice; only the equivalent choice produces a graph that lies exactly on the original.

Review: Chapter 20, Equivalent Expressions (page 289)

5. Correct answer: C M-C2005

Equivalent expressions · Advanced Math · Easier

C 5

By the remainder theorem, dividing a polynomial $p(x)$ by $x - a$ leaves the remainder $p(a)$. Here $a = 2$: $p(2) = 2(8) - 5(4) + 2 + 7 = 16 - 20 + 2 + 7 = 5$. Long division gives the same result, because $2x^3 - 5x^2 + x + 7 = (x - 2)(2x^2 - x - 1) + 5$.

1. Remainder theorem: the remainder of $p(x) \div (x - 2)$ is $p(2)$.
2. Evaluate: $p(2) = 16 - 20 + 2 + 7 = 5$.

A This evaluates the polynomial at $x = -2$. Dividing by $x - 2$ calls for $x = 2$, the value that makes $x - 2$ equal 0.

B A remainder of 0 would mean $x - 2$ is a factor. Evaluating $p(2)$ gives 5, so $x - 2$ is not a factor.

D 7 is the constant term, which equals $p(0)$. The remainder for division by $x - 2$ is $p(2)$, not $p(0)$.

Calculator option: Define $p(x) = 2x^3 - 5x^2 + x + 7$ in the calculator and evaluate $p(2)$, or read the value at $x = 2$ from a table; it is 5, the remainder.

Review: Chapter 20, Equivalent Expressions (page 289)

6. Correct answer: A M-C2006

Equivalent expressions · Advanced Math · Medium

A $2x(x - 6)(x + 2)$

Factor out the greatest common factor first: $2x^3 - 8x^2 - 24x = 2x(x^2 - 4x - 12)$. Then factor the trinomial: the numbers with product -12 and sum -4 are -6 and 2, so x^2

$- 4x - 12 = (x - 6)(x + 2)$. The full factorization is $2x(x - 6)(x + 2)$.

1. Factor out $2x$: $2x(x^2 - 4x - 12)$.
2. Find two numbers with product -12 and sum -4: -6 and 2.
3. Result: $2x(x - 6)(x + 2)$.

B This drops the common factor x. Its product is $2x^2 - 8x - 24$, a quadratic expression rather than a cubic one.

C The signs are reversed: $(x + 6)(x - 2) = x^2 + 4x - 12$, so this product has the middle term $+8x^2$.

D The numbers -3 and 4 multiply to -12 but add to 1, not -4, so this product is $2x^3 + 2x^2 - 24x$.

Calculator option: Graph the original expression together with a choice; an equivalent choice produces a curve that overlaps the original completely.

Review: Chapter 20, Equivalent Expressions (page 289)

7. Correct answer: D M-C2007

Equivalent expressions · Advanced Math · Medium

D $x^{13/6}$

Rewrite each radical as a rational exponent, where the root is the denominator: $\sqrt[3]{(x^5)} = x^{5/3}$ and $\sqrt{x} = x^{1/2}$. When you multiply powers of the same base, add the exponents: $5/3 + 1/2 = 10/6 + 3/6 = 13/6$. So the product is $x^{13/6}$.

1. Rewrite the radicals: $\sqrt[3]{(x^5)} = x^{5/3}$ and $\sqrt{x} = x^{1/2}$.
2. Add the exponents with a common denominator: $10/6 + 3/6 = 13/6$.
3. Result: $x^{13/6}$.

A This multiplies the exponents, $(5/3)(1/2) = 5/6$. Multiplying powers of the same base calls for adding the exponents.

B This writes $\sqrt[3]{(x^5)}$ as $x^{3/5}$. In a rational exponent the denominator is the root, so $\sqrt[3]{(x^5)} = x^{5/3}$.

C This adds the fractions by adding numerators and denominators. With a common denominator, $5/3 + 1/2 = 13/6$.

Review: Chapter 20, Equivalent Expressions (page 289)

8. Correct answer: A M-C2008

Equivalent expressions · Advanced Math · Medium

A $(x - 3)/(x + 2)$

Factor the numerator and the denominator: $x^2 - 9 = (x - 3)(x + 3)$ and $x^2 + 5x + 6 = (x + 2)(x + 3)$. Cancel the common factor $x + 3$, which is never 0 when $x > 0$. The result is $(x - 3)/(x + 2)$. Only whole factors cancel, never individual terms.

1. Factor: $(x - 3)(x + 3)$ over $(x + 2)(x + 3)$.
2. Cancel the common factor $x + 3$.
3. Result: $(x - 3)/(x + 2)$.

B This cancels the x^2 terms, but x^2 is a term in a sum, not a factor. Only factors shared by the whole numerator and denominator cancel.

C This treats $x^2 - 9$ as $(x + 3)^2$. A difference of squares factors as $(x - 3)(x + 3)$.

D This factors the denominator as $(x - 2)(x - 3)$, which expands to $x^2 - 5x + 6$, and then cancels $x - 3$.

Calculator option: Graph the original expression and $(x - 3)/(x + 2)$; for $x > 0$ the two graphs coincide.

Review: Chapter 20, Equivalent Expressions (page 289)

9. Correct answer: 7/2 or 3.5 M-C2009

Equivalent expressions · Advanced Math · Medium

Rewrite the radical with a rational exponent: $\sqrt{(x^3)} = x^{3/2}$. Multiplying powers of the same base adds the exponents: $x^2 \cdot x^{3/2} = x^{2 + 3/2} = x^{7/2}$, so $k = 7/2$, which can also be entered as 3.5. Entering 3 comes from multiplying 2 by 3/2; 8/3 comes from writing $\sqrt{(x^3)}$ as $x^{2/3}$.

1. Rewrite the radical: $\sqrt{(x^3)} = x^{3/2}$.
2. Multiply powers by adding exponents: $x^2 \cdot x^{3/2} = x^{2 + 3/2}$.
3. Add: $2 + 3/2 = 7/2$, so $k = 7/2$ (or 3.5).

Calculator option: Graph $y = x^2\sqrt{(x^3)}$ and $y = x^k$ with k on a slider; the graphs match for $x > 0$ when $k = 3.5$.

Review: Chapter 20, Equivalent Expressions (page 289)

10. Correct answer: B M-C2010

Equivalent expressions · Advanced Math · Medium

B -7

Expand the left side: $(ax + 4)(2x - 3) = 2ax^2 - 3ax + 8x - 12$. For the equation to hold for all x, matching coefficients must be equal. The x^2 terms give $2a = 10$, so $a = 5$. The x terms give $b = -3a + 8 = -15 + 8 = -7$. The constants already match: $-12 = -12$.

1. Expand: $2ax^2 + (8 - 3a)x - 12$.
2. Match the x^2 coefficients: $2a = 10$, so $a = 5$.
3. Match the x coefficients: $b = 8 - 3(5) = -7$.

A This writes the product $4(2x)$ as $-8x$, combining -15 and -8. The product of 4 and $2x$ is $+8x$.

C 5 is the value of a, found from the x^2 coefficients. The question asks for b, the coefficient of x.

D This drops the negative sign on $-3ax$, adding 15 and 8. The term $ax(-3)$ is $-3ax$, which is $-15x$ when $a = 5$.

Review: Chapter 20, Equivalent Expressions (page 289)

11. Correct answer: D M-C2011

Equivalent expressions · Advanced Math · Medium

D $x + 4 + 2/(x + 3)$

Rewrite the numerator using a multiple of $x + 3$. Because $(x + 3)(x + 4) = x^2 + 7x + 12$, the numerator is $(x + 3)(x + 4)$

+ 2. Dividing each part by x + 3 gives x + 4 + 2/(x + 3). Check with x = 1: 22/4 = 5.5, and 5 + 2/4 = 5.5.

1. Divide the leading terms: $x(x + 3) = x^2 + 3x$, leaving 4x + 14.
2. Divide again: 4(x + 3) = 4x + 12, leaving a remainder of 2.
3. Result: x + 4 + 2/(x + 3).

A This keeps 14 as the remainder. Part of the 14 is already used by the quotient, since (x + 3)(x + 4) has constant 12, leaving 2.

B This subtracts in the wrong order. The numerator is 2 more than (x + 3)(x + 4), because 14 − 12 = 2, so the remainder is +2.

C This uses 3 instead of −3 in synthetic division, which divides by x − 3 (quotient x + 10, remainder 44), and then writes the remainder over x + 3.

Calculator option: Graph the original expression and a choice; the equivalent choice traces exactly the same curve.

Review: Chapter 20, Equivalent Expressions (page 289)

12. Correct answer: C M-C2012

Equivalent expressions · Advanced Math · Medium

C *(5x + 5)/((x − 1)(x + 4))*

Use the common denominator (x − 1)(x + 4). Multiply each numerator by the factor its denominator is missing: 2(x + 4) + 3(x − 1) = 2x + 8 + 3x − 3 = 5x + 5. The sum is (5x + 5)/((x − 1)(x + 4)). Check with x = 2: 2/1 + 3/6 = 2.5, and 15/6 = 2.5.

1. Common denominator: (x − 1)(x + 4).
2. New numerator: 2(x + 4) + 3(x − 1) = 5x + 5.
3. Result: (5x + 5)/((x − 1)(x + 4)).

A This adds the numerators and adds the denominators. Fractions can be added only after they are rewritten with a common denominator.

B This multiplies 3 by (x − 1) to get 3x + 3. The product is 3x − 3, which makes the numerator 5x + 5.

D The product (x − 1)(x + 4) is $x^2 + 3x - 4$, not $x^2 - 4$; multiplying only the first and last terms skips the middle term.

Review: Chapter 20, Equivalent Expressions (page 289)

13. Correct answer: A M-C2013

Equivalent expressions · Advanced Math · Medium

A $(x - 4)^2 - 6$

Complete the square. Half of −8 is −4, and $(x - 4)^2 = x^2 - 8x + 16$. That is 6 more than the given expression, so subtract 6: $x^2 - 8x + 10 = (x - 4)^2 - 6$. This form also shows the expression's minimum value, −6, which occurs at x = 4.

1. Take half of the x-coefficient: half of −8 is −4, and $(x - 4)^2 = x^2 - 8x + 16$.
2. Adjust the constant: 10 − 16 = −6.
3. Result: $(x - 4)^2 - 6$.

B This forgets that $(x - 4)^2$ already contains +16. Adding 10 to it gives $x^2 - 8x + 26$.

C The sign inside the parentheses is wrong: $(x + 4)^2$ produces +8x. The coefficient −8 requires $(x - 4)^2$.

D This uses the whole coefficient, 8, instead of half of it. $(x - 8)^2 - 54$ expands to $x^2 - 16x + 10$.

Calculator option: Graph $y = x^2 - 8x + 10$; its vertex at (4, −6) matches the form $(x - 4)^2 - 6$.

Review: Chapter 20, Equivalent Expressions (page 289)

14. Correct answer: 9 M-C2014

Equivalent expressions · Advanced Math · Harder

Expand each part: $(2x - 5)^2 = 4x^2 - 20x + 25$ and $(x + 1)(x - 1) = x^2 - 1$. Subtracting changes both signs of the second part, giving $3x^2 - 20x + 26$, so a + b + c = 9. Shortcut: a + b + c is the value at x = 1, which is $(-3)^2 - 0 = 9$. Entering 29 drops the −20x term; entering 7 subtracts 1 instead of adding it.

1. Expand: $(2x - 5)^2 = 4x^2 - 20x + 25$.
2. Expand: $(x + 1)(x - 1) = x^2 - 1$.
3. Subtract: $4x^2 - 20x + 25 - x^2 + 1 = 3x^2 - 20x + 26$.
4. Add the coefficients: 3 + (−20) + 26 = 9.

Review: Chapter 20, Equivalent Expressions (page 289)

15. Correct answer: D M-C2015

Equivalent expressions · Advanced Math · Harder

D a^2/b^2

Apply the outer exponent to each factor: $(a^3b^{-2})^{1/2} = a^{3/2}b^{-1}$. Dividing powers of the same base subtracts exponents: for a, 3/2 − (−1/2) = 2; for b, −1 − 1 = −2. The result is a^2b^{-2}, which equals a^2/b^2.

1. Apply the exponent 1/2 to each factor: $a^{3/2}b^{-1}$.
2. Divide by $a^{-1/2}b$ by subtracting exponents: $a^{3/2 - (-1/2)}b^{-1 - 1}$.
3. Simplify: $a^2b^{-2} = a^2/b^2$.

A This loses the power of b. Dividing b^{-1} by b^1 gives $b^{-1 - 1} = b^{-2}$, which leaves b^2 in the denominator.

B This subtracts 1/2 instead of −1/2 in the exponent of a. Subtracting a negative exponent adds: 3/2 − (−1/2) = 2.

C This leaves b^{-2} unchanged inside the parentheses. The exponent 1/2 applies to every factor: $(b^{-2})^{1/2} = b^{-1}$.

Review: Chapter 20, Equivalent Expressions (page 289)

16. Correct answer: B M-C2016

Equivalent expressions · Advanced Math · Harder

B *(x − y − 1)(x + y − 5)*

Each group is a perfect square: $x^2 - 6x + 9 = (x - 3)^2$ and $y^2 - 4y + 4 = (y - 2)^2$. The expression is a difference of squares, $A^2 - B^2 = (A - B)(A + B)$, with $A = x - 3$ and $B = y - 2$. So it equals $(x - 3 - y + 2)(x - 3 + y - 2) = (x - y - 1)(x + y - 5)$.

1. Recognize the perfect squares: $(x - 3)^2 - (y - 2)^2$.
2. Apply $A^2 - B^2 = (A - B)(A + B)$ with $A = x - 3$ and $B = y - 2$.
3. Simplify each factor: $(x - y - 1)(x + y - 5)$.

A This distributes the minus sign incorrectly in the first factor, writing $-(y - 2)$ as $-y - 2$. Since $-(y - 2) = -y + 2$, the constant is $-3 + 2 = -1$.

C This writes $y^2 - 4y + 4$ as $(y + 2)^2$. The middle term $-4y$ requires $(y - 2)^2$.

D This treats $A^2 - B^2$ as $(A - B)^2$. A difference of squares factors into a difference times a sum.

Review: Chapter 20, Equivalent Expressions (page 289)

17. Correct answer: 5 M-C2017

Equivalent expressions · Advanced Math · Harder

Factor and cancel first: $(5x^2 + 10x)/(x^2 - 4) = 5x(x + 2)/((x - 2)(x + 2)) = 5x/(x - 2)$. Both fractions now share the denominator $x - 2$: $(5x - 10)/(x - 2) = 5(x - 2)/(x - 2) = 5$. If a sign slips and the numerator becomes $5x + 10$, nothing cancels, which signals an error. Check at $x = 3$: $75/5 - 10 = 5$.

1. Factor: $5x(x + 2)$ over $(x - 2)(x + 2)$.
2. Cancel $x + 2$: $5x/(x - 2)$.
3. Subtract over the common denominator: $(5x - 10)/(x - 2)$.
4. Factor and cancel: $5(x - 2)/(x - 2) = 5$.

Calculator option: Graph the expression for $x > 2$; it is the horizontal line $y = 5$.

Review: Chapter 20, Equivalent Expressions (page 289)

18. Correct answer: C M-C2018

Equivalent expressions · Advanced Math · Harder

C *3*

By the remainder theorem, the remainder when $p(x)$ is divided by $x + 2$ equals $p(-2)$, because $x + 2 = 0$ when $x = -2$. So $p(-2) = 2(-8) + k(4) - 5(-2) + 4 = -16 + 4k + 10 + 4 = 4k - 2$. Setting $4k - 2 = 10$ gives $4k = 12$ and $k = 3$.

1. Remainder theorem: the remainder equals $p(-2)$.
2. Evaluate: $p(-2) = -16 + 4k + 10 + 4 = 4k - 2$.
3. Set $4k - 2 = 10$: $4k = 12$, so $k = 3$.

A This evaluates $p(2)$ instead of $p(-2)$. Dividing by $x + 2$ calls for $x = -2$, the value that makes the divisor zero.

B This sets $p(-2)$ equal to 0, as if $x + 2$ were a factor. The remainder is 10, so $p(-2)$ must equal 10.

D This treats $-5(-2)$ as -10. A negative times a negative is positive, so $-5(-2) = +10$.

Review: Chapter 20, Equivalent Expressions (page 289)

CHAPTER 21

Nonlinear Equations and Systems

A linear equation usually leads straight to one answer. A nonlinear equation can have zero, one, two or more real solutions, and some values your algebra produces may not work at all. This chapter gives you one method for every type. The College Board calls the skill "Nonlinear equations in one variable and systems of equations in two variables." It belongs to the Advanced Math domain, about 35 percent of the Math section.

What the question looks like

Most questions give an equation or a system and ask for its solutions in a specific form:

- "What is the positive solution to the given equation?" (often a student-produced response)
- "What are all the solutions to the given equation?"
- "How many distinct real solutions does the given equation have?"
- "If (x, y) is a solution to the given system, what is the value of y?"
- "In the given equation, k is a constant. The equation has exactly one real solution. What is the value of k?"

Context questions, such as a thrown ball's height or a growing bacteria culture, ask when a quantity reaches a given value.

How to recognize it

Look at where the variable sits. If it is squared, under a root, in a denominator, between absolute value bars or in an exponent, the equation is nonlinear. A system belongs here when at least one of its equations is nonlinear, most often a line paired with a parabola.

Chapter 20 rewrites expressions without solving anything, and Chapter 22 asks about a function's features, such as its vertex or growth rate. If the task is to find the values of x that make an equation true, or the points where two graphs meet, you are in this chapter.

The method

1. **Name the type.** Use the table below to choose your first move.
2. **Reach a standard form.** For a quadratic, move every term to one side so the other side is 0.
3. **Pick the fastest tool.** Factor if a factor pair appears within a few seconds. Otherwise complete the square or use the quadratic formula. If the question asks only *how many* solutions, use the discriminant and skip solving.
4. **Solve, then check.** Whenever you square both sides, multiply by an expression that contains x, or split an absolute value whose other side contains x, substitute each candidate into the original equation.
5. **Answer the question asked.** Reread the last sentence: the positive solution, the sum of the solutions, or the value of y.

Equation type	First move	Watch for
Quadratic	Set one side to 0, then factor or use the formula	Never divide both sides by x; you lose x = 0
Radical	Isolate the root, then square both sides	Extraneous solutions: check every candidate
Rational	List excluded values, then multiply by the common denominator	Candidates that make a denominator 0
Absolute value	Split into A = c and A = −c	No solution if c is negative; check if c contains x
Exponential	Write both sides as powers of one base, then set the exponents equal	Rewrite 8 as 2^3 and 1/9 as 3^{-2}
Line and parabola	Substitute the line into the quadratic	Find both coordinates when asked for a point

Three tools for $ax^2 + bx + c = 0$, where $a \neq 0$.

- *Factoring* works when you can find two numbers whose product is ac and whose sum is b.
- *Completing the square* is fastest when a = 1 and b is even: move c to the right side, then add $(b/2)^2$ to both sides.
- *The quadratic formula* always works: $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$.

The expression under the root, $b^2 - 4ac$, is the **discriminant**. If it is positive, the equation has two real solutions; if it is 0, exactly one; if it is negative, none. Neither the quadratic formula nor the discriminant is on the reference sheet, so memorize both.

NOTE An extraneous solution is a value your algebra produces that fails the original equation. The three moves named in step 4 of the method can create one, which is why each of them requires a check.

Worked examples

Worked Example 1: Factoring and the positive solution

What is the positive solution to $3x^2 + 4x - 4 = 0$?

Thinking. Here ac = 3 × (−4) = −12. You need two numbers with product −12 and sum 4: 6 and −2. Split the middle term and group: $3x^2 + 6x - 2x - 4 = 3x(x + 2) - 2(x + 2) = (3x - 2)(x + 2)$. Setting each factor equal to 0 gives x = 2/3 or x = −2.

Answer. 2/3. As a student-produced response, the fraction 2/3 is the safest entry. A decimal too long to fit is cut off or rounded to fill the space (here .6666 or .6667); follow the College Board's directions on test day.

Traps. Entering −2 answers the wrong question. Entering 2 forgets that 3x = 2 still needs division by 3.

Worked Example 2: Completing the square, checked by the formula

Solve $x^2 + 4x - 2 = 0$.

Thinking. The x^2 coefficient is 1 and b = 4 is even, so complete the square. Move the constant: $x^2 + 4x = 2$. Half of 4 is 2, and $2^2 = 4$, so add 4 to both sides: $x^2 + 4x + 4 = 6$, which is $(x + 2)^2 = 6$. Take the square root of both sides and keep both signs: $x + 2 = \pm\sqrt{6}$, so $x = -2 \pm \sqrt{6}$.

Check with the formula. $x = (-4 \pm \sqrt{(16 + 8)})/2 = (-4 \pm \sqrt{24})/2$. Since $\sqrt{24} = 2\sqrt{6}$, this simplifies to $-2 \pm \sqrt{6}$.

Traps. Adding 4 to only one side; writing only $+\sqrt{6}$; dividing only the -4 by 2, which gives the wrong answer $-2 \pm 2\sqrt{6}$.

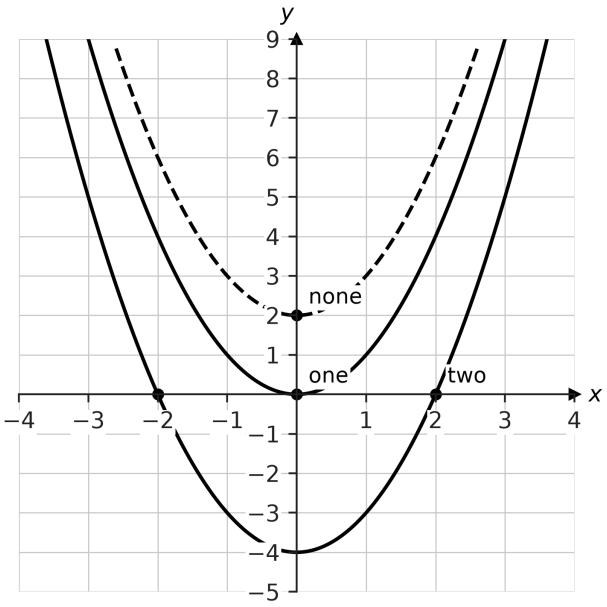
Worked Example 3: Counting solutions with the discriminant

How many distinct real solutions does $2x^2 - 3x + 5 = 0$ have?

Thinking. You are not asked for the solutions, so do not find them. With $a = 2$, $b = -3$ and $c = 5$, the discriminant is $(-3)^2 - 4(2)(5) = 9 - 40 = -31$, which is negative.

Answer. Zero real solutions. The graph of $y = 2x^2 - 3x + 5$ never reaches the x-axis.

In the figure, three parabolas of the same shape differ only in their constant. The x-intercepts of $y = ax^2 + bx + c$ are the real solutions of $ax^2 + bx + c = 0$, and the labels show how many each graph has.



Traps. Writing $(-3)^2$ as -9; computing the discriminant before moving every term to one side.

Worked Example 4: A radical equation with an extraneous solution

Solve $\sqrt{(x + 10)} = x - 2$.

Thinking. The root is already isolated, so square both sides: $x + 10 = x^2 - 4x + 4$. Move every term to one side: $x^2 - 5x - 6 = 0$, so $(x - 6)(x + 1) = 0$, and the candidates are 6 and -1.

Check. For $x = 6$: $\sqrt{16} = 4$ and $6 - 2 = 4$, so 6 works. For $x = -1$: $\sqrt{9} = 3$, but $-1 - 2 = -3$. A square root is never negative, so -1 is extraneous. Squaring hid the problem, because 3^2 and $(-3)^2$ are both 9.

Answer. $x = 6$ only.

Worked Example 5: A rational equation with an excluded value

Solve $x^2/(x - 3) = 9/(x - 3)$.

Thinking. First note that x cannot be 3, because 3 makes both denominators 0. Multiply both sides by $x - 3$: $x^2 = 9$, so $x = 3$ or $x = -3$. Reject 3.

Answer. $x = -3$. Check: $9/(-6)$ equals $9/(-6)$.

TRAP An answer choice that lists both 3 and −3 is built for students who skip the excluded-value check.

Worked Example 6: Absolute value and exponential equations

Absolute value. Solve $|3x - 2| = 10$. The expression inside the bars equals 10 or −10. If $3x - 2 = 10$, then $x = 4$. If $3x - 2 = -10$, then $3x = -8$ and $x = -8/3$.

Both check, because the right side, 10, is positive and contains no x. An equation such as $|3x - 2| = -10$ has no solution at all.

Exponential. Solve $4^{x+1} = 8$. Write both sides as powers of 2: $4^{x+1} = (2^2)^{x+1} = 2^{2x+2}$, and $8 = 2^3$. Equal powers of the same base have equal exponents, so $2x + 2 = 3$ and $x = 1/2$.

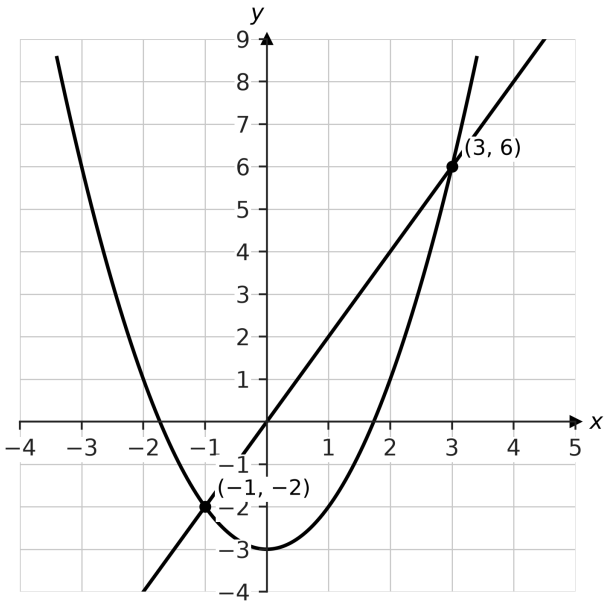
Traps. Solving only the positive case; writing $(2^2)^{x+1}$ as 2^{x+3} instead of multiplying the exponents.

Worked Example 7: A line and a parabola

What are the solutions (x, y) of the system $y = x^2 - 3$ and $y = 2x$?

Thinking. Both equations give y, so set the right sides equal: $x^2 - 3 = 2x$. Rearrange to $x^2 - 2x - 3 = 0$, which factors as $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$. Use the simpler equation, $y = 2x$, to find each y-value.

Answer. (3, 6) and (−1, −2), the two points where the graphs cross.



Graphing check. On the built-in graphing calculator, graph both equations and select the intersection points. A line can meet a parabola at two points, touch it at exactly one, or miss it. These cases match the three signs of the discriminant after you substitute.

Worked Example 8 (harder module): A constant that makes the graphs touch

In the system $y = 2x^2$ and $y = 8x + c$, c is a constant. The graphs intersect at exactly one point. What is the value of c?

Thinking. Set the right sides equal: $2x^2 = 8x + c$, so $2x^2 - 8x - c = 0$. Exactly one intersection point means exactly one solution, so the discriminant is 0: $(-8)^2 - 4(2)(-c) = 64 + 8c = 0$. Therefore $c = -8$.

Check. With $c = -8$ the equation becomes $2x^2 - 8x + 8 = 2(x - 2)^2 = 0$, so $x = 2$ is the only solution and the single point is $(2, 8)$. A slider for c on the graphing calculator shows the line just touching the parabola at $c = -8$, but the discriminant gives the exact value faster.

Trap. Treating the constant term as $+c$ instead of $-c$ leads to the sign error $c = 8$.

Common traps

Trap	How to spot it
Dividing both sides by x	You solved $x^2 = 5x$ and got only $x = 5$. Factor instead: $x(x - 5) = 0$ gives 0 and 5.
Keeping only the positive root	Solving $x^2 = 49$ gives $x = \pm 7$. The symbol $\sqrt{49}$ alone means the nonnegative root, 7.
Keeping an extraneous solution	You squared both sides or cleared a denominator. Substitute every candidate into the original equation.
Factoring before one side is 0	$(x - 2)(x + 1) = 4$ does not mean $x - 2 = 4$. Expand, subtract 4, then factor.
Sign slip with $-b$	The formula begins with $-b$. If $b = -5$, the numerator begins with $+5$.
Answering the wrong quantity	The question wants the positive solution, a sum, or y. Underline the last sentence.
Forgetting a case	An absolute value equation with a positive right side usually gives two candidates.

Harder-module variations

- **A constant controls the number of solutions.** Write the discriminant in terms of the constant and set it greater than 0 (two solutions), equal to 0 (one) or less than 0 (none). If the constant multiplies x^2, also test the value that makes it 0, because the equation then becomes linear.
- **Sum or product without solving.** For $ax^2 + bx + c = 0$, the solutions add to $-b/a$ and multiply to c/a. For $2x^2 - 6x - 9 = 0$, the sum is 3 and the product is $-9/2$.
- **A quadratic in disguise.** In $x^4 - 13x^2 + 36 = 0$, let $u = x^2$. Then $u^2 - 13u + 36 = (u - 4)(u - 9) = 0$, so $x = \pm 2$ or $x = \pm 3$. The same substitution handles $2(x + 1)^2 - 3(x + 1) = 0$ with $u = x + 1$.
- **One known solution.** When one solution is given and a constant is unknown, substitute the known solution to find the constant, then factor or use the product c/a.
- **Two quadratics.** When both equations in a system are quadratic, set them equal. The x^2 terms often cancel or combine into a simpler equation.
- **Context limits.** A negative time or distance can satisfy the equation but not the situation. Questions often say "$t > 0$" so that the answer is unique.

Check yourself

1. What does a negative discriminant tell you about a quadratic equation and its graph?
2. Which algebra moves can create extraneous solutions?

3. How do you solve an absolute value equation when the right side is positive, and what happens when it is negative?
4. In a system of a line and a parabola, how do you get a single equation in x?
5. Why should you never divide both sides of $x^2 = 3x$ by x?
6. Answers: (1) The equation has no real solutions, and the parabola does not touch the x-axis. (2) Squaring both sides, multiplying by an expression that contains x, and splitting an absolute value whose other side contains x. (3) Solve the two cases A = c and A = −c; a negative right side gives no solution. (4) Set the two expressions for y equal and move every term to one side. (5) Dividing by x loses the solution x = 0.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/
nonlinear-equations-and-systems

Practice Set

18 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2101

What are the solutions to the equation $x^2 - 3x - 28 = 0$?

A −7 and 4 **B** −4 and 7 **C** −2 and 14 **D** 4 and 7

2 M-C2102

What are all the solutions to the equation $|x - 5| = 9$?

A 14 only **B** −14 and 4 **C** −9 and 9 **D** −4 and 14

3 M-C2103

What is the solution to the equation $\sqrt{(x + 6)} = 4$?

A −2 **B** 2 **C** 10 **D** 22

4 M-C2104

What value of x is the solution to the equation $5^{2x - 1} = 125$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

5 M-C2105

$y = x^2 - 1$

$y = x + 5$

Which ordered pair (x, y) is a solution to the given system of equations?

A (3, 8) **B** (−3, 8) **C** (2, 7) **D** (8, 3)

6 M-C2106

What is the positive solution to the equation $2x^2 + 5x - 12 = 0$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

7 M-C2107

$4x^2 + 9 = 12x$

How many distinct real solutions does the given equation have?

A Zero **B** Exactly one **C** Exactly two **D** Infinitely many

8 M-C2108

The height h, in feet, of a stream of water from a ground-level fountain is modeled by $h = -x^2 + 6x$, where x is the horizontal distance, in feet, from the nozzle. At which horizontal distances is the water exactly 4 feet high?

A $3 - \sqrt{5}$ and $3 + \sqrt{5}$ **B** $3 - 2\sqrt{5}$ and $3 + 2\sqrt{5}$ **C** $3 - \sqrt{13}$ and $3 + \sqrt{13}$ **D** $-3 - \sqrt{5}$ and $-3 + \sqrt{5}$

9 M-C2109

$\sqrt{(2x + 7)} = x + 2$

What are all the solutions to the given equation?

A −3 only **B** −3 and 1 **C** 1 only **D** −1 and 3

10 M-C2110

In a hypothetical experiment, the number of cells N in a bacteria culture t hours after the start is modeled by $N = 500(2)^{t/3}$. According to the model, how many hours after the start will the culture contain 16,000 cells?

A 5 **B** 8 **C** 12 **D** 15

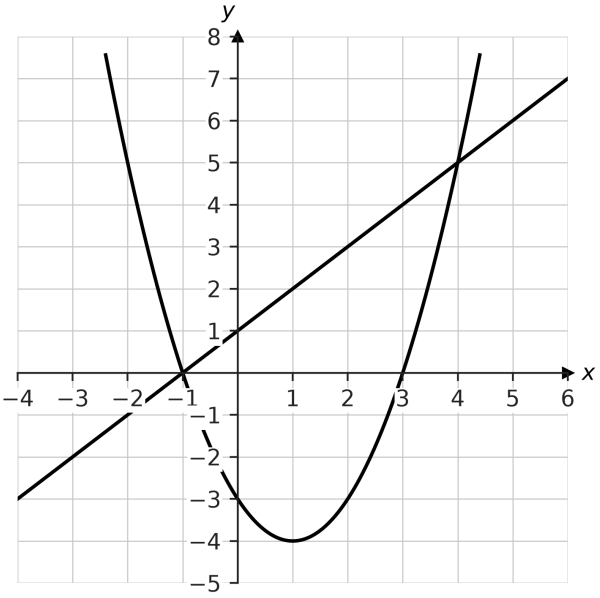
11 M-C2111

A ball is thrown upward from a platform, and its height, in feet, t seconds after the throw is modeled by $h = -16t^2 + 48t + 6$. At the same moment, a drone begins rising from the same platform, and its height, in feet, t seconds later is modeled by $h = 8t + 6$. At what time $t > 0$, in seconds, are the ball and the drone at the same height?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

12 M-C2112



The graphs of a parabola and a line in the xy-plane are shown. Which ordered pair (x, y) is a solution to the system of equations represented by the two graphs?

A (0, 1) **B** (1, −4) **C** (3, 0) **D** (4, 5)

13 M-C2113

The equation $x^2 + 8x = 5$ is rewritten in the form $(x + p)^2 = q$, where p and q are constants. What is the value of q?

A 9 **B** 11 **C** 21 **D** 69

14 M-C2114

$kx^2 - 6x + 3 = 0$

In the given equation, k is a constant. The equation has no real solutions. Which of the following could be the value of k?

A −4 **B** 0 **C** 3 **D** 4

15 M-C2115

$|x - 4| = 2x + 1$

What are all the solutions to the given equation?

A −5 only **B** 1 only

C −5 and 1 **D** There are no solutions.

16 M-C2116

$(3x)/(x - 4) = 12/(x - 4) + x$

What is the solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

17 M-C2117

$x^2 + kx - 21 = 0$

In the given equation, k is a constant. If $x = 3$ is one solution of the equation, what is the other solution?

A −7 **B** −3 **C** 4 **D** 7

18 M-C2118

$y = x^2 + 4$

$y = 2x + b$

In the given system of equations, b is a constant. The graphs of the equations in the xy-plane intersect at exactly one point. What is the value of b?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

Practice Set Explanations

1. Correct answer: B M-C2101

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Easier

B *−4 and 7*

Factor the trinomial. You need two numbers whose product is −28 and whose sum is −3: −7 and 4. So $x^2 - 3x - 28 = (x - 7)(x + 4) = 0$. Setting each factor equal to 0 gives $x = 7$ or $x = -4$. Check: $7^2 - 3(7) - 28 = 49 - 21 - 28 = 0$.

1. Find two numbers with product −28 and sum −3: −7 and 4.
2. Write the equation as $(x - 7)(x + 4) = 0$.
3. Set each factor equal to 0: $x = 7$ or $x = -4$.

A These are the numbers inside the factors, not the solutions. The factor $x - 7$ equals 0 when $x = 7$, so each sign must be reversed.

C −2 and 14 come from $(x + 2)(x - 14)$, which expands to $x^2 - 12x - 28$. The middle term does not match −3x.

D This ignores the signs. The product of 4 and 7 is 28, not −28, and $x = 4$ gives $16 - 12 - 28 = -24$, not 0.

Calculator option: Graph $y = x^2 - 3x - 28$ and read the x-intercepts, (−4, 0) and (7, 0).

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

2. Correct answer: D M-C2102

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Easier

D *−4 and 14*

An absolute value equals 9 when the expression inside the bars is 9 or −9. Case 1: $x - 5 = 9$, so $x = 14$. Case 2: $x - 5 = -9$, so $x = -4$. Both values check: $|14 - 5| = 9$ and $|-4 - 5| = |-9| = 9$.

1. Split into two cases: $x - 5 = 9$ or $x - 5 = -9$.
2. Solve each case: $x = 14$ or $x = -4$.
3. Check both values in the original equation.

A This solves only the case $x - 5 = 9$. The expression inside the bars can also equal −9, which gives a second solution, $x = -4$.

B These values solve $|x + 5| = 9$, so the sign of 5 was reversed. Substituting $x = 4$ into the given equation gives $|4 - 5| = 1$, not 9.

C These are the two possible values of $x - 5$, not of x. Add 5 to each one to finish solving for x.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

3. Correct answer: C M-C2103

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Easier

C *10*

Square both sides to undo the square root: $x + 6 = 16$, so $x = 10$. Check in the original equation: $\sqrt{(10 + 6)} = \sqrt{16} = 4$. Because the check works, 10 is a true solution and not an extraneous one.

1. Square both sides: $x + 6 = 16$.
2. Subtract 6 from both sides: $x = 10$.
3. Check: $\sqrt{16} = 4$.

A This comes from $x + 6 = 4$, which skips squaring. The square root is undone by squaring both sides, which turns 4 into 16.

B This comes from $x + 6 = 8$, which doubles 4 instead of squaring it. Squaring 4 gives 16, so $x + 6 = 16$.

D This adds 6 to 16 instead of subtracting it. From $x + 6 = 16$, subtract 6 from both sides to get $x = 10$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

4. Correct answer: 2 M-C2104

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Easier

Write both sides as powers of 5. Since $125 = 5^3$, the equation becomes $5^{2x - 1} = 5^3$. Equal powers of the same base have equal exponents, so $2x - 1 = 3$, $2x = 4$ and $x = 2$. A common wrong entry is 3, which is the exponent rather than x. Another is 63, which comes from setting $2x - 1$ equal to 125 instead of to the exponent 3.

1. Rewrite 125 as 5^3: $5^{2x - 1} = 5^3$.
2. Set the exponents equal: $2x - 1 = 3$.
3. Solve: $2x = 4$, so $x = 2$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

5. Correct answer: A M-C2105

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Easier

A *(3, 8)*

Substitute $x + 5$ for y in the first equation: $x^2 - 1 = x + 5$, so $x^2 - x - 6 = 0$ and $(x - 3)(x + 2) = 0$. When $x = 3$, $y = 3 + 5 = 8$. The pair (3, 8) checks in both equations: $3^2 - 1 = 8$ and $3 + 5 = 8$. The other solution, (−2, 3), is not offered.

1. Set the expressions for y equal: $x^2 - 1 = x + 5$.
2. Rearrange and factor: $x^2 - x - 6 = (x - 3)(x + 2) = 0$, so $x = 3$ or $x = -2$.
3. Find y from the line: $x = 3$ gives $y = 8$, and $x = -2$ gives $y = 3$.
4. Only (3, 8) appears among the choices.

B (−3, 8) satisfies $y = x^2 - 1$, but $-3 + 5 = 2$, not 8. A solution to a system must satisfy both equations.

C (2, 7) lies on the line $y = x + 5$, but $2^2 - 1 = 3$, not 7, so the point is not on the parabola.

D This reverses the coordinates. With $x = 8$, the line gives $y = 13$ and the parabola gives $y = 63$, so (8, 3) fails both.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

6. Correct answer: 3/2 or 1.5 M-C2106

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

Factor by grouping. Since ac = −24, find two numbers with product −24 and sum 5: 8 and −3. Then $2x^2 + 8x - 3x - 12 = 2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4) = 0$, so x = 3/2 or x = −4. The positive solution is 3/2, entered as 3/2 or 1.5. Entering −4 gives the negative solution, and entering 3 forgets to divide by 2.

1. Multiply a and c: 2 × (−12) = −24; use 8 and −3, which add to 5.
2. Group: $2x(x + 4) - 3(x + 4) = (2x - 3)(x + 4) = 0$.
3. Solve: x = 3/2 or x = −4; the positive solution is 3/2.

Calculator option: Graph $y = 2x^2 + 5x - 12$ and read the positive x-intercept, 1.5.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

7. Correct answer: B M-C2107

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

B *Exactly one*

First write the equation as $ax^2 + bx + c = 0$: $4x^2 - 12x + 9 = 0$, so a = 4, b = −12 and c = 9. The discriminant is $b^2 - 4ac = 144 - 4(4)(9) = 144 - 144 = 0$, which means exactly one real solution. Indeed, $4x^2 - 12x + 9 = (2x - 3)^2$, which equals 0 only when x = 3/2.

1. Rewrite as $4x^2 - 12x + 9 = 0$: a = 4, b = −12, c = 9.
2. Compute the discriminant: 144 − 144 = 0.
3. A discriminant of 0 means exactly one real solution, x = 3/2.

A Zero real solutions requires a negative discriminant. Writing $(-12)^2$ as −144 produces one, but a squared number is never negative.

C Two solutions would need a positive discriminant. Moving 9 incorrectly so that c = −9 gives 144 + 144, which is a sign error.

D Infinitely many solutions happen only when both sides are identical expressions. A quadratic equation has at most two real solutions.

Calculator option: Graph $y = 4x^2 - 12x + 9$; the parabola touches the x-axis at exactly one point, (1.5, 0).

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

8. Correct answer: A M-C2108

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A *$3 - \sqrt{5}$ and $3 + \sqrt{5}$*

Set the height equal to 4: $-x^2 + 6x = 4$, so $x^2 - 6x + 4 = 0$. Complete the square: $x^2 - 6x + 9 = 5$, so $(x - 3)^2 = 5$ and $x = 3 \pm \sqrt{5}$. The quadratic formula agrees: $x = (6 \pm \sqrt{(36 - 16)})/2 = (6 \pm 2\sqrt{5})/2 = 3 \pm \sqrt{5}$. Both distances, about 0.76 and 5.24 feet, are positive.

1. Set h = 4: $-x^2 + 6x = 4$.
2. Rearrange: $x^2 - 6x + 4 = 0$.
3. Complete the square: $(x - 3)^2 = 5$.
4. Take square roots: $x = 3 \pm \sqrt{5}$.

B This divides only the 6 by 2. In $(6 \pm 2\sqrt{5})/2$, both terms of the numerator must be divided by 2, which gives $3 \pm \sqrt{5}$.

C This comes from $x^2 - 6x - 4 = 0$, a sign error made when moving 4 across the equals sign. The constant term should be +4.

D This uses b instead of −b in the quadratic formula. Since b = −6, the formula begins with −b = 6, so the solutions are centered at 3.

Calculator option: Graph $y = -x^2 + 6x$ and y = 4. The intersections at x ≈ 0.764 and x ≈ 5.236 match $3 - \sqrt{5}$ and $3 + \sqrt{5}$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

9. Correct answer: C M-C2109

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C *1 only*

Square both sides: $2x + 7 = x^2 + 4x + 4$, so $x^2 + 2x - 3 = 0$ and $(x + 3)(x - 1) = 0$. Check both candidates. For x = 1: $\sqrt{9} = 3$ and 1 + 2 = 3, so 1 works. For x = −3: $\sqrt{1} = 1$, but −3 + 2 = −1, so −3 is extraneous. The only solution is 1.

1. Square both sides: $2x + 7 = x^2 + 4x + 4$.
2. Rearrange and factor: $x^2 + 2x - 3 = (x + 3)(x - 1) = 0$.
3. Check x = 1: $\sqrt{9} = 3$ and 1 + 2 = 3, so it works.
4. Check x = −3: $\sqrt{1} = 1$, but −3 + 2 = −1, so reject it.

A This keeps the extraneous value and rejects the true one. At x = −3 the right side is −1, but a square root is never negative.

B These solve the squared equation, but squaring can create extraneous values. The value x = −3 fails the original equation.

D These are the numbers inside the factors (x + 3)(x − 1), with their signs left unchanged. Neither −1 nor 3 satisfies the squared equation.

Calculator option: Graph $y = \sqrt{(2x + 7)}$ and y = x + 2. They intersect only once, at (1, 3), which shows that −3 is extraneous.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

10. Correct answer: D M-C2110

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D *15*

Set the model equal to 16,000: $500(2)^{t/3} = 16{,}000$. Divide both sides by 500: $2^{t/3} = 32$. Since $32 = 2^5$, the exponents are equal: $t/3 = 5$, so $t = 15$ hours. Check: the culture doubles every 3 hours, and 15 hours is 5 doublings, so it reaches $500 \times 32 = 16{,}000$ cells.

1. Set $500(2)^{t/3} = 16{,}000$ and divide by 500: $2^{t/3} = 32$.
2. Write 32 as 2^5, so $t/3 = 5$.
3. Multiply by 3: $t = 15$.

A 5 is the value of $t/3$, the number of doublings. Multiply by 3 hours per doubling to get the time in hours.

B This adds 3 instead of multiplying by 3. From $t/3 = 5$, multiply both sides by 3 to get $t = 15$.

C This counts only four doublings. Going from 500 to 16,000 takes five: 1,000, 2,000, 4,000, 8,000 and 16,000.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

11. Correct answer: 2.5 or 5/2 M-C2111

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Set the two heights equal: $-16t^2 + 48t + 6 = 8t + 6$. Subtract $8t + 6$ from both sides: $-16t^2 + 40t = 0$, so $-8t(2t - 5) = 0$. The solutions are $t = 0$, the moment both leave the platform, and $t = 2.5$. At 2.5 seconds both are 26 feet high. Entering 0 ignores the condition $t > 0$, and entering 26 gives the height instead of the time.

1. Set the height models equal: $-16t^2 + 48t + 6 = 8t + 6$.
2. Simplify: $-16t^2 + 40t = 0$.
3. Factor: $-8t(2t - 5) = 0$, so $t = 0$ or $t = 2.5$.
4. Use the solution with $t > 0$: $t = 2.5$ seconds.

Calculator option: Graph $y = -16x^2 + 48x + 6$ and $y = 8x + 6$, with x standing for time; the graphs meet at (0, 6) and (2.5, 26).

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

12. Correct answer: D M-C2112

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D *(4, 5)*

A solution to a system is a point that lies on both graphs, which is an intersection point. The graphs cross at (−1, 0) and (4, 5), and only (4, 5) is offered. You can confirm it algebraically: the graphs are $y = x^2 - 2x - 3$ and $y = x + 1$, and both give $y = 5$ when $x = 4$.

1. A solution of the system is an intersection point of the two graphs.
2. Read the intersections from the grid: (−1, 0) and (4, 5).
3. Check the choices: only (4, 5) is an intersection point.

A (0, 1) is the y-intercept of the line. The parabola crosses the y-axis at (0, −3), so this point is not shared.

B (1, −4) is the vertex of the parabola. The line is at $y = 2$ when $x = 1$, so this point is not on both graphs.

C (3, 0) is an x-intercept of the parabola, but the line passes through (3, 4), so this point lies on only one graph.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

13. Correct answer: C M-C2113

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C *21*

Complete the square. Half of 8 is 4, and $4^2 = 16$. Add 16 to both sides: $x^2 + 8x + 16 = 5 + 16$, so $(x + 4)^2 = 21$. Thus $p = 4$ and $q = 21$. Check by expanding: $(x + 4)^2 = x^2 + 8x + 16$, and subtracting 16 from both sides returns $x^2 + 8x = 5$.

1. Take half of the x-coefficient and square it: $(8/2)^2 = 16$.
2. Add 16 to both sides: $x^2 + 8x + 16 = 21$.
3. Factor the left side: $(x + 4)^2 = 21$, so $q = 21$.

A This adds half of 8, which is 4, instead of the square of half of 8. Completing the square requires adding $(8/2)^2 = 16$.

B This subtracts 5 from 16. The 16 must be added to both sides, so the right side becomes $5 + 16 = 21$.

D This adds $8^2 = 64$ instead of $(8/2)^2 = 16$. The expansion of $(x + 4)^2$ has 16 as its constant term, not 64.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

14. Correct answer: D M-C2114

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D *4*

A quadratic equation has no real solutions when its discriminant is negative. Here $b^2 - 4ac = (-6)^2 - 4(k)(3) = 36 - 12k$. Solve $36 - 12k < 0$: $12k > 36$, so $k > 3$. Of the choices, only 4 is greater than 3. Check: with $k = 4$ the discriminant is $36 - 48 = -12$, which is negative.

1. Identify $a = k$, $b = -6$ and $c = 3$.
2. Write the discriminant: $(-6)^2 - 4(k)(3) = 36 - 12k$.
3. No real solutions requires $36 - 12k < 0$, so $k > 3$.
4. Only 4 satisfies $k > 3$.

A With $k = -4$ the discriminant is $36 + 48 = 84$, which is positive, so the equation has two real solutions.

B With $k = 0$ the equation becomes $-6x + 3 = 0$, a linear equation with exactly one solution, $x = 1/2$.

C With $k = 3$ the discriminant is $36 - 36 = 0$, so the equation has exactly one real solution, not zero.

Calculator option: Graph $y = kx^2 - 6x + 3$ with a slider for k. The parabola stops reaching the x-axis once k is greater than 3, which confirms the inequality.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

15. Correct answer: B M-C2115

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B *1 only*

Split into two cases, then check each candidate, because the right side contains x and could be negative. Case 1: $x - 4 = 2x + 1$ gives $x = -5$. Case 2: $x - 4 = -(2x + 1)$ gives $3x = 3$, so $x = 1$. Check $x = 1$: $|-3| = 3$ and $2(1) + 1 = 3$. Check $x = -5$: $|-9| = 9$, but $2(-5) + 1 = -9$. Only $x = 1$ works.

1. Case 1: $x - 4 = 2x + 1$, so $x = -5$.
2. Case 2: $x - 4 = -(2x + 1)$, so $3x = 3$ and $x = 1$.
3. Check: $x = 1$ gives $3 = 3$; $x = -5$ gives $9 = -9$, which is false.
4. The only solution is $x = 1$.

A At $x = -5$ the right side is -9, and an absolute value cannot be negative. This choice keeps the extraneous value and drops the valid one.

C Both case results must be checked in the original equation. The value $x = -5$ fails because $|-5 - 4| = 9$, while $2(-5) + 1 = -9$.

D The value $x = 1$ satisfies the equation, since both sides equal 3. Rejecting every candidate overlooks the case that checks.

Calculator option: Graph $y = |x - 4|$ and $y = 2x + 1$; the graphs intersect only at (1, 3).

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

16. Correct answer: 3 M-C2116

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The denominators are 0 when $x = 4$, so 4 cannot be a solution. Multiply every term by $x - 4$: $3x = 12 + x(x - 4)$, so $3x = 12 + x^2 - 4x$. Rearrange: $x^2 - 7x + 12 = 0$, or $(x - 3)(x - 4) = 0$. Reject $x = 4$, which makes the denominators 0. The solution is $x = 3$. Entering 4 keeps the excluded value, and entering 7 adds the two candidates.

1. Note the excluded value: $x \neq 4$.
2. Multiply each term by $x - 4$: $3x = 12 + x^2 - 4x$.
3. Rearrange and factor: $x^2 - 7x + 12 = (x - 3)(x - 4) = 0$.
4. Reject $x = 4$; the solution is $x = 3$.

Calculator option: Graph each side as its own function; the graphs meet only at $x = 3$, because neither side is defined at $x = 4$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

17. Correct answer: A M-C2117

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A *−7*

Substitute the known solution to find k: $3^2 + 3k - 21 = 0$, so $3k = 12$ and $k = 4$. The equation is $x^2 + 4x - 21 = 0$, which factors as $(x - 3)(x + 7) = 0$, so the other solution is -7. A faster check uses the product of the solutions, $c/a = -21$: since $3 \times (-7) = -21$, the other solution is -7.

1. Substitute $x = 3$: $9 + 3k - 21 = 0$, so $k = 4$.
2. Rewrite the equation: $x^2 + 4x - 21 = 0$.
3. Factor: $(x - 3)(x + 7) = 0$.
4. The other solution is $x = -7$.

B This assumes the two solutions are opposites. That happens only when there is no x-term, but here $k = 4$, not 0.

C 4 is the value of k, not a solution. Substituting $x = 4$ gives $16 + 16 - 21 = 11$, not 0.

D This reverses the sign. With $k = 4$, $x = 7$ gives $49 + 28 - 21 = 56$, not 0; the factor $x + 7$ gives $x = -7$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

18. Correct answer: 3 M-C2118

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Substitute to get one equation in x: $x^2 + 4 = 2x + b$, so $x^2 - 2x + (4 - b) = 0$. One intersection point means exactly one solution, so the discriminant is 0: $(-2)^2 - 4(1)(4 - b) = 0$, which gives $4b - 12 = 0$ and $b = 3$. Then $(x - 1)^2 = 0$, and the single point is (1, 5). Entering 4 sets the constant term, not the discriminant, equal to 0.

1. Set the expressions for y equal: $x^2 + 4 = 2x + b$.
2. Rearrange: $x^2 - 2x + (4 - b) = 0$.
3. For exactly one solution, set the discriminant to 0: $4 - 4(4 - b) = 0$.
4. Solve: $4b - 12 = 0$, so $b = 3$.

Calculator option: Graph $y = x^2 + 4$ and $y = 2x + b$ with a slider for b; the line just touches the parabola when $b = 3$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

CHAPTER 22

Nonlinear Functions

A function is a rule that gives exactly one output for each input. This chapter covers three nonlinear families: quadratic functions, whose graphs are parabolas; exponential functions, which grow or shrink by a constant percent; and polynomial functions with several zeros. It also covers function notation and the shifts and reflections that move a graph. The College Board names this skill "Nonlinear functions," and like Chapter 21 it belongs to the Advanced Math domain.

What the question looks like

Typical stems include:

- "The function f is defined by f(x) = ... What is the value of f(−2)?"
- "What is the minimum value of f?" or "What is the maximum height, in feet, of the ball?"
- "Which function gives the number of cells h hours after the start?"
- "Which statement best describes how the population changes, according to the model?"
- "Which equivalent form of the function displays the minimum value as a constant or coefficient?"
- "The graph of y = f(x) is shown. Which of the following could define f?"

Common contexts include the height of a ball or a fountain's spray, a population that changes by a percent each year, a machine that loses value, and a bacteria culture that doubles.

How to recognize it

The question asks about a function's features or behavior, not the solutions of one equation. Watch for the words *vertex*, *maximum*, *minimum*, *zeros*, *x-intercepts*, *initial value*, *growth rate*, *percent per year*, *doubles*, *half*, *shifted* and *reflected*. A graph or table of a curve, or notation such as f(x + 3) or g(x) = −f(x), also signals this skill.

The method

1. **Identify the family.** A squared variable as the highest power means quadratic; a variable in an exponent means exponential; a product of several linear factors means polynomial.
2. **Match the feature to the form that shows it.** Use the tables below.
3. **Translate the context.** Decide what the input and the output mean, with units, before you compute.
4. **Compute or rewrite.** Evaluate, find a vertex, complete the square, or build the model.
5. **Interpret.** State the answer as a quantity in context, such as "30 feet at 1.25 seconds," and confirm that it is the quantity the question asks for.

Quadratic functions: three forms

Form	Example	What it shows directly
Standard: $ax^2 + bx + c$	$x^2 - 2x - 8$	y-intercept c; vertex at x = −b/(2a)

Form	Example	What it shows directly
Vertex: $a(x - h)^2 + k$	$(x - 1)^2 - 9$	vertex (h, k); maximum or minimum value k; axis of symmetry x = h
Factored: $a(x - r)(x - s)$	$(x - 4)(x + 2)$	zeros r and s; vertex halfway between them

In every form, $a > 0$ means the parabola opens upward and has a minimum; $a < 0$ means it opens downward and has a maximum. The axis of symmetry is the vertical line through the vertex, and inputs the same distance from it on either side give equal outputs.

Exponential functions

An exponential function has the form $f(t) = a(b)^t$, with $b > 0$ and $b \neq 1$. In the growth and decay models here, $a > 0$ is the initial value, f(0). The base b is the growth factor: the number the output is multiplied by each time t increases by 1.

- Growth by a rate r per period: $f(t) = a(1 + r)^t$. A 6% yearly increase gives a base of 1.06.
- Decay by a rate r per period: $f(t) = a(1 - r)^t$. A 15% yearly decrease gives a base of 0.85.
- Doubling every d units: $f(t) = a(2)^{t/d}$. Halving every d units, a half-life of d: $f(t) = a(1/2)^{t/d}$.

A linear function adds the same *amount* in each equal step; an exponential function multiplies by the same *factor*. In a table, check differences for linear and ratios for exponential.

Function notation and transformations

f(3) means "the output when the input is 3." In f(x + 3), the input is x + 3. The table shows how each change moves the graph of y = f(x) and a sample vertex at (2, −4).

New function	What happens to the graph	Vertex (2, −4) moves to
f(x) + 3	Up 3	(2, −1)
f(x) − 3	Down 3	(2, −7)
f(x − 3)	Right 3	(5, −4)
f(x + 3)	Left 3	(−1, −4)
−f(x)	Reflected across the x-axis	(2, 4)
f(−x)	Reflected across the y-axis	(−2, −4)

Changes inside the parentheses act on x and work opposite to their sign. Changes outside act on y and work as written.

Polynomial zeros and factors

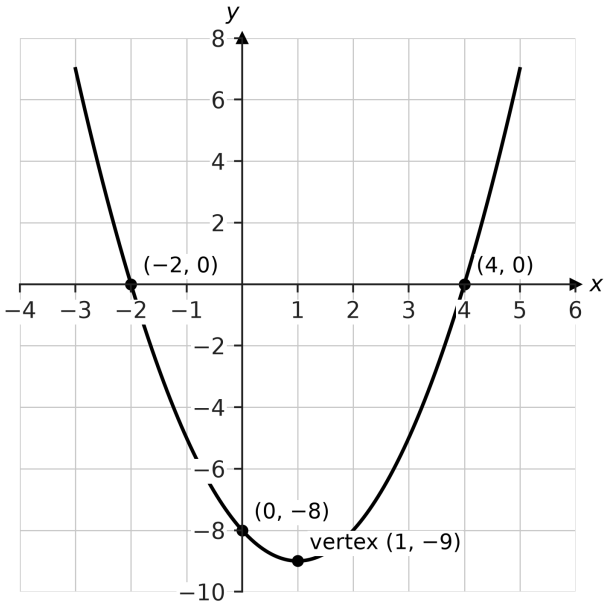
If p(r) = 0, then x − r is a factor of p(x), and (r, 0) is an x-intercept of the graph. The number of copies of that factor is the zero's **multiplicity**. An even multiplicity, such as 2 or 4, makes the graph touch the x-axis and turn back; an odd multiplicity, such as 1 or 3, makes it cross.

Worked examples

Worked Example 1: Reading the three forms

The function $f(x) = x^2 - 2x - 8$ can also be written as $(x - 4)(x + 2)$ and as $(x - 1)^2 - 9$. What are its zeros, its minimum value and its y-intercept?

Thinking. The factored form shows the zeros, $x = 4$ and $x = -2$. The vertex form shows the vertex $(1, -9)$; since $a = 1$ is positive, -9 is the minimum value, reached at $x = 1$. The standard form shows the y-intercept, $(0, -8)$. Notice that $x = 1$ lies halfway between -2 and 4.



Trap. "Minimum value" means the output -9, not the input 1.

Worked Example 2: The maximum height of a ball

A ball's height, in feet, t seconds after it is thrown is $h(t) = -16t^2 + 40t + 5$. What is the maximum height, and what does the 5 mean?

Thinking. Because $a = -16$ is negative, the parabola opens downward and its vertex is a maximum. The vertex is at $t = -b/(2a) = -40/(-32) = 1.25$ seconds. Then $h(1.25) = -16(1.5625) + 40(1.25) + 5 = -25 + 50 + 5 = 30$.

Answer. The maximum height is 30 feet, reached 1.25 seconds after the throw. The 5 is $h(0)$, the height in feet at the moment of the throw.

> **TIP** To check on a graphing calculator, graph the function and select its highest point, $(1.25, 30)$.

Worked Example 3: Building an exponential model

A hypothetical bacteria culture starts with 50 cells and doubles every 20 minutes. Write a model, and find the number of cells after 2 hours.

Thinking. The initial value is 50 and the factor is 2 per 20 minutes, so $N(t) = 50(2)^{t/20}$, with t in minutes. Two hours is 120 minutes, and $120/20 = 6$ doublings: $N(120) = 50(2)^6 = 50 \times 64 = 3{,}200$ cells.

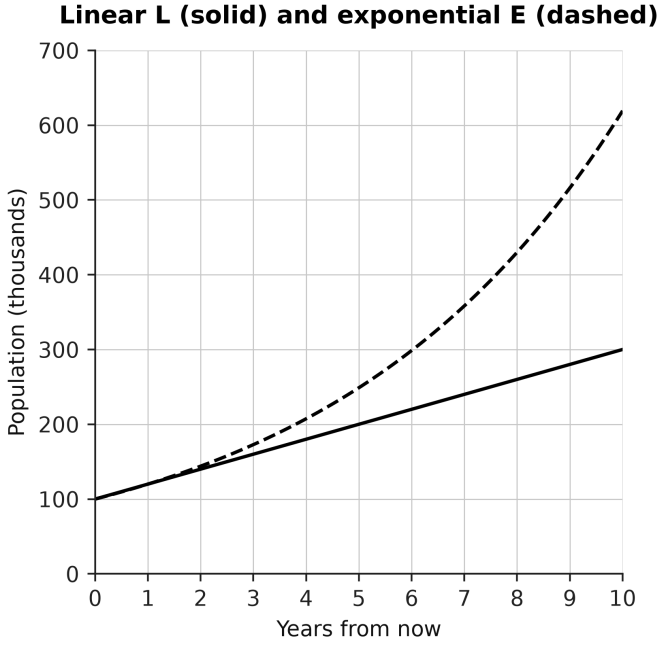
A decay model to compare. If a machine's value after t years is $V(t) = 18{,}000(0.88)^t$ dollars, then 18,000 is the purchase price, and $0.88 = 1 - 0.12$ means the value falls by 12% each year.

Traps. Writing the exponent as 20t, which would double the culture 20 times each minute; mixing hours and minutes.

Worked Example 4: Linear or exponential?

Two hypothetical models predict a city's population, in thousands, t years from now: $L(t) = 100 + 20t$ and $E(t) = 100(1.2)^t$. Which model predicts the larger population after 5 years?

Thinking. Compare one-year steps. L adds 20 thousand each year: 100, 120, 140, 160. E multiplies by 1.2 each year: 100, 120, 144, 172.8. The models agree at t = 0 and t = 1; after that the exponential model pulls ahead, and the gap keeps widening.



Answer. $L(5) = 200$ and $E(5) = 100(1.2)^5 \approx 248.8$, so the exponential model predicts the larger population.

Trap. Deciding from the first step alone.

Worked Example 5: Notation and shifts

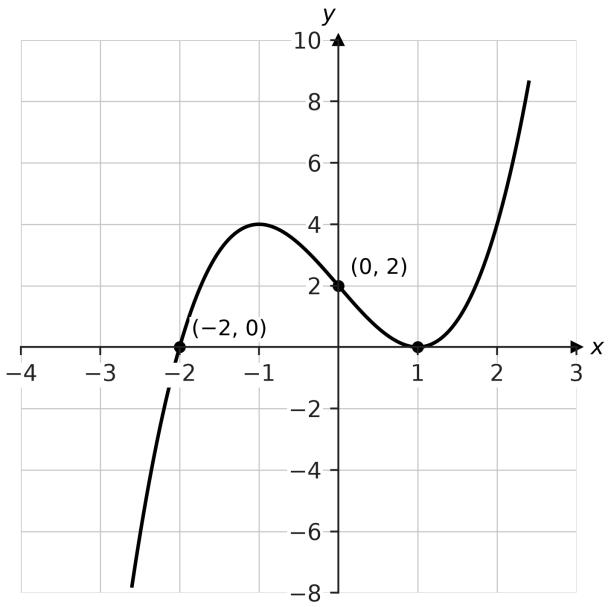
Let $f(x) = x^2 - 4x$, whose vertex is (2, −4). (a) What is the vertex of the graph of $g(x) = f(x - 1) + 3$? (b) What is $f(a + 1)$ in terms of a?

Thinking. (a) The $x - 1$ inside shifts the graph 1 unit right, and the $+ 3$ outside shifts it 3 units up, so the vertex moves to (3, −1). (b) Replace every x with $(a + 1)$: $f(a + 1) = (a + 1)^2 - 4(a + 1) = a^2 + 2a + 1 - 4a - 4 = a^2 - 2a - 3$.

Traps. Shifting left for $x - 1$; treating $f(a + 1)$ as $f(a) + 1$, which is $a^2 - 4a + 1$, a different expression.

Worked Example 6: Zeros and factors from a graph

The graph of a polynomial p is shown. Which function could define p?



Thinking. The graph crosses the x-axis at $x = -2$ and touches it at $x = 1$ without crossing. The zero -2 has odd multiplicity and the zero 1 has even multiplicity. A simplest candidate is $p(x) = (x + 2)(x - 1)^2$. Its y-intercept is $p(0) = (2)(1) = 2$, and its plotted shape matches the figure. Touching alone does not prove that a factor occurs exactly twice.

Trap. Reversing the signs. A zero at -2 gives the factor $x + 2$, not $x - 2$.

Worked Example 7 (harder module): Symmetry beats algebra

The graph of $y = a(x - 2)^2 - 5$, where a is a constant, passes through the point $(4, 3)$. What is the y-intercept of the graph?

Thinking. The axis of symmetry is $x = 2$. The input $x = 0$ is 2 units left of the axis and $x = 4$ is 2 units right, so they give the same output. The y-intercept is $(0, 3)$.

Algebra check. Substitute $(4, 3)$: $3 = a(2)^2 - 5$, so $4a = 8$ and $a = 2$. At $x = 0$, $y = 2(-2)^2 - 5 = 8 - 5 = 3$.

Why this is harder. The unknown constant blocks quick graphing, and symmetry turns three steps into one.

Common traps

Trap	How to spot it
Giving the input when asked for the value	"Minimum value" and "maximum height" are outputs: the y-coordinate of the vertex.
Misreading the sign in vertex form	In $(x - 3)^2$ the vertex x-coordinate is 3; in $(x + 3)^2$ it is -3.
Treating the base as the rate	0.85 means 85% remains, so the decrease is 15%. A base of 1.06 means 6% growth.
Building the exponent upside down	Doubling every 5 years is $2^{t/5}$, not 2^{5t}. Test $t = 5$: the factor should be 2.
Splitting a percent across periods	44% growth per 2 years is not 22% per year; the yearly factor is $\sqrt{1.44} = 1.2$, or 20%.

Trap	How to spot it
Shifting the wrong way	f(x + 2) moves the graph 2 units left, not right.
Linear thinking about exponential change	A 15% yearly drop does not remove the same dollar amount each year.

Harder-module variations

- **A rate over a different period.** Convert with powers, not by dividing or multiplying the percent. If a factor F applies every n periods, the factor b for one period satisfies $b^n = F$. In the other direction, a 10% monthly increase is a yearly factor of $(1.1)^{12} \approx 3.14$, not $1 + 12(0.10) = 2.2$.
- **Unknown constants in a model.** Use a given point or the time of the maximum to find the constant first, then answer the question.
- **Shifted inputs.** Given a formula for f(x + 2), choose the x that makes x + 2 equal the input you need, then substitute it.
- **Choosing a form.** "Displays the vertex" or "the minimum" calls for vertex form, "displays the zeros" for factored form, and "displays the initial value" for c in standard form or a in $a(b)^t$.
- **Matching a graph.** Use the intercepts, the sign of the leading coefficient, and touching versus crossing zeros to eliminate choices quickly.

Check yourself

1. Which form of a quadratic shows its zeros, and which shows its maximum or minimum value?
2. What does the base 0.91 tell you in $V(t) = 5{,}000(0.91)^t$?
3. A 200-gram sample of a substance has a half-life of 6 hours. Write a model for the amount left after t hours.
4. If the vertex of the graph of f is (−2, 5), what is the vertex of the graph of y = f(x − 4) − 1?
5. How does a squared factor show up on the graph of a polynomial?
6. Answers: (1) Factored form shows the zeros; vertex form shows the maximum or minimum value. (2) The value keeps 91% each period, so it decreases by 9% per period. (3) $A(t) = 200(1/2)^{t/6}$, in grams. (4) (2, 4). (5) The graph touches the x-axis at that zero and turns back instead of crossing.

ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/nonlinear-functions

Practice Set

18 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2201

The function f is defined by $f(x) = 3x^2 - 2x + 1$. What is the value of $f(-2)$?

A −7 **B** 9 **C** 17 **D** 41

2 M-C2202

The function f is defined by $f(x) = 2(x - 3)^2 + 5$. What is the minimum value of f?

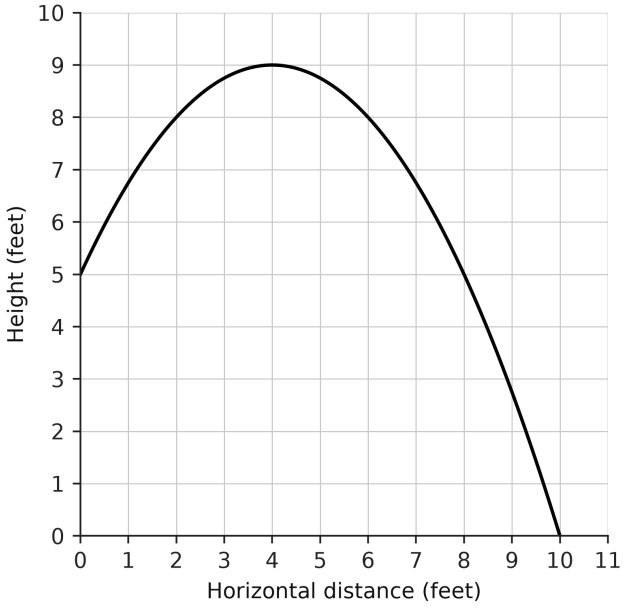
A −3 **B** 2 **C** 3 **D** 5

3 M-C2203

The number of fish in a lake t years after the start of a hypothetical study is modeled by $P(t) = 4{,}500(0.92)^t$. According to the model, which statement best describes how the fish population changes?

A It decreases by 8% each year.
B It decreases by 92% each year.
C It decreases by 0.08% each year.
D It decreases by 360 fish each year.

4 M-C2204



The graph models the height h, in feet, of water sprayed from the nozzle of a hypothetical fountain as a function of the horizontal distance x, in feet, from the nozzle. According to the graph, how far from the nozzle, in feet, does the water reach the ground?

A 4 **B** 5 **C** 9 **D** 10

5 M-C2205

The function f is defined by $f(x) = (x - 2)(x + 5)(x - 7)$. Which of the following is an x-intercept of the graph of $y = f(x)$ in the xy-plane?

A (−7, 0) **B** (0, 70) **C** (−5, 0) **D** (5, 0)

6 M-C2206

The function f is defined by $f(x) = (x + 1)(x - 7)$. What is the minimum value of f?

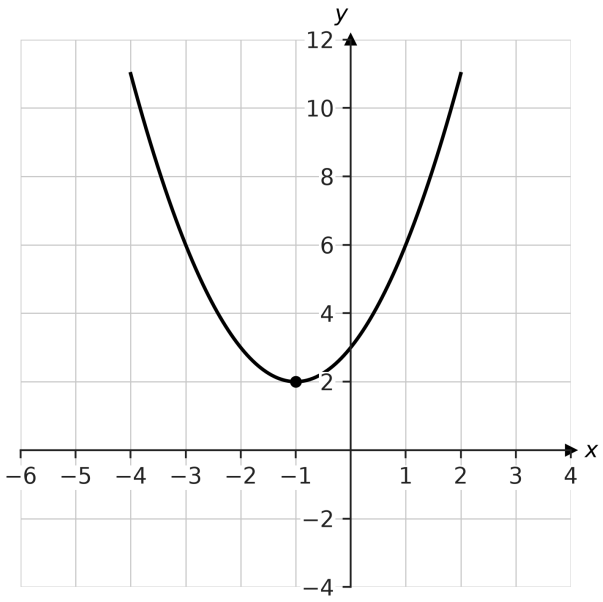
A −16 **B** −7 **C** 3 **D** 16

7 M-C2207

A hypothetical bacteria culture starts with 600 cells, and the number of cells triples every 4 hours. Which function gives the number of cells, N(h), in the culture h hours after the start?

A $N(h) = 600(3)^{4h}$ **B** $N(h) = 600(4)^{h/3}$ **C** $N(h) = 600(3)^{h/4}$ **D** $N(h) = 600 + 300h$

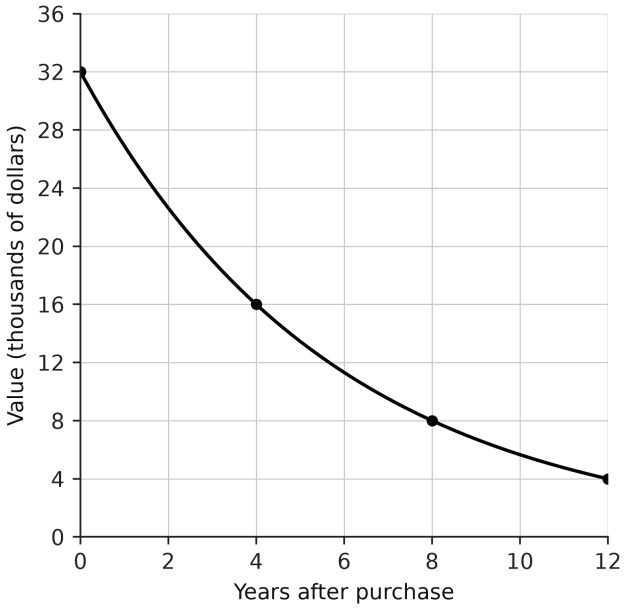
8 M-C2208



The graph of $y = f(x)$ is shown. The function g is defined by $g(x) = f(x + 3) - 4$. What is the vertex of the graph of $y = g(x)$?

A (2, −2) **B** (−4, −2) **C** (−4, 6) **D** (−1, −2)

9 M-C2209



The graph shows a model of the value V, in thousands of dollars, of a hypothetical piece of factory equipment t years after it was purchased. Which statement is best supported by the graph?

A The equipment loses $4,000 of value each year.

B The value of the equipment is cut in half every 4 years.

C The equipment is worth $16,000 eight years after purchase.

D The equipment has no value left eight years after purchase.

---

10 M-C2210

A hypothetical car is worth $24,000 when it is new, and its value decreases by 20% each year. What is the value of the car, in dollars, 2 years after it is new?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

11 M-C2211

A model rocket is launched upward, and its height h, in meters, t seconds after launch is modeled by $h(t) = -4.9t^2 + 29.4t + 2$. What is the maximum height, in meters, of the rocket?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

12 M-C2212

x	f(x)
0	4
1	−1
2	−7
3	3
4	9

The table shows some values of the function f. The function g is defined by $g(x) = -f(x - 1)$. What is the value of $g(3)$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

13 M-C2213

$f(x) = x^2 - 10x + 21$

Which of the following is an equivalent form of the given function that displays the minimum value of f as a constant or coefficient?

A $f(x) = (x - 3)(x - 7)$ **B** $f(x) = (x + 5)^2 - 4$ **C** $f(x) = x(x - 10) + 21$ **D** $f(x) = (x - 5)^2 - 4$

14 M-C2214

The population of a hypothetical town increases by 21% every 10 years. If the population grows exponentially, by what percent does it increase every 5 years?

A 10% **B** 10.5% **C** 21% **D** 42%

15 M-C2215

For the function f, $f(x + 2) = x^2 + 3x$ for all values of x. What is the value of $f(5)$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

16 M-C2216

Bacteria culture (hypothetical data)

Time (hours)	Number of cells
0	320
2	480
4	720
6	1,080

The table shows the number of cells in a bacteria culture at several times after the start of an experiment. Which function is consistent with every value in the table, where N(t) is the number of cells t hours after the start?

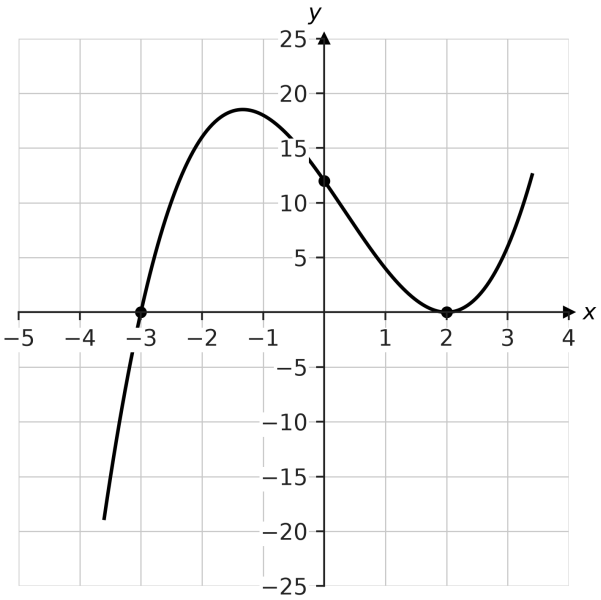
A $N(t) = 320(1.5)^{2t}$ **B** $N(t) = 320 + 80t$ **C** $N(t) = 320(1.5)^{t/2}$ **D** $N(t) = 320(1.25)^{t}$

17 M-C2217

A ball is thrown upward, and its height h, in feet, t seconds after it is thrown is modeled by $h(t) = -16t^2 + bt + 4$, where b is a constant. The ball reaches its maximum height 1.5 seconds after it is thrown. What is the maximum height, in feet, of the ball?

A 36 **B** 40 **C** 48 **D** 112

18 M-C2218



The graph of the polynomial function p is shown. Which of the following could define p?

A $p(x) = (x - 3)(x + 2)^2$ **B** $p(x) = (x + 3)(x - 2)$ **C** $p(x) = (x + 3)^2(x - 2)$ **D** $p(x) = (x + 3)(x - 2)^2$

Practice Set Explanations

1. Correct answer: C M-C2201

Nonlinear functions · Advanced Math · Easier

C *17*

Substitute −2 for every x, using parentheses: $f(-2) = 3(-2)^2 - 2(-2) + 1 = 3(4) + 4 + 1 = 12 + 4 + 1 = 17$. The exponent applies to −2 first, and the result is then multiplied by 3.

1. Substitute $x = -2$: $f(-2) = 3(-2)^2 - 2(-2) + 1$.
2. Evaluate the power first: $(-2)^2 = 4$, so the first term is 12.
3. Add: $12 + 4 + 1 = 17$.

A This treats $(-2)^2$ as −4. A negative number squared is positive, so $(-2)^2 = 4$ and the first term is 12, not −12.

B This makes a sign error in the middle term. Since −2 times −2 is +4, the sum is 12 + 4 + 1, not 12 − 4 + 1.

D This squares the product 3 × (−2) to get 36. The exponent applies only to x, so the first term is 3 × 4 = 12.

Review: Chapter 22, Nonlinear Functions (page 313)

2. Correct answer: D M-C2202

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D *5*

The function is in vertex form, $a(x - h)^2 + k$, with $h = 3$ and $k = 5$. Because $a = 2$ is positive, the parabola opens upward, and its vertex (3, 5) is the lowest point. The squared term is never negative, so f(x) is never less than 5. The minimum value is 5.

1. Match $f(x) = 2(x - 3)^2 + 5$ to $a(x - h)^2 + k$: $a = 2$, $h = 3$, $k = 5$.
2. Since $a > 0$, the parabola opens upward, so the vertex is the lowest point.
3. The minimum value is $k = 5$.

A −3 misreads the sign inside the parentheses. The vertex's x-coordinate is 3, and in any case the minimum value is an output, not an input.

B 2 is the leading coefficient. It determines how steep the parabola is and which way it opens, not its lowest value.

C 3 is the x-value where the minimum occurs. The minimum value of f is the output there, the y-coordinate of the vertex.

Calculator option: Graph $y = 2(x - 3)^2 + 5$ and select the vertex, (3, 5).

Review: Chapter 22, Nonlinear Functions (page 313)

3. Correct answer: A M-C2203

Nonlinear functions · Advanced Math · Easier

A *It decreases by 8% each year.*

In an exponential model $a(1 - r)^t$, the base $1 - r$ is the fraction that remains after each period. Here $0.92 = 1 - 0.08$, so each year the population keeps 92% of its size and loses 8%. Because the population is multiplied by the same factor each year, the loss is a constant percent, not a constant number of fish.

1. Compare the base 0.92 with $1 - r$: $r = 0.08$.
2. Convert the rate to a percent: $0.08 = 8\%$.
3. Interpret: the population decreases by 8% each year.

B 92% is the share that remains each year. Losing 92% each year would leave only 8%, which would require a base of 0.08.

C 0.08 is the decimal form of the rate. Written as a percent it is 8%, so 0.08% is 100 times too small.

D 360 fish is 8% of 4,500, the loss in the first year only. Later losses are smaller because each is 8% of a smaller population.

Review: Chapter 22, Nonlinear Functions (page 313)

4. Correct answer: D M-C2204

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D *10*

The water reaches the ground where its height is 0, which is where the graph meets the horizontal axis. The curve starts at a height of 5 feet, rises to 9 feet at $x = 4$, and comes back down to a height of 0 at $x = 10$. So the water lands 10 feet from the nozzle.

1. The water is on the ground when $h = 0$.
2. Find where the graph meets the horizontal axis: $x = 10$.
3. The water lands 10 feet from the nozzle.

A 4 feet is the horizontal distance at which the water is highest, the x-coordinate of the vertex, not where it lands.

B 5 feet is the height of the nozzle, where the graph meets the vertical axis. It is a height, not a horizontal distance.

C 9 feet is the maximum height of the water. The question asks for the horizontal distance at which the height is 0.

Review: Chapter 22, Nonlinear Functions (page 313)

5. Correct answer: C M-C2205

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C *(−5, 0)*

An x-intercept is a point where $f(x) = 0$. A product equals 0 when one of its factors equals 0, so the zeros are $x = 2$, $x = -5$ and $x = 7$, and the x-intercepts are (2, 0), (−5, 0) and (7, 0). Of the choices, only (−5, 0) is one. Check: $f(-5) = (-7)(0)(-12) = 0$.

1. Set $f(x) = 0$: $(x - 2)(x + 5)(x - 7) = 0$.
2. Solve each factor: $x = 2$, $x = -5$ or $x = 7$.
3. The x-intercepts are (2, 0), (−5, 0) and (7, 0); only (−5, 0) is listed.

A The factor $x - 7$ equals 0 when $x = 7$, not −7. This reverses the sign of that zero.

B (0, 70) is the y-intercept, since f(0) = (−2)(5)(−7) = 70. An x-intercept must have a y-coordinate of 0.

D The factor x + 5 equals 0 when x = −5, not 5. In fact f(5) = (3)(10)(−2) = −60, not 0.

Calculator option: Graph y = (x − 2)(x + 5)(x − 7) and select the points where the graph crosses the x-axis.

Review: Chapter 22, Nonlinear Functions (page 313)

6. Correct answer: A M-C2206

Nonlinear functions · Advanced Math · Medium

A *−16*

The zeros of f are x = −1 and x = 7. A parabola is symmetric, so its vertex lies halfway between the zeros, at x = (−1 + 7)/2 = 3. Because the parabola opens upward, the minimum value is the output there: f(3) = (3 + 1)(3 − 7) = (4)(−4) = −16.

1. Find the zeros: x = −1 and x = 7.
2. The vertex is midway between them: x = (−1 + 7)/2 = 3.
3. Evaluate: f(3) = (4)(−4) = −16.

B −7 is f(0), the y-intercept. The vertex is at x = 3, where the function reaches a lower value.

C 3 is the x-coordinate of the vertex. The minimum value is the output at the vertex, f(3), not the input.

D This drops a sign: 3 − 7 = −4, so (4)(−4) = −16. The minimum cannot be positive, because f is negative between its zeros.

Calculator option: Graph y = (x + 1)(x − 7) and select the vertex, (3, −16).

Review: Chapter 22, Nonlinear Functions (page 313)

7. Correct answer: C M-C2207

Nonlinear functions · Advanced Math · Medium

C $N(h) = 600(3)^{h/4}$

Tripling means multiplying by 3, so the base is 3. The culture triples once every 4 hours, so after h hours it has tripled h/4 times, and the exponent is h/4. Check: $N(4) = 600(3)^1 = 1{,}800$ and $N(8) = 600(3)^2 = 5{,}400$, which triples every 4 hours as required.

1. Initial amount: 600 cells.
2. Growth factor per period: 3, because the culture triples.
3. Number of periods after h hours: h/4.
4. Model: $N(h) = 600(3)^{h/4}$.

A The exponent 4h would make the culture triple 4 times every hour. Then N(1) would be 600 × 81 = 48,600, far too fast.

B This switches the roles of 3 and 4. It describes a culture that quadruples every 3 hours, not one that triples every 4 hours.

D This linear model adds 1,200 cells every 4 hours. It matches the first 4 hours but gives N(8) = 3,000 instead of 5,400.

Calculator option: Make a table for each choice at h = 0, 4 and 8, and keep the choice that gives 600, 1,800 and 5,400.

Review: Chapter 22, Nonlinear Functions (page 313)

8. Correct answer: B M-C2208

Nonlinear functions · Advanced Math · Medium

B *(−4, −2)*

From the graph, the vertex of f is (−1, 2). Replacing x with x + 3 shifts the graph 3 units left, and subtracting 4 after applying f shifts it 4 units down. The vertex moves to (−1 − 3, 2 − 4) = (−4, −2). Check: g(−4) = f(−1) − 4 = 2 − 4 = −2.

1. Read the vertex of f from the graph: (−1, 2).
2. f(x + 3) shifts the graph 3 units left: (−4, 2).
3. Subtracting 4 shifts it 4 units down: (−4, −2).

A This shifts the graph 3 units right. In f(x + 3) the graph moves left, because x = −4 now produces the old input −1.

C This shifts the graph 4 units up. Subtracting 4 after applying f moves every output down by 4, so the y-coordinate is −2.

D This applies only the vertical shift. The + 3 inside the parentheses also moves the graph 3 units left.

Review: Chapter 22, Nonlinear Functions (page 313)

9. Correct answer: B M-C2209

Nonlinear functions · Advanced Math · Medium

B *The value of the equipment is cut in half every 4 years.*

Read the marked points from the graph: (0, 32), (4, 16), (8, 8) and (12, 4). Each 4-year step cuts the value in half, from \$32,000 to \$16,000 to \$8,000 to \$4,000. Equal ratios over equal time steps mean exponential decay, here with the value halving every 4 years.

1. Read the marked points: (0, 32), (4, 16), (8, 8) and (12, 4).
2. Compare consecutive values: each is half of the one before.
3. The points are 4 years apart, so the value halves every 4 years.

A The value drops \$16,000 in the first 4 years but only \$8,000 in the next 4 years, so the yearly loss is not constant.

C \$16,000 is the value 4 years after purchase. At 8 years the graph shows 8, which means \$8,000.

D This extends the first 4-year drop as a straight line. The curve flattens instead and shows \$8,000 at 8 years.

Review: Chapter 22, Nonlinear Functions (page 313)

10. Correct answer: 15360 M-C2210

Nonlinear functions · Advanced Math · Medium

A 20% yearly decrease leaves 80% of the value each year, so the value after t years is $V(t) = 24{,}000(0.8)^t$. After 2 years, $V(2) = 24{,}000(0.8)^2 = 24{,}000(0.64) = 15{,}360$ dollars. Enter 15360 with no comma or dollar sign. Entering 14400 subtracts 40% of the original value at once, ignoring that the second year's loss is 20% of a smaller amount.

1. Each year the car keeps 100% − 20% = 80% of its value.
2. Model: $V(t) = 24{,}000(0.8)^t$.
3. Evaluate: $V(2) = 24{,}000 \times 0.64 = 15{,}360$.

Review: Chapter 22, Nonlinear Functions (page 313)

11. Correct answer: 46.1 M-C2211

Nonlinear functions · Advanced Math · Medium

The maximum of a downward-opening parabola is at its vertex. The vertex occurs at $t = -b/(2a) = -29.4/(2 \times (-4.9)) = 3$ seconds. The maximum height is $h(3) = -4.9(9) + 29.4(3) + 2 = -44.1 + 88.2 + 2 = 46.1$ meters. Entering 3 gives the time of the maximum, and entering 2 gives the launch height.

1. Identify $a = -4.9$ and $b = 29.4$.
2. Time of the maximum: $t = -29.4/(2 \times (-4.9)) = 3$.
3. Height at that time: $h(3) = -44.1 + 88.2 + 2 = 46.1$.

Calculator option: Graph $y = -4.9x^2 + 29.4x + 2$ and select the highest point, (3, 46.1).

Review: Chapter 22, Nonlinear Functions (page 313)

12. Correct answer: 7 M-C2212

Nonlinear functions · Advanced Math · Medium

Work from the inside out. First, $g(3) = -f(3 - 1) = -f(2)$. The table gives $f(2) = -7$, so $g(3) = -(-7) = 7$. Entering −3 uses f(3) and ignores the shift inside the parentheses. Entering −7 forgets the negative sign in front of f, which reflects every output across the x-axis.

1. Find the input to f: $x - 1 = 3 - 1 = 2$.
2. Read the table: $f(2) = -7$.
3. Apply the negative sign: $g(3) = -(-7) = 7$.

Review: Chapter 22, Nonlinear Functions (page 313)

13. Correct answer: D M-C2213

Nonlinear functions · Advanced Math · Medium

D $f(x) = (x - 5)^2 - 4$

Vertex form, $a(x - h)^2 + k$, displays the minimum value k when $a > 0$. Complete the square: $x^2 - 10x + 21 = (x^2 - 10x + 25) - 25 + 21 = (x - 5)^2 - 4$. The minimum value, −4, appears as the constant, and it occurs at $x = 5$.

1. Take half of −10 and square it: 25.
2. Add and subtract 25: $(x^2 - 10x + 25) - 25 + 21$.
3. Write the result as $(x - 5)^2 - 4$; the minimum value is −4.

A This form is equivalent and displays the zeros, 3 and 7, but the minimum value, −4, does not appear in it.

B This form is not equivalent: $(x + 5)^2 - 4$ expands to $x^2 + 10x + 21$. The sign inside the parentheses should be negative.

C This form is equivalent, but its constant 21 is the y-intercept, not the minimum value. The minimum value is −4.

Calculator option: Graph the given function and each choice; equivalent forms trace the same curve. Then check which equivalent choice shows the vertex value −4.

Review: Chapter 22, Nonlinear Functions (page 313)

14. Correct answer: A M-C2214

Nonlinear functions · Advanced Math · Harder

A *10%*

Over 10 years the population is multiplied by 1.21. Let b be the 5-year growth factor. Two 5-year periods make one 10-year period, so $b^2 = 1.21$ and $b = \sqrt{1.21} = 1.1$. A factor of 1.1 means a 10% increase every 5 years. Check: $1.1 \times 1.1 = 1.21$.

1. Let b be the 5-year growth factor; two 5-year periods equal 10 years, so $b^2 = 1.21$.
2. Solve: $b = \sqrt{1.21} = 1.1$.
3. Convert: a factor of 1.1 is a 10% increase every 5 years.

B This halves 21%, ignoring compounding. Two increases of 10.5% give $1.105^2 \approx 1.221$, a 22.1% increase over 10 years, not 21%.

C 21% is the increase over 10 years. Over a period half as long, the population grows by a smaller percent.

D This doubles 21%, as if 5 years were longer than 10 years. A shorter period must have a smaller percent increase.

Calculator option: Compute $1.21^{0.5}$ directly; the result, 1.1, confirms a 10% increase.

Review: Chapter 22, Nonlinear Functions (page 313)

15. Correct answer: 18 M-C2215

Nonlinear functions · Advanced Math · Harder

Find the x that makes the input x + 2 equal to 5: $x + 2 = 5$ gives $x = 3$. Then $f(5) = 3^2 + 3(3) = 9 + 9 = 18$. Another route rewrites f: $f(x) = (x - 2)^2 + 3(x - 2) = x^2 - x - 2$, so $f(5) = 25 - 5 - 2 = 18$. Entering 40 substitutes 5 for x directly, and entering 70 uses $x = 7$.

1. Set the input equal to 5: $x + 2 = 5$, so $x = 3$.
2. Substitute $x = 3$ into $x^2 + 3x$: $9 + 9 = 18$.
3. So $f(5) = 18$.

Review: Chapter 22, Nonlinear Functions (page 313)

16. Correct answer: C M-C2216

Nonlinear functions · Advanced Math · Harder

C $N(t) = 320(1.5)^{t/2}$

Check differences and ratios. The differences, 160, 240 and 360, are not constant, so the model is not linear. The ratios are constant: 480/320 = 720/480 = 1,080/720 = 1.5. The count is multiplied by 1.5 every 2 hours, so the exponent is t/2 and $N(t) = 320(1.5)^{t/2}$. Check: $N(6) = 320(1.5)^3 = 1{,}080$.

1. Compute the ratios of consecutive counts: each one is 1.5.
2. The time step is 2 hours, so the factor 1.5 applies every 2 hours.
3. Use the initial value 320: $N(t) = 320(1.5)^{t/2}$.
4. Check t = 6: 320 × 3.375 = 1,080.

A The exponent 2t applies the factor 1.5 twice every hour. Then $N(2) = 320(1.5)^4 = 1{,}620$, not 480.

B This linear model fits the first 2 hours only. At t = 4 it gives 640, but the table shows 720.

D This splits the 50% increase into 25% per hour, ignoring compounding. Then $N(2) = 320(1.25)^2 = 500$, not 480.

Calculator option: Enter the table as points, then graph each choice; only $320(1.5)^{x/2}$ passes through all four points.

Review: Chapter 22, Nonlinear Functions (page 313)

17. Correct answer: B M-C2217

Nonlinear functions · Advanced Math · Harder

B *40*

The vertex of h is at t = −b/(2a) = −b/(2 × (−16)) = b/32. Setting b/32 = 1.5 gives b = 48. The maximum height is h(1.5) = −16(2.25) + 48(1.5) + 4 = −36 + 72 + 4 = 40 feet.

1. The vertex is at t = −b/(2 × (−16)) = b/32.
2. Set b/32 = 1.5, so b = 48.
3. Evaluate: h(1.5) = −36 + 72 + 4 = 40.

A This leaves out the starting height. The ball is released 4 feet above the ground, so h(1.5) = −36 + 72 + 4 = 40.

C 48 is the value of the constant b, not a height. Use b = 48 in h(t) and evaluate at t = 1.5.

D This treats $-16(1.5)^2$ as +36. The leading term is negative, so it subtracts 36: −36 + 72 + 4 = 40.

Review: Chapter 22, Nonlinear Functions (page 313)

18. Correct answer: D M-C2218

Nonlinear functions · Advanced Math · Harder

D $p(x) = (x + 3)(x - 2)^2$

The graph crosses at x = −3 and touches without crossing at x = 2. Thus x + 3 is a factor of odd multiplicity, and x − 2 is a factor of even multiplicity. Only choice D has both features. Check its y-intercept: $p(0) = (3)(-2)^2 = 12$, matching the graph.

1. Read the x-intercepts: the graph crosses at −3 and touches at 2.
2. Use odd multiplicity at −3 and even multiplicity at 2; among the choices, only $(x + 3)(x - 2)^2$ fits.
3. Check the y-intercept: $(0 + 3)(0 - 2)^2 = 12$, as shown.

A This reverses the signs. Its zeros are 3 and −2, but the graph meets the x-axis at −3 and 2.

B This quadratic has the right zeros but would cross the x-axis at both of them. It also gives p(0) = −6, not 12.

C This puts the squared factor on the wrong zero. Its graph would touch the x-axis at −3 and cross at 2, the reverse of the figure.

Calculator option: Graph each choice and compare it with the figure; only $(x + 3)(x - 2)^2$ matches the intercepts and the touch at x = 2.

Review: Chapter 22, Nonlinear Functions (page 313)

CHAPTER 23

Ratios, Rates, Proportions, Units and Percentages

Problem-Solving and Data Analysis makes up about 15% of the Math section, or 5 to 7 scored questions per section. This chapter covers two of its skills: Ratios, rates, proportional relationships, and units, and Percentages. The arithmetic is rarely hard. Points are lost to setup errors: dividing the wrong way, using the wrong base for a percent, or answering a middle step. Two habits prevent most of them: write a unit beside every number, and turn every percent into a multiplier.

What the questions look like

Most questions in these two skills are short word problems. About 30% of all Math questions are set in a real-world context, and most of those use 50 words or fewer. Some are four-option multiple choice; others are student-produced responses (SPR), where you type your own answer. Typical tasks:

- Find a unit rate, such as dollars per pound or points per game, and use it to predict a total.
- Convert a quantity from one unit to another, sometimes with squared or cubed units.
- Solve a proportion or apply a scale from a map, a model or a recipe.
- Use density or another rate that links two quantities.
- Find a percent of a number, or find the whole when a part and its percent are given.
- Find a percent change, combine two percent changes, or work backward to an original value.

How to recognize them

A **rate** compares two quantities with different units, such as 12 miles per 2 hours. A **unit rate** gives the amount for exactly one unit, such as 6 miles per hour. A **proportion** is an equation that sets two ratios equal. The wording usually tells you which tool to use.

Signal in the question	What it usually means
"per," "for each," "every"	A rate. Divide to find the amount for one unit.
"at this rate," "at the same rate," "the same ratio"	A proportion. Scale a known pair of values.
"a scale of 1 centimeter : 2 meters"	Multiply lengths by the scale. Square it for areas.
Two different units for the same kind of quantity	A conversion. Chain conversion factors.
"density," "grams per cubic centimeter"	Mass = density × volume.
"% of," "is what percent of"	A percent of a number, or the whole from a part.
"increased by," "decreased by," "discount," "markup"	A percent change. Use a multiplier.
"original," "before the change"	A reverse percent. Divide by the multiplier.

The method

For ratios, rates and units

1. Write the unit beside every number in the problem.
2. Decide what unit the answer must have, such as liters or meters per second.
3. Write each conversion fact as a fraction equal to 1, such as (1,000 meters)/(1 kilometer). Put the unwanted unit on the opposite side of the fraction from where it currently appears, so it cancels.
4. Multiply along the chain and cancel units. If the leftover unit is not the one the question asks for, a factor is upside down.
5. Check the size. Converting to a smaller unit gives a bigger number, and more of one quantity in a proportion should usually mean more of the other.

TIP For square units, apply a length conversion twice; for cubic units, three times. Since 1 yard = 3 feet, 1 square yard = 9 square feet and 1 cubic yard = 27 cubic feet.

For percentages

1. Turn every percent into a decimal: p% of a quantity is (p/100) × quantity.
2. An increase of p% multiplies by (1 + p/100). A decrease of p% multiplies by (1 − p/100). For example, a 6% increase multiplies by 1.06, and a 6% decrease multiplies by 0.94.
3. For successive changes, multiply the multipliers. Never add or subtract the percents.
4. To find an original value, divide the final value by the multiplier.
5. For percent change, compute (new − original)/original, then write the result as a percent.

NOTE The reference sheet lists no percent rules, so learn these by heart.

Worked examples

Example 1: Comparing unit rates

A 24-ounce jar of peanut butter costs $4.20, and a 40-ounce jar costs $6.40. How many cents less per ounce does the larger jar cost?

Thinking. "Per ounce" asks for unit rates, so divide dollars by ounces for each jar. Small jar: 4.20 ÷ 24 = 0.175 dollar per ounce, or 17.5 cents. Large jar: 6.40 ÷ 40 = 0.16 dollar per ounce, or 16 cents.

Answer. 17.5 − 16 = 1.5 cents per ounce.

Why the traps fail. Comparing the total prices ($2.20 apart) ignores the different sizes. Dividing ounces by dollars gives ounces per dollar, a rate that answers a different question.

Example 2: A conversion with cubed units

A rectangular sandbox is 9 feet long, 6 feet wide and 1.5 feet deep. Sand is sold by the cubic yard. How many cubic yards of sand fill the sandbox? (1 yard = 3 feet)

Thinking. Convert each length before multiplying: 9 feet = 3 yards, 6 feet = 2 yards and 1.5 feet = 0.5 yard. The volume is 3 × 2 × 0.5 = 3 cubic yards.

Check by a second method. In feet, the volume is 9 × 6 × 1.5 = 81 cubic feet. One cubic yard is 3 × 3 × 3 = 27 cubic feet, and 81 ÷ 27 = 3.

Why the trap fails. Dividing 81 by 3 gives 27 cubic yards, nine times too much. A cubic unit needs the length factor three times.

Example 3: A proportion from a scale

On a hiking map, 2 centimeters represent 5 kilometers. Two campsites are 7.2 centimeters apart on the map. How far apart are they, in kilometers?

Thinking. Keep map distance on top in both ratios: 2/5 = 7.2/d. Cross-multiplying gives 2d = 36, so d = 18 kilometers. A faster route is the unit rate: 5 ÷ 2 = 2.5 kilometers per centimeter, and 2.5 × 7.2 = 18.

Why the trap fails. Writing 7.2 × 2/5 = 2.88 inverts the scale. A longer map distance must give a longer real distance, and 2.88 kilometers is less than the 5 kilometers that only 2 centimeters represent.

Example 4: Density with a unit change

A cooking oil has a density of 0.92 gram per milliliter. What is the mass, in kilograms, of 1.5 liters of the oil? (1 liter = 1,000 milliliters and 1 kilogram = 1,000 grams)

Thinking. Density is mass per unit of volume, so mass = density × volume. The volume must be in milliliters to match the density: 1.5 liters = 1,500 milliliters. The mass is 0.92 × 1,500 = 1,380 grams, which is 1.38 kilograms.

Why the traps fail. Using 1.5 directly gives 0.92 × 1.5 = 1.38 grams, which mixes liters with a per-milliliter rate. Dividing 1,500 by 0.92 gives about 1,630, which is not a mass at all; the units do not cancel.

Example 5: Percent change depends on the base

A library lent 1,250 e-books in March and 1,600 in April. By what percent did e-book loans increase?

Thinking. The change is 1,600 − 1,250 = 350 loans. Divide by the original value, the March total: 350 ÷ 1,250 = 0.28, a 28% increase.

A useful contrast. If loans fell from 1,600 back to 1,250, the change would still be 350, but the base would be 1,600: 350 ÷ 1,600 ≈ 0.219, a decrease of about 21.9%. The same 350 gives a different percent because the base changed.

TRAP If both values are already percents, such as 20% rising to 25%, the change is 5 **percentage points** but a 25% increase, because 5 ÷ 20 = 0.25. Read which one the question wants.

Example 6: Working backward to an original price

After a 15% discount, a jacket costs $68. What was the price before the discount?

Thinking. A 15% discount multiplies the original price p by 0.85, so 0.85p = 68. Dividing gives p = 68 ÷ 0.85 = 80 dollars.

Check. 15% of $80 is $12, and $80 − $12 = $68.

Why the trap fails. Adding 15% back, 68 × 1.15 = $78.20, uses the sale price as the base, but the 15% was taken from the original price. Indeed, 15% off $78.20 is about $66.47, not $68.

Example 7: Harder-module example: an unknown percent applied twice

A quantity is increased by p% and the result is then decreased by p%. The final value is 96% of the original value. What is the value of p?

Thinking. Write each change as a multiplier and multiply: $(1 + p/100)(1 - p/100) = 0.96$. The left side is a difference of squares: $1 - (p/100)^2 = 0.96$. So $(p/100)^2 = 0.04$, which gives $p/100 = 0.2$ and $p = 20$.

Check. $1.20 \times 0.80 = 0.96$.

What this teaches. For a positive starting value and $0 < p < 100$, a rise and a fall by p% end below the start in either order: their combined multiplier is $1 - (p/100)^2$. Reversing the order leaves that product unchanged. Unequal percents can cancel exactly, so multiply the multipliers rather than guess.

Common traps

Trap	How to spot it
Dividing the wrong way in a rate	The leftover unit is upside down, such as hours per mile when the question asks for miles per hour.
Using the linear factor for area or volume	The question mentions square or cubic units, so the factor must be squared or cubed.
Inverting a proportion	Your answer moved in the wrong direction: more muffins should need more flour, not less.
Using the new value as the base	Percent change always divides by the original value.
Adding successive percents	Two changes happen one after the other, so multiply the multipliers.
Multiplying when you should divide	The question gives the value after a change and asks for the value before it.
Mixing percentage points and percent	Both values are percents; their difference is in points, not a percent change.
Answering a middle step	The choices include the unit rate, the amount of the increase, or the area in the wrong unit. Reread the last sentence of the question.

Harder-module variations

Harder questions use the same tools with more structure:

- **Letters instead of numbers.** If a is 150% of b and b is 40% of c, then $a = 1.5 \times 0.4 \times c = 0.6c$, so a is 60% of c. Multiply the multipliers just as you would with numbers.
- **Parts and wholes.** If a club consists only of sophomores and juniors in a ratio of 3 : 5 and has 48 members, the parts total $3 + 5 = 8$, so there are $48 \times 3/8 = 18$ sophomores. Using 3/5 of 48 is the trap.
- **Long conversion chains.** A rate such as gallons per minute may need to become liters per hour. Write every factor, cancel units, and calculate only at the end.
- **Compound units.** Density, cost per square foot or people per square mile can sit inside a larger problem. Treat each as a rate.

- **Reverse percent with two steps.** Divide by each multiplier in reverse order, or combine the multipliers first and divide once.
- **SPR entry.** Never type a percent sign, dollar sign or comma. If the answer is 25%, enter 25. If it is $1,380, enter 1380.

CHECK After any percent question, run a 5-second test with a starting value of 100. It catches most base and direction errors.

Check yourself

1. How do you convert square feet to square yards, and why?
2. A price rises 10% and then falls 10%. Is the final price higher, lower or the same as the original?
3. When you compute a percent change, which value goes in the denominator?
4. A value after a 20% increase is 90. What do you divide 90 by to find the original value, and what is it?
5. In a rate question, how can units warn you that your setup is wrong?
6. Answers: (1) Divide by 9, because 1 square yard is 3 feet by 3 feet. (2) Lower: 1.1 × 0.9 = 0.99, which is 1% less than the original. (3) The original value. (4) Divide by 1.2; the original value is 75. (5) If the units do not cancel to the unit the question asks for, a factor is upside down or missing.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

Ratios, Rates, Proportions and Units
skillbyskillprep.com/skills/ratios-rates-proportions-units

Percentages
skillbyskillprep.com/skills/percentages

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2301

A hypothetical basketball player scored 180 points in the first 12 games of a 30-game season. If the player continues to score at the same average rate, how many points will the player score in the remaining 18 games?

A 15 **B** 120 **C** 180 **D** 270

---

2 M-C2302

In a hypothetical survey of 280 students, 35% said they walk to school. How many of the surveyed students said they walk to school?

A 98 **B** 182 **C** 245 **D** 800

---

3 M-C2303

A smoothie recipe calls for 1.25 cups of yogurt. A cook has only a tablespoon measure. Using 1 cup = 16 tablespoons, how many tablespoons of yogurt does the recipe call for?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

4 M-C2304

The total rainfall at a hypothetical weather station was 40 inches in one year. The next year, the total rainfall was 15% greater. What was the total rainfall, in inches, in the second year?

A 6 **B** 34 **C** 46 **D** 55

---

5 M-C2305

At a hypothetical track meet, one sprinter ran 400 meters in 50 seconds, and a teammate ran 300 meters in 40 seconds. How much greater, in meters per second, was the first sprinter's average speed than the teammate's average speed?

A 0.5 **B** 7.5 **C** 8 **D** 10

---

6 M-C2306

A hypothetical weather station recorded a wind speed of 54 kilometers per hour. What is this speed in meters per second? (1 kilometer = 1,000 meters and 1 hour = 3,600 seconds)

A 0.9 **B** 15 **C** 194.4 **D** 900

---

7 M-C2307

A hypothetical recipe for 12 muffins uses 2.25 cups of flour. How many cups of flour are needed to make 20 muffins using the same ratio of flour to muffins?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

8 M-C2308

In a hypothetical survey, 45 students said they listen to music while studying. These 45 students were 18% of all the students surveyed. How many students were surveyed?

A 8.1 **B** 53.1 **C** 250 **D** 810

9 M-C2309

A hypothetical batch of cookie dough has a density of 1.2 grams per cubic centimeter. A baker presses the dough into a rectangular slab that is 20 centimeters long, 15 centimeters wide and 2 centimeters thick. What is the mass, in grams, of the slab?

A 44.4 **B** 360 **C** 500 **D** 720

10 M-C2310

A hypothetical indoor practice area for a school's track team is a rectangle that measures 72 feet by 45 feet. What is the area of the practice area, in square yards? (1 yard = 3 feet)

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

11 M-C2311

In a hypothetical survey at a school, 12% of students said they volunteer every week. A year later, a similar survey at the same school found that 15% of students said they volunteer every week. By what percent did the share of students who volunteer every week increase?

A 3% **B** 15% **C** 20% **D** 25%

12 M-C2312

A hypothetical stadium raised the price of a season ticket by 25%. The following year, it lowered the new price by 20%. The final price is what percent of the original price?

A 80% **B** 90% **C** 100% **D** 105%

13 M-C2313

A scale drawing of a hypothetical high school baseball field uses the scale 1 inch : 40 feet. The infield on the drawing has an area of 4.5 square inches. What is the actual area of the infield, in square yards? (1 yard = 3 feet)

A 20 **B** 800 **C** 2,400 **D** 7,200

14 M-C2314

During a hypothetical storm, rain fell at a constant rate of 18 millimeters per hour onto a flat rooftop with an area of 40 square meters. How many liters of rain fell on the rooftop in 25 minutes? (1 meter = 1,000 millimeters and 1 cubic meter = 1,000 liters)

A 0.3 **B** 300 **C** 720 **D** 18,000

15 M-C2315

The annual rainfall recorded at a hypothetical weather station was 12% less in Year 2 than in Year 1, and 25% greater in Year 3 than in Year 2. The rainfall in Year 3 was 770 millimeters. What was the rainfall, in millimeters, in Year 1?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

Practice Set Explanations

1. Correct answer: D M-C2301

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Easier

D *270*

Find the unit rate, then scale it. The player averaged 180 ÷ 12 = 15 points per game. For the remaining 18 games, that rate gives 15 × 18 = 270 points. Check: 18 games is 1.5 times as many as 12 games, and 1.5 × 180 = 270.

1. Let r be the points per game: r = 180 ÷ 12 = 15.
2. Multiply the rate by the 18 remaining games: 15 × 18 = 270 points.

A 15 is the unit rate, in points per game. The question asks for the total over the remaining 18 games, so the rate must still be multiplied by 18.

B 120 comes from 180 × 12/18, which flips the proportion. More games at the same rate must produce more points, not fewer.

C 180 repeats the total for the first 12 games. The remaining stretch has 18 games, which is more games, so its total must be larger.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

2. Correct answer: A M-C2302

Percentages · Problem-Solving and Data Analysis · Easier

A *98*

To find a percent of a number, multiply by the decimal form of the percent. Since 35% = 0.35, the number of walkers is 0.35 × 280 = 98 students. Check with a benchmark: 10% of 280 is 28, so 35% is 3.5 × 28 = 98.

1. Write the percent as a decimal: 35% = 0.35.
2. Multiply by the number surveyed: 0.35 × 280 = 98 students.

B 182 is 65% of 280, the number of students who did not say they walk. It answers the opposite question.

C 245 comes from subtracting 35 from 280, treating the percent as a number of students instead of a fraction of the group.

D 800 comes from dividing 280 by 0.35. Dividing finds a whole when only a part is known; here the whole is known, so multiply.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

3. Correct answer: 20 M-C2303

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Easier

Multiply by a conversion factor written so that cups cancel: 1.25 cups × (16 tablespoons)/(1 cup) = 20 tablespoons. A common wrong answer is 12.8, from dividing 16 by 1.25. Because 1.25 cups is more than 1 cup, the answer must be more than 16 tablespoons.

1. Write the conversion factor with cups in the denominator: (16 tablespoons)/(1 cup).
2. Multiply: 1.25 × 16 = 20 tablespoons.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

4. Correct answer: C M-C2304

Percentages · Problem-Solving and Data Analysis · Easier

C *46*

An increase of 15% means the new amount is 100% + 15% = 115% of the original. Multiply by the growth multiplier: 40 × 1.15 = 46 inches. Equivalently, 15% of 40 is 6, and 40 + 6 = 46 inches.

1. Convert the increase to a multiplier: 1 + 0.15 = 1.15.
2. Multiply the first year's total: 40 × 1.15 = 46 inches.

A 6 inches is the amount of the increase, 15% of 40. The question asks for the second year's total, so the increase must be added to 40.

B 34 inches is the result of a 15% decrease, 40 × 0.85. The rainfall was greater in the second year, so the total must exceed 40.

D 55 comes from adding 15 to 40, treating 15% as 15 inches. A percent increase is a fraction of the original amount.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

5. Correct answer: A M-C2305

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

A *0.5*

Compute each unit rate separately, then subtract. The first sprinter averaged 400 ÷ 50 = 8 meters per second, and the teammate averaged 300 ÷ 40 = 7.5 meters per second. The difference is 8 − 7.5 = 0.5 meter per second.

1. First sprinter: 400 ÷ 50 = 8 meters per second.
2. Teammate: 300 ÷ 40 = 7.5 meters per second.
3. Difference: 8 − 7.5 = 0.5 meter per second.

B 7.5 meters per second is the teammate's speed alone. The question asks how much greater the first sprinter's speed was, which requires a subtraction.

C 8 meters per second is the first sprinter's speed alone. Comparing the two speeds requires the difference between the two unit rates.

D 10 comes from dividing the difference in distances, 100 meters, by the difference in times, 10 seconds. Each runner's rate must be found first.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

6. Correct answer: B M-C2306

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

B *15*

Chain conversion factors so that unwanted units cancel: 54 kilometers per hour × (1,000 meters)/(1 kilometer) × (1 hour)/(3,600 seconds) = 54,000 ÷ 3,600 = 15 meters per second. Kilometers cancel in the first factor and hours cancel in the second.

1. Convert kilometers to meters: 54 × 1,000 = 54,000 meters per hour.
2. Convert hours to seconds: 54,000 ÷ 3,600 = 15 meters per second.

A 0.9 is 54 ÷ 60, which changes hours to minutes only and never changes kilometers to meters. Both units must be converted.

C 194.4 comes from multiplying 54 by 3.6. Converting kilometers per hour to meters per second multiplies by 1,000 and divides by 3,600, a net division by 3.6.

D 900 is 54,000 ÷ 60, a speed in meters per minute. Hours were converted to minutes rather than to seconds.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

7. Correct answer: 3.75 or 15/4 M-C2307

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

Set up a proportion with the same units in matching positions: 2.25/12 = c/20, where c is the cups needed. Cross-multiplying gives 12c = 45, so c = 3.75 cups. A common wrong answer is 1.35, from 2.25 × 12/20; more muffins need more flour, so the answer must exceed 2.25.

1. Let c be the cups of flour needed for 20 muffins.
2. Write the proportion: 2.25/12 = c/20.
3. Cross-multiply: 12c = 2.25 × 20 = 45, so c = 45 ÷ 12 = 3.75.

Calculator option: Type 2.25/12 = x/20 into the graphing calculator; it draws the vertical line x = 3.75, which is the solution.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

8. Correct answer: C M-C2308

Percentages · Problem-Solving and Data Analysis · Medium

C *250*

Translate the sentence into an equation: 45 is 18% of n, so 0.18n = 45. Dividing both sides by 0.18 gives n = 250 students. Check: 18% of 250 is 0.18 × 250 = 45, which matches the survey.

1. Let n be the number of students surveyed. Since 18% of n is 45, 0.18n = 45.
2. Divide: n = 45 ÷ 0.18 = 250.

A 8.1 is 18% of 45. The 45 students are the part, not the whole, so the unknown total must be larger than 45.

B 53.1 is 45 increased by 18%. The problem gives a part and the percent it represents; it does not describe an increase.

D 810 comes from multiplying 45 by 18 instead of dividing 45 by 0.18. Writing the equation 0.18n = 45 first prevents this.

Calculator option: Type 0.18x = 45; the calculator draws the vertical line x = 250, confirming the total without rearranging.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

9. Correct answer: D M-C2309

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

D *720*

Density is mass per unit of volume, so mass = density × volume. The slab's volume is 20 × 15 × 2 = 600 cubic centimeters. The mass is 1.2 grams per cubic centimeter × 600 cubic centimeters = 720 grams.

1. Find the volume: V = 20 × 15 × 2 = 600 cubic centimeters.
2. Multiply by the density: mass = 1.2 × 600 = 720 grams.

A 44.4 comes from adding the dimensions, 20 + 15 + 2 = 37, and multiplying by 1.2. Volume requires multiplying the three dimensions.

B 360 is 1.2 × 300, which uses only the area of the top face, 20 × 15 = 300 square centimeters, and ignores the 2-centimeter thickness.

C 500 comes from 600 ÷ 1.2, dividing the volume by the density. Density is grams per cubic centimeter, so multiply by the volume to cancel cubic centimeters.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

10. Correct answer: 360 M-C2310

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

Convert each side to yards, then multiply: 72 feet = 24 yards and 45 feet = 15 yards, so the area is 24 × 15 = 360 square yards. Equivalently, 72 × 45 = 3,240 square feet, and each square yard holds 3 × 3 = 9 square feet, so 3,240 ÷ 9 = 360. A common wrong answer is 1,080, from dividing by 3 instead of 9.

1. Convert each side: 72 ÷ 3 = 24 yards and 45 ÷ 3 = 15 yards.
2. Multiply: 24 × 15 = 360 square yards.
3. Check: 72 × 45 = 3,240 square feet, and 3,240 ÷ 9 = 360.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

11. Correct answer: D M-C2311

Percentages · Problem-Solving and Data Analysis · Medium

D *25%*

Percent change compares the change with the original value: (new − original)/original. The share rose by 15 − 12 = 3 percentage points, and 3/12 = 0.25, so the share increased by 25%. A rise of 3 percentage points and a rise of 25 percent describe the same change in two ways.

1. Find the change: 15 − 12 = 3 percentage points.
2. Divide by the original share: 3 ÷ 12 = 0.25.
3. Write as a percent: 0.25 = 25%.

A The change is 3 percentage points, not a relative increase of 3%. Percent change divides that difference by the original share of 12%.

B 15% is the new share itself, not the change. The question asks how much the share grew relative to its starting value.

C 20% comes from 3/15, which divides by the new share. Percent change always uses the original value, 12%, as the base.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

12. Correct answer: C M-C2312

Percentages · Problem-Solving and Data Analysis · Harder

C *100%*

Successive percent changes multiply. A 25% increase multiplies the price by 1.25, and a 20% decrease multiplies the new price by 0.80. The combined multiplier is 1.25 × 0.80 = 1.00, so the final price is 100% of the original. Testing a $100 ticket confirms it: $100 becomes $125, then $100.

1. Let p be the original price. After the increase, the price is 1.25p.
2. After the decrease, the price is 0.80 × 1.25p = 1.00p.
3. 1.00p is 100% of the original price.

A 80% applies only the 20% decrease to the original price. The decrease was applied to the already increased price, which was 125% of the original.

B 90% comes from 0.75 × 1.20, a 25% decrease followed by a 20% increase. The directions of the two changes are reversed.

D 105% comes from combining the percents directly, 25 − 20 = 5. The two changes apply to different base amounts, so they multiply rather than add.

Calculator option: Graph y = 0.8(1.25x) together with y = x; the two graphs are the same line, so the final price always equals the original.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

13. Correct answer: B M-C2313

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Harder

B *800*

Areas scale by the square of the linear scale. Each square inch on the drawing represents 40 × 40 = 1,600 square feet, so the infield is 4.5 × 1,600 = 7,200 square feet. One square yard is 3 × 3 = 9 square feet, so the area is 7,200 ÷ 9 = 800 square yards.

1. Each inch represents 40 feet, so each square inch represents 40^2 = 1,600 square feet.
2. Actual area: 4.5 × 1,600 = 7,200 square feet.
3. Convert: 7,200 ÷ 9 = 800 square yards.

A 20 applies the scale only once, 4.5 × 40 = 180, and then divides by 9. An area needs the linear scale factor squared.

C 2,400 converts square feet to square yards by dividing by 3. A square yard contains 9 square feet, so the divisor must be 9.

D 7,200 is the area in square feet. The question asks for square yards, so one more conversion is needed.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

14. Correct answer: B M-C2314

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Harder

B *300*

In 25 minutes, which is 25/60 hour, the rain depth is 18 × 25/60 = 7.5 millimeters, or 0.0075 meter. The volume of rain is area × depth = 40 × 0.0075 = 0.3 cubic meter, and 0.3 × 1,000 = 300 liters.

1. Convert the time: 25 minutes = 25/60 hour, so the depth is 18 × 25/60 = 7.5 millimeters = 0.0075 meter.
2. Volume: V = 40 × 0.0075 = 0.3 cubic meter.
3. Convert: 0.3 × 1,000 = 300 liters.

A 0.3 is the volume in cubic meters. The question asks for liters, so the volume must be multiplied by 1,000 liters per cubic meter.

C 720 liters is the rain for a full hour, 18 millimeters over 40 square meters. Only 25 minutes, or 5/12 of an hour, passed.

D 18,000 treats 25 minutes as 25 hours by multiplying 18 millimeters by 25. Minutes must be converted to hours before the hourly rate is used.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

15. Correct answer: 700 M-C2315

Percentages · Problem-Solving and Data Analysis · Harder

Work backward by dividing by each multiplier. Year 2 rainfall was 770 ÷ 1.25 = 616 millimeters, and Year 1 rainfall was 616 ÷ 0.88 = 700 millimeters. Check forward: 700 × 0.88 = 616 and 616 × 1.25 = 770. A common wrong answer is 646.8, from 770 × 0.75 × 1.12, which reverses each percent instead of dividing by the multipliers.

1. Let r be the Year 1 rainfall. Then Year 2 = 0.88r and Year 3 = 1.25 × 0.88r = 1.1r.
2. Set the Year 3 expression equal to the data: 1.1r = 770.
3. Divide: r = 770 ÷ 1.1 = 700 millimeters.

Calculator option: Type 0.88 × 1.25x = 770; the calculator draws the vertical line x = 700, which confirms the answer without rearranging.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

CHAPTER 24

One-Variable and Two-Variable Data

This chapter covers two Problem-Solving and Data Analysis skills. One-variable data: distributions and measures of center and spread asks you to summarize a single list of values: where its center is and how spread out it is. Two-variable data: models and scatterplots asks you to describe how two quantities move together, usually with a line or curve of best fit.

These questions reward reading a display correctly more than long calculation. Most mistakes come from using the wrong number, such as treating a count as a data value or a data point as a prediction, so the method below has you name what each number means before you use it.

What the questions look like

You will see data in several forms: a list, a frequency table, a dot plot, a histogram, a box plot or a scatterplot. Typical tasks:

- Find the mean, median or mode from a list or a display.
- Decide how the mean or median changes when a value is added, removed or changed.
- Compare spread using the range, the interquartile range or the standard deviation.
- Read a box plot and decide which statements must be true.
- Interpret the slope or y-intercept of a line of best fit in context.
- Use a line of best fit to make a prediction, or compare a data point with the line.
- Decide whether a linear or an exponential model fits a data set better.

How to recognize them

The **mean** is the sum of the values divided by the number of values. The **median** is the middle value when the values are in order. The **mode** is the most common value. An **outlier** is a value far from the rest of the data. A **distribution** is the overall pattern of the values: where they cluster, how far they spread and whether one side stretches out farther than the other.

Signal in the question	What it usually means
"mean," "average," "median," "typical value"	Measure of center
"range," "interquartile range," "standard deviation," "spread," "variability"	Measure of spread
"dot plot," "histogram," "frequency," "number of students"	Counts tell how many values, not what the values are
"box plot," "quartile"	Five-number summary
"line of best fit," "model," "predicted," "based on the line"	Use the equation, not a data point
"best interpretation of the slope" or "of the y-intercept"	Translate a number in the model into words with units
"increases by the same percent," "grows by a factor"	Exponential model

The method

One-variable data

1. **Count the values.** Call the count n. In a dot plot, count the dots. In a frequency table or histogram, add the counts.
2. **Find the median position.** If n is odd, the median is value number (n + 1)/2. If n is even, it is the mean of values n/2 and n/2 + 1. Use running totals of the counts to find where those positions fall.
3. **Find the mean with totals.** For exact values, multiply each value by its count, add the products, and divide by n. A histogram with intervals usually does not give an exact mean or median; its counts can locate the intervals containing the middle observations.
4. **For changes, think in totals.** Mean × count = total. Adding or removing a value changes the total and the count.
5. **For spread, ask how far values typically sit from the center.** The **range** is maximum − minimum. The **interquartile range (IQR)** is Q3 − Q1, the spread of the middle half. The **standard deviation** measures the typical distance of values from the mean.

Measure	What it uses	Pulled by an outlier?
Mean	Every value	Yes, strongly
Median	The middle position	Very little
Range	The two extreme values	Yes, strongly
Interquartile range	The middle half of the values	Very little
Standard deviation	Every value's distance from the mean	Yes

The **box plots in this book** show the minimum, first quartile (Q1), median, third quartile (Q3) and maximum. Each section marks about one quarter of the ordered data, though ties can put more values at a boundary. Other box plots may mark outliers separately, so read their whisker convention. A box plot alone does not give the mean or sample size.

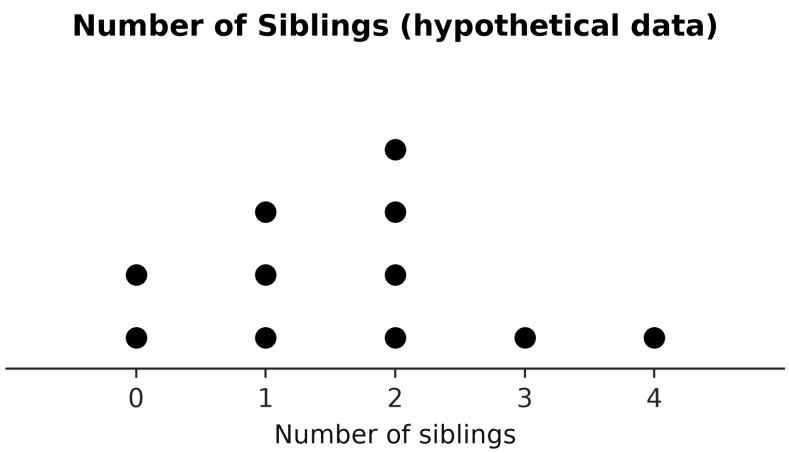
Two-variable data

1. **Read both axes.** Name x, y and their units before reading the choices.
2. **Interpret the line.** In y = mx + b, the slope m is the predicted change in y for each increase of 1 in x. The y-intercept b is the predicted y when x = 0, which is meaningful only if x = 0 makes sense in context.
3. **Predict from the equation.** Substitute the x-value into the line's equation. Do not read a nearby data point.
4. **Compare a point with the line.** Compute actual − predicted. A positive result means the point lies above the line; a negative result means it lies below. This difference is called a **residual**.
5. **Choose a model by its pattern.** If y changes by about the same amount for each step in x, a linear model fits. If y changes by about the same factor, or percent, for each step, an exponential model fits.

Worked examples

Example 1: Center from a dot plot

The dot plot shows the number of siblings for each of 11 students in a hypothetical club. Find the median and the mean.



Thinking. There are 2 + 3 + 4 + 1 + 1 = 11 dots, so n = 11 and the median is value number (11 + 1)/2 = 6. Running totals: 2 values at 0, then 5 values through 1, then 9 values through 2. The 6th value is 2, so the median is 2.

For the mean, multiply and add: 0(2) + 1(3) + 2(4) + 3(1) + 4(1) = 18. The mean is 18 ÷ 11 ≈ 1.64.

Why the traps fail. The tallest column has 4 dots, but 4 is a count, not a number of siblings. The middle of the scale, also 2 here, matches the median only by chance; never use it as a shortcut.

Example 2: How one new value moves the mean

A student's mean score on 5 tests is 84. What score on a 6th test would raise the mean to exactly 86?

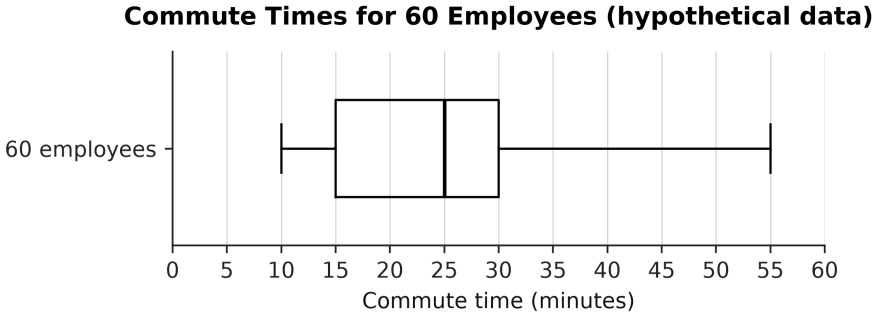
Thinking. Work with totals. Five tests at a mean of 84 total 5 × 84 = 420 points. Six tests at a mean of 86 must total 6 × 86 = 516 points. The 6th score must be 516 − 420 = 96.

Why the trap fails. Guessing 88, two points above the new mean, ignores the count. To lift the mean of six scores by 2 points, the new score must be 6 × 2 = 12 points above the old mean of 84.

TIP Removing a value works the same way. Subtract it from the total, reduce the count by 1, and divide again. For the median, recount the middle position, because n has changed.

Example 3: Reading a box plot

The box plot summarizes the commute times of 60 employees at a hypothetical company.



Thinking. Read the five numbers: minimum 10, Q1 15, median 25, Q3 30 and maximum 55 minutes. The IQR is 30 − 15 = 15 minutes and the range is 55 − 10 = 45 minutes. The box holds the middle half of the values, so roughly 30 of the 60 employees commute 15 to 30 minutes.

What the plot cannot tell you. It does not show the mean. The long right whisker suggests the data stretch out toward large values, which often pulls the mean above the median, but a box plot alone cannot confirm that.

Example 4: Comparing standard deviations without computing

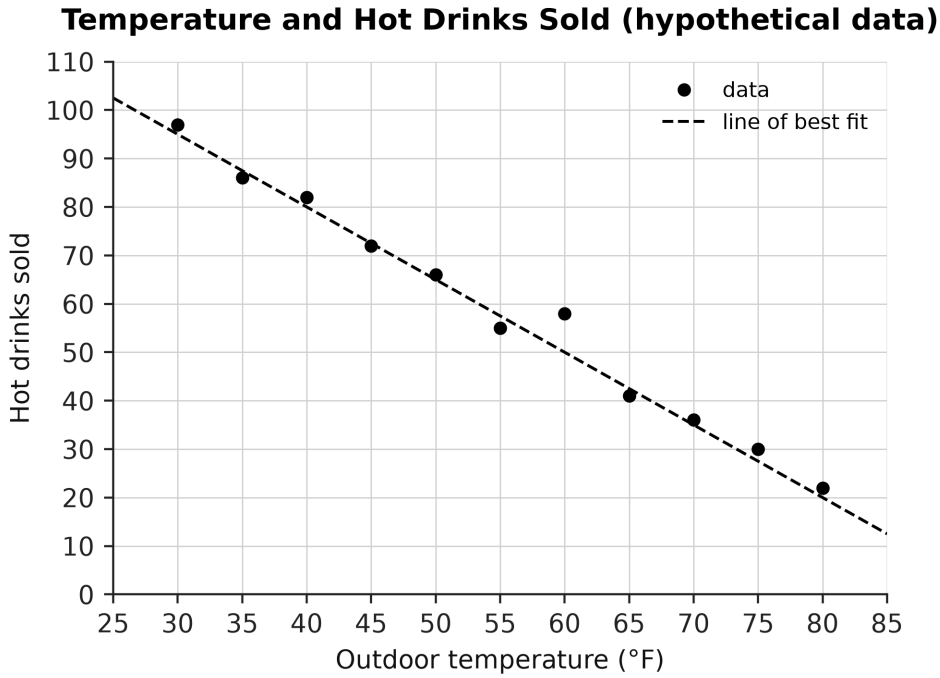
Set A is 10, 20, 30, 40, 50. Set B is 28, 29, 30, 31, 32. Which has the greater standard deviation?

Thinking. Both sets have a mean and median of 30. In Set A, values sit 10 or 20 units from the mean; in Set B, they sit only 1 or 2 units away. Set A is far more spread out, so its standard deviation is greater.

Two follow-up facts. Adding 5 to every value in Set A shifts the mean and median up by 5 but leaves the standard deviation and range unchanged, because the distances between values stay the same. Doubling every value doubles the mean, the median, the range and the standard deviation.

Example 5: Slope, prediction and a residual

The scatterplot shows a hypothetical cafe's hot-drink sales on 11 days, with the line of best fit $y = -1.5x + 140$, where x is the outdoor temperature in degrees Fahrenheit.



Slope. For each increase of 1°F, the line predicts 1.5 fewer hot drinks sold.

Prediction. At 60°F, the line predicts $-1.5(60) + 140 = 50$ drinks.

Residual. On the 60°F day, the cafe actually sold 58 drinks. Actual − predicted = 58 − 50 = 8, so that point lies 8 drinks above the line.

Intercept. The value 140 is the predicted sales at 0°F, far colder than any day in the data, so treat it with caution.

Example 6: Harder-module example: linear or exponential?

A hypothetical online video's total views were recorded each week: 160, 240, 360, 540 and 810 in weeks 0 through 4. Which kind of model fits, and what is its equation?

Thinking. Check differences first: 80, 120, 180 and 270. They grow, so a linear model is a poor fit. Now check ratios: 240/160 = 1.5, 360/240 = 1.5, and so on. A constant ratio means exponential growth. Each week multiplies the views by 1.5, a 50% increase, so the model is $y = 160(1.5)^x$.

A harder twist. Suppose the same data had been recorded every 2 weeks instead. Then 1.5 would be the 2-week factor, and the weekly factor b would satisfy $b^2 = 1.5$, so $b \approx 1.22$. Always match the factor to the time unit in the question.

Common traps

Trap	How to spot it
Reading a count as a data value	In dot plots, frequency tables and histograms, heights and counts tell how many, not what value.
Forgetting the even-count median	With an even n, average the two middle values.
Assuming a removal leaves the median alone	Removing any value changes n, so the middle position moves.
Mixing up range and IQR	Range uses the whisker ends; IQR uses the box ends.
Equating the same mean with the same spread	Look at how far values sit from the center.
Predicting from a data point	"Predicted" or "based on the line" means substitute into the equation.
Reversing actual and predicted	Residual = actual − predicted. Above the line is positive.
Over-reading the intercept	If x = 0 is outside the data, the intercept may have no sensible meaning.
Calling the slope a guarantee	The slope describes a predicted, average change, not an exact change for every case.

Harder-module variations

- **A missing frequency.** A table shows counts with one unknown, k, and gives the mean. Write total ÷ count = mean with k in both the total and the count, then solve.
- **Combining two groups.** If 20 students average 80 and 30 students average 70, the combined mean is (20 × 80 + 30 × 70) ÷ 50 = 74, not 75. The larger group pulls harder.
- **"Must be true" statements.** For box plots and histograms, sort each statement into "shown," "contradicted" or "cannot be determined." Only a statement the display guarantees can be correct.
- **Differences between predictions.** Two predictions from the same line differ by slope × (difference in x). The intercept cancels.
- **Comparing two models.** Compute each model's predictions at the data's x-values. The better model is the one whose predictions are closer to the actual values.

NOTE The graphing calculator can find a list's mean, median and standard deviation and fit linear or exponential regressions. Use it to check, but know the reasoning, because many questions ask what a number means rather than what it is.

Check yourself

1. A frequency table describes 20 values. Which positions determine the median?
2. Five numbers have a mean of 12. What is their total?
3. Which two numbers in a box plot give the interquartile range?
4. A data point lies below the line of best fit. Is actual − predicted positive or negative?
5. A quantity grows by the same percent each year. Which kind of model fits?
6. Answers: (1) The 10th and 11th values; average them. (2) 60. (3) Q3 and Q1; the IQR is Q3 − Q1. (4) Negative. (5) An exponential model.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

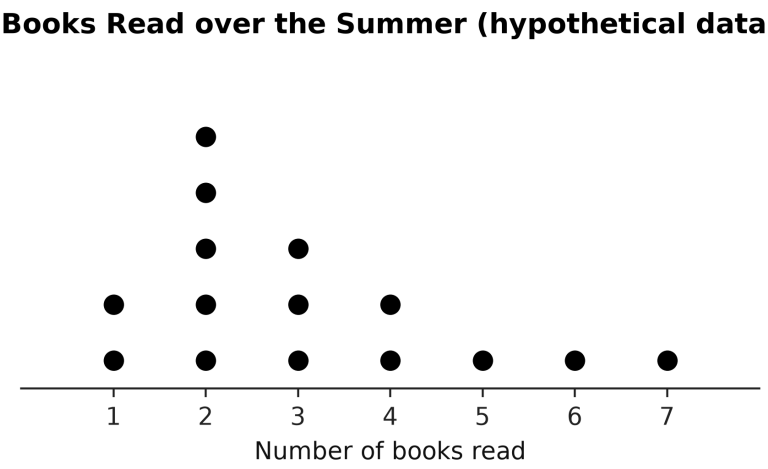
One-Variable Data
skillbyskillprep.com/skills/one-variable-data

Two-Variable Data and Scatterplots
skillbyskillprep.com/skills/two-variable-data-scatterplots

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2401



The dot plot shows the number of books each of 15 students read over the summer, based on a hypothetical survey. What is the mean number of books read by these students, rounded to the nearest tenth?

A 2 **B** 3 **C** 3.1 **D** 4

2 M-C2402

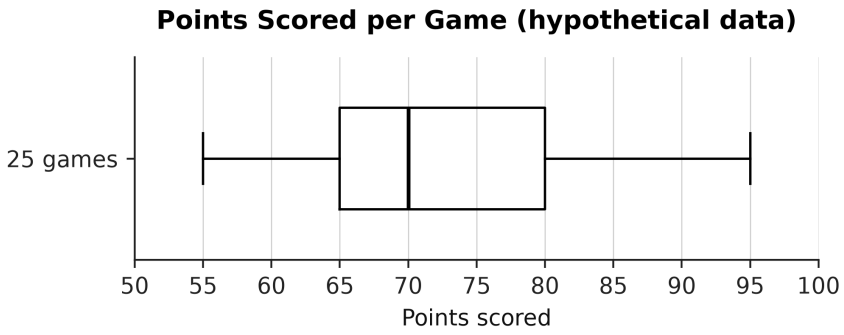
Goals Scored per Game (hypothetical data)

Goals scored	Number of games
0	3
1	4
2	2
3	1
4	1
5	1

The table shows the number of goals a hypothetical soccer team scored in each of its 12 games. What is the median number of goals scored per game?

A 1 **B** 1.5 **C** 2.5 **D** 6

3 M-C2403

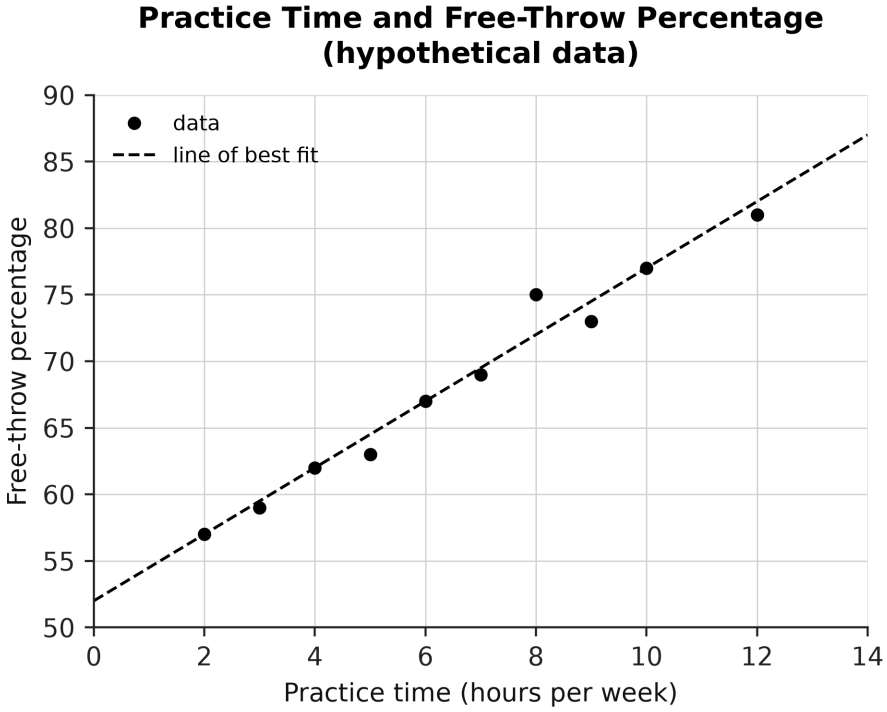


The box plot summarizes the points scored by a hypothetical basketball team in each of its 25 games. What is the interquartile range of the points scored?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

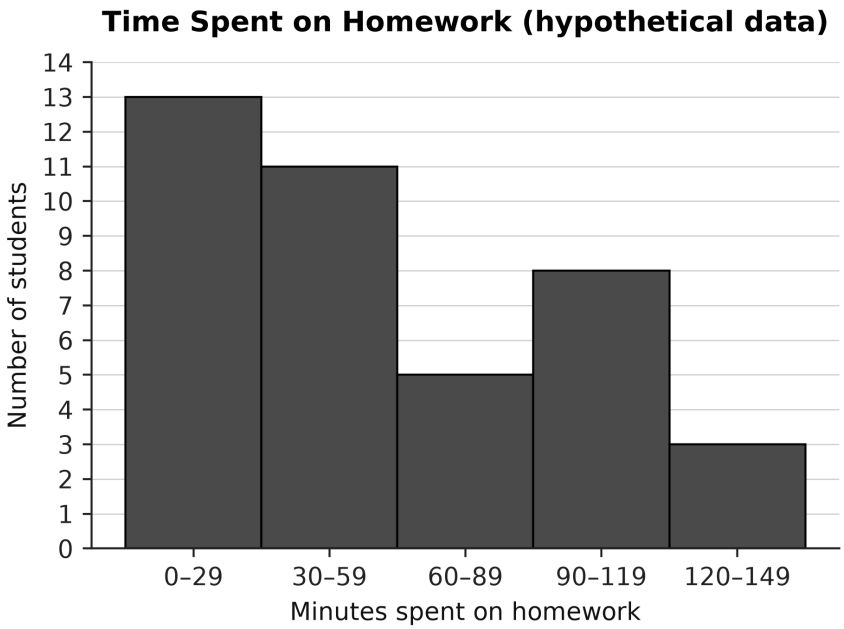
4 M-C2404



The scatterplot shows the weekly practice time x, in hours, and the free-throw percentage y for 10 hypothetical basketball players, along with a line of best fit with equation $y = 2.5x + 52$. Based on the line of best fit, what is the predicted free-throw percentage for a player who practices 8 hours per week?

A 20 **B** 60 **C** 72 **D** 75

5 M-C2405



The histogram summarizes the number of minutes each of 40 students spent on homework one night, based on a hypothetical survey. Which interval contains the median number of minutes?

A 0–29 minutes **B** 30–59 minutes **C** 60–89 minutes **D** 90–119 minutes

6 M-C2406

A hypothetical weather station recorded the following amounts of rain, in millimeters, on 9 rainy days: 4, 6, 7, 7, 9, 10, 12, 13, 58. If the value 58 is removed from the data set, which statement about the mean and median of the remaining values is true?

A The mean decreases, and the median stays the same.

B The mean decreases, and the median increases.

C The mean and the median decrease by the same amount.

D The mean and the median both decrease, and the mean decreases by more.

7 M-C2407

For 12 hypothetical high school runners, the line of best fit $y = -0.25x + 21$ models the relationship between x, the number of weeks a runner has trained, and y, the runner's time, in minutes, to complete a 5-kilometer race. Which choice is the best interpretation of −0.25 in this context?

A The predicted race time for a runner with 0 weeks of training is 0.25 minute.

B For each additional week of training, the predicted race time decreases by 0.25 minute.

C For each additional minute of race time, the predicted number of weeks of training decreases by 0.25.

D Each runner's actual race time decreased by exactly 0.25 minute for every week of training.

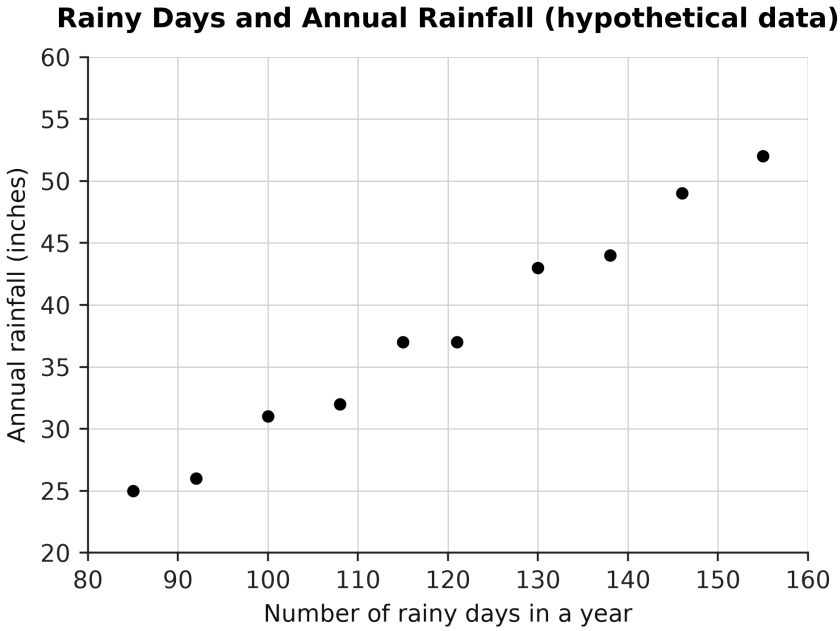
8 M-C2408

The mean of the daily high temperatures at a hypothetical weather station for 6 days was 71°F. When the high temperature for a 7th day was included, the mean for the 7 days was 73°F. What was the high temperature, in degrees Fahrenheit, on the 7th day?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

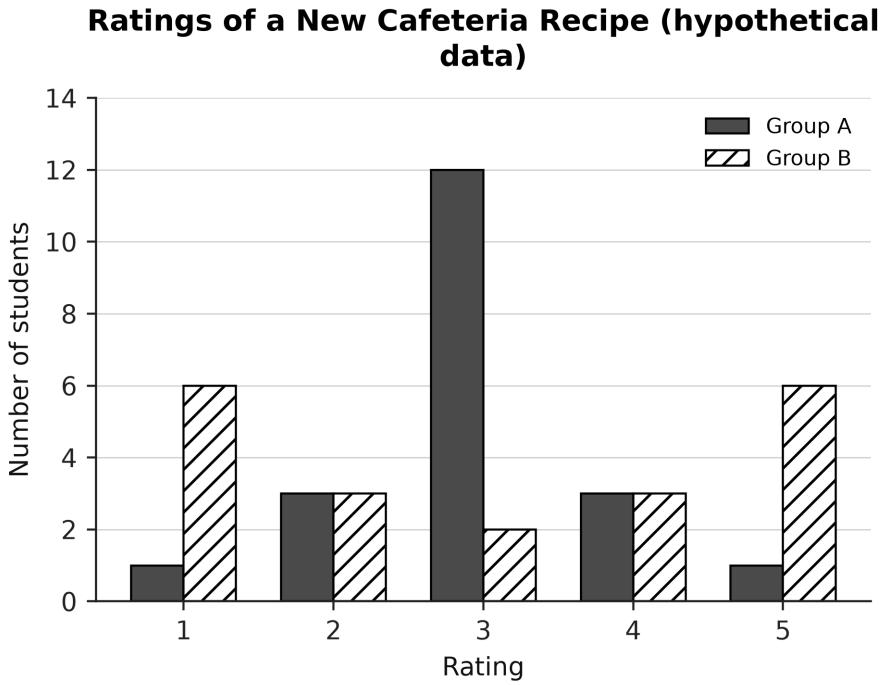
9 M-C2409



The scatterplot shows the number of rainy days x and the annual rainfall y, in inches, for 10 hypothetical towns. Which equation is the most appropriate linear model for the data?

A $y = 0.4x - 10$ **B** $y = -0.4x + 70$ **C** $y = 0.4x + 30$ **D** $y = 2.5x - 220$

10 M-C2410



Two hypothetical groups of 20 students each rated a new cafeteria recipe on a scale from 1 to 5. The bar graph shows the results. Both groups have a mean rating of 3. Which statement about the standard deviations of the two groups' ratings is true?

A Group A's standard deviation is greater, because Group A has the tallest bar.

B The standard deviations are equal, because both groups have a mean rating of 3.

C The standard deviations are equal, because both groups' ratings range from 1 to 5.

D Group B's standard deviation is greater, because more of Group B's ratings are far from the mean.

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11 M-C2411

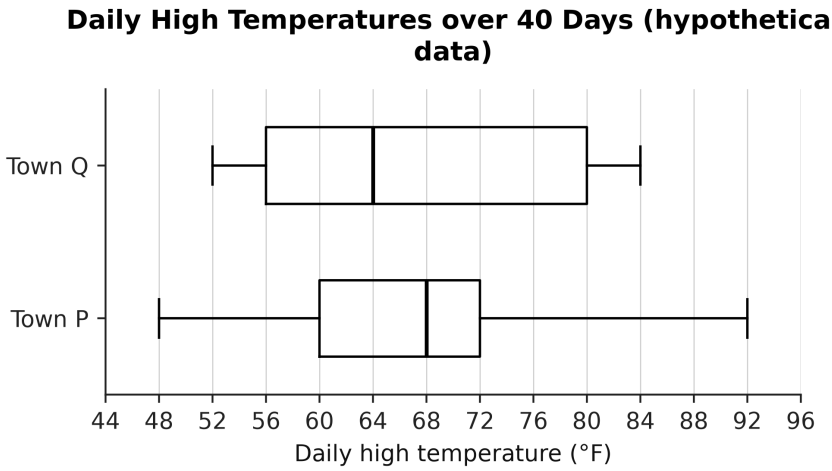
A line of best fit, $y = 6x - 310$, models the relationship between the daily high temperature x, in degrees Fahrenheit, and the number of visitors y at a hypothetical public pool. On a day when the high temperature was 85°F, the pool had 214 visitors. What is the actual number of visitors minus the number of visitors predicted by the line of best fit?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

12 M-C2412



The box plots summarize the daily high temperatures, in degrees Fahrenheit, recorded in two hypothetical towns on the same 40 days. Which statement must be true?

A The interquartile range of Town Q's temperatures is greater than that of Town P's temperatures.

B The mean of Town P's temperatures is greater than the mean of Town Q's temperatures.

C More than half of Town Q's temperatures were greater than 68°F.

D The range of Town Q's temperatures is greater than the range of Town P's temperatures.

---

13 M-C2413

For a set of hypothetical weather stations, the line of best fit $y = 0.12x - 30$ models the relationship between a station's elevation x, in meters, and its average annual snowfall y, in centimeters. Based on the model, how many more centimeters of average annual snowfall are predicted for a station whose elevation is 500 meters greater than that of another station?

A 30 **B** 60 **C** 90 **D** 4,167

---

14 M-C2414

Registered Players in a Youth Soccer League (hypothetical data)

Years after start, x	Registered players
0	50
1	61
2	73
3	87
4	104

The table shows the number of registered players in a hypothetical youth soccer league x years after the league started. Two models are proposed: Model L, $y = 50 + 13x$, and Model E, $y = 50(1.2)^x$. Which choice identifies the model that better fits the data for $x = 1$ through $x = 4$ and gives a valid reason?

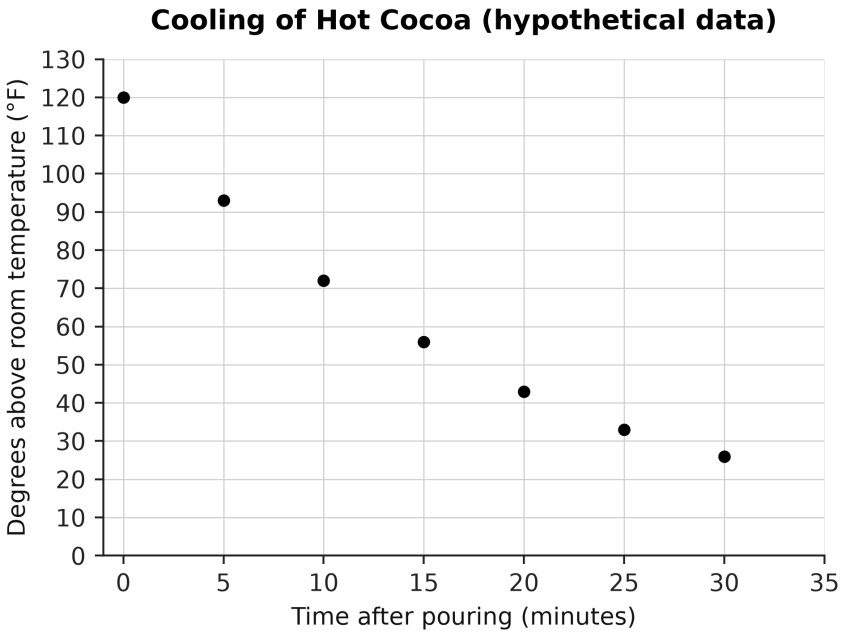
A Model L, because the number of registered players increased every year.

B Model E, because an exponential model always fits increasing data better than a linear model does.

C Model L, because the number of registered players increased by about the same amount each year.

D Model E, because for each of $x = 1$ through $x = 4$, its prediction is closer to the actual number than Model L's prediction is.

15 M-C2415



The scatterplot shows y, the difference in degrees Fahrenheit between the temperature of a hypothetical cup of hot cocoa and the room's temperature, x minutes after the cocoa was poured. Which equation best models the data?

A $y = 120 - 3.1x$ **B** $y = 120(0.77)^x$ **C** $y = 120(0.95)^x$ **D** $y = 120(1.05)^x$

Practice Set Explanations

1. Correct answer: C M-C2401

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

C *3.1*

The mean is the total divided by the number of values. Multiply each value by its number of dots and add: 1(2) + 2(5) + 3(3) + 4(2) + 5(1) + 6(1) + 7(1) = 47 books. There are 15 students, so the mean is 47 ÷ 15 ≈ 3.13, or 3.1 books to the nearest tenth.

1. Multiply each value by its count and add: 2 + 10 + 9 + 8 + 5 + 6 + 7 = 47.
2. Count the dots: n = 15.
3. Divide: 47 ÷ 15 ≈ 3.13, which is 3.1 to the nearest tenth.

A 2 is the mode, the value with the most dots. The mean uses every value, including the larger values on the right side of the plot.

B 3 is the median, the 8th of the 15 ordered values. The mean is pulled slightly higher by the students who read 5, 6 and 7 books.

D 4 is the middle of the scale from 1 to 7. The center of the number line is not a measure of the center of the data.

Calculator option: Type the 15 values as one list, repeating each value once per dot, and apply the mean function; it returns about 3.13.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

2. Correct answer: A M-C2402

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

A *1*

List the 12 values in order: 0, 0, 0, 1, 1, 1, 1, 2, 2, 3, 4, 5. With an even number of values, the median is the mean of the 6th and 7th values. Both are 1, so the median is 1 goal per game.

1. Total games: 3 + 4 + 2 + 1 + 1 + 1 = 12, so the median is the mean of the 6th and 7th values.
2. Running totals: 0 goals fills games 1 to 3, and 1 goal fills games 4 to 7.
3. The 6th and 7th values are both 1, so the median is 1.

B 1.5 is the median of the frequency column, which lists numbers of games. Frequencies count how often a value occurs; they are not goal values.

C 2.5 is the median of the six goal values listed, 0 through 5, ignoring how many games had each value.

D 6 is the position of the first middle value, 12 ÷ 2. The median is the value found at the middle positions, not the position number.

Calculator option: Type the 12 values as a list and apply the median function to confirm the answer of 1.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

3. Correct answer: 15 M-C2403

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

The interquartile range (IQR) is the third quartile minus the first quartile, which is the width of the box. Here IQR = 80 − 65 = 15 points. A common wrong answer is 40, the range 95 − 55, which uses the ends of the whiskers instead of the ends of the box.

1. Read the quartiles from the box: Q1 = 65 and Q3 = 80.
2. Subtract: IQR = 80 − 65 = 15.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

4. Correct answer: C M-C2404

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Easier

C *72*

A prediction comes from the line of best fit, not from a data point. Substitute x = 8 into the equation: y = 2.5(8) + 52 = 20 + 52 = 72. The predicted free-throw percentage is 72%.

1. Use the line: y = 2.5x + 52.
2. Substitute x = 8: y = 2.5(8) + 52 = 72.

A 20 is only the slope term, 2.5 × 8. The y-intercept, 52, must be added to get the predicted value.

B 60 comes from adding 8 to 52, which skips multiplying the number of hours by the slope of 2.5.

D 75 is the actual percentage for the player shown at 8 hours. The question asks for the value predicted by the line.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

5. Correct answer: B M-C2405

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Medium

B *30–59 minutes*

With 40 values, the median is the mean of the 20th and 21st values in order. Counting from the left, the first interval holds values 1 through 13 and the second holds values 14 through 24. Both the 20th and 21st values fall in the 30–59 minute interval, so the median does too.

1. n = 40, so the median is the mean of the 20th and 21st values.
2. Running totals: 13 after 0–29, then 13 + 11 = 24 after 30–59.

3. Values 14 through 24 lie in 30–59, which includes the 20th and 21st values.

A 0–29 minutes has the tallest bar, so it is the most common interval. It holds only the 13 smallest values, short of the 20th and 21st.

C 60–89 minutes is the middle of the five intervals. The median depends on how many values fall in each interval, not on where the intervals sit.

D 90–119 minutes has the middle bar height, 8, of the five counts 3, 5, 8, 11 and 13. The median of the counts is not the median of the data.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

6. Correct answer: D M-C2406

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Medium

D *The mean and the median both decrease, and the mean decreases by more.*

With all 9 values, the mean is 126 ÷ 9 = 14 and the median is the 5th value, 9. Without 58, the mean is 68 ÷ 8 = 8.5, and the median is the mean of the 4th and 5th values, (7 + 9) ÷ 2 = 8. The mean drops by 5.5, but the median drops by only 1.

1. Original: sum = 126 and n = 9, so the mean is 14; the median is the 5th value, 9.
2. New: sum = 126 − 58 = 68 and n = 8, so the mean is 8.5.
3. New median: (4th value + 5th value) ÷ 2 = (7 + 9) ÷ 2 = 8.
4. The mean falls by 5.5 and the median falls by 1.

A Removing a value changes the count, so the middle position moves. With 8 values left, the median becomes (7 + 9) ÷ 2 = 8, not 9.

B Removing the greatest value can only shift the middle toward smaller values, so the median cannot increase. It falls from 9 to 8.

C The mean falls by 5.5 but the median falls by only 1. An extreme value pulls the mean much more than it pulls the median.

Calculator option: Enter the list and compare the mean and median functions before and after deleting 58; this checks all four values quickly.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

7. Correct answer: B M-C2407

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Medium

B *For each additional week of training, the predicted race time decreases by 0.25 minute.*

In y = mx + b, the slope m is the change in y for each 1-unit increase in x. Here x is weeks of training and y is the predicted race time in minutes, so a slope of −0.25 means that each additional week of training is associated with a predicted race time 0.25 minute lower.

1. Identify the slope: m = −0.25, in minutes per week.
2. Interpret: when x increases by 1 week, the predicted y decreases by 0.25 minute.

A The predicted time for 0 weeks of training is the y-intercept, 21 minutes. The slope describes a rate of change, not a starting value.

C This reverses the roles of the variables. The slope gives the change in race time, y, for each week of training, x.

D A line of best fit describes predicted values, not exact changes for every runner. Actual times scatter above and below the line.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

8. Correct answer: 85 M-C2408

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Medium

Work with totals, because a mean is a total divided by a count. The 6-day total is 6 × 71 = 426 degrees, and the 7-day total is 7 × 73 = 511 degrees. The 7th day's high is 511 − 426 = 85°F. Common wrong answers are 2, the change in the mean, and 75, from adding that change to 73.

1. 6-day total: 6 × 71 = 426.
2. 7-day total: 7 × 73 = 511.
3. 7th value: 511 − 426 = 85.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

9. Correct answer: A M-C2409

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Medium

A $y = 0.4x - 10$

The points rise from left to right, so the slope is positive. Two points near the trend, (100, 31) and (146, 49), give a slope of 18 ÷ 46 ≈ 0.39. Testing y = 0.4x − 10 gives 30 at x = 100 and 50 at x = 150, both close to the data, so choice A fits.

1. Choose two points near the trend: (100, 31) and (146, 49).
2. Slope: (49 − 31) ÷ (146 − 100) = 18 ÷ 46 ≈ 0.4.
3. Test y = 0.4x − 10: at x = 100, y = 30, and at x = 150, y = 50, matching the data.

B A negative slope describes points that fall from left to right. These points rise, so the slope of the model must be positive.

C The slope is reasonable, but at x = 100 this line gives 70 inches, far above the data near 31. The y-intercept is too high.

D 2.5 is run over rise, the reciprocal of the slope. At x = 150 this line gives 155 inches, far above every point.

Calculator option: Enter the 10 points in a table and run a linear regression; it gives a slope of about 0.40 and a y-intercept of about −9.5, matching choice A.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

10. Correct answer: D M-C2410

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Medium

D *Group B's standard deviation is greater, because more of Group B's ratings are far from the mean.*

Standard deviation measures how far values typically are from the mean. Twelve of Group A's 20 ratings are 3, exactly at the mean, while 12 of Group B's 20 ratings are 1 or 5, two points from the mean. Group B's ratings are more spread out, so its standard deviation is greater.

1. Both groups have mean 3.
2. Group A: 12 of 20 ratings equal 3, so most values are 0 points from the mean.
3. Group B: 12 of 20 ratings are 1 or 5, which are 2 points from the mean, so Group B is more spread out.

A A tall bar at the mean means many values sit at the center. That concentration makes Group A's standard deviation smaller, not greater.

B Equal means say nothing about spread. Two data sets can share a center while one is far more spread out than the other.

C The range uses only the two extreme values. Standard deviation depends on every value, and Group B has many more ratings at the extremes.

Calculator option: Enter each group's 20 ratings as a list and apply the sample standard deviation function: Group A gives about 0.86 and Group B about 1.69. The population version gives about 0.84 and 1.64 and the same comparison.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

11. Correct answer: 14 M-C2411

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Medium

The predicted number comes from the line: y = 6(85) − 310 = 510 − 310 = 200 visitors. The actual number was 214, so actual minus predicted is 214 − 200 = 14. The positive result means the data point lies above the line. A common wrong answer is −14, which subtracts in the reverse order.

1. Predicted visitors: y = 6(85) − 310 = 200.
2. Actual minus predicted: 214 − 200 = 14.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

12. Correct answer: A M-C2412

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Harder

A *The interquartile range of Town Q's temperatures is greater than that of Town P's temperatures.*

Use only what a box plot shows: the minimum, quartiles, median and maximum. Town Q's IQR is 80 − 56 = 24 degrees and Town P's is 72 − 60 = 12 degrees, so choice A must be true. The means are not shown, and choices C and D contradict the plots.

1. Town P: IQR = 72 − 60 = 12; range = 92 − 48 = 44.
2. Town Q: IQR = 80 − 56 = 24; range = 84 − 52 = 32.
3. Means are not shown, and Town Q's median of 64 rules out choice C, so only choice A must be true.

B A box plot does not show the mean. Town P's higher median does not guarantee a higher mean, because the mean depends on every value.

C Town Q's median is 64°F, so no more than half of its temperatures are above 64°F. More than half cannot be above 68°F.

D Town Q's range is 84 − 52 = 32 degrees, while Town P's range is 92 − 48 = 44 degrees. Town P has the greater range.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

13. Correct answer: B M-C2413

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Harder

B *60*

Two predictions from the same line differ only through the slope term. The slope 0.12 means 0.12 centimeter more predicted snowfall per meter of elevation, so 500 more meters gives 0.12 × 500 = 60 centimeters more. The intercept appears in both predictions and cancels: [0.12(x + 500) − 30] − [0.12x − 30] = 60.

1. Let the lower station have elevation x; the other has elevation x + 500.
2. Difference in predictions: [0.12(x + 500) − 30] − [0.12x − 30] = 0.12 × 500.
3. 0.12 × 500 = 60 centimeters.

A 30 is the predicted snowfall at an elevation of exactly 500 meters, 0.12(500) − 30. The question asks for a difference between two stations.

C 90 subtracts the intercept, 60 − (−30). The intercept is the same for both stations, so it cancels out of the difference.

D 4,167 comes from 500 ÷ 0.12, dividing by the slope instead of multiplying. The slope is centimeters per meter, so multiply by meters.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

14. Correct answer: D M-C2414

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Harder

D *Model E, because for each of x = 1 through x = 4, its prediction is closer to the actual number than Model L's prediction is.*

Compare each model with the table. Model L predicts 63, 76, 89 and 102, which miss by 2, 3, 2 and 2 players. Model E predicts 60, 72, 86.4 and 103.68, which miss by 1, 1, 0.6 and 0.32. Model E is closer every year, which fits the growing yearly increases of 11, 12, 14 and 17.

1. Model L at x = 1, 2, 3, 4: 63, 76, 89, 102; differences from the data: 2, 3, 2, 2.
2. Model E at x = 1, 2, 3, 4: 60, 72, 86.4, 103.68; differences: 1, 1, 0.6, 0.32.
3. Model E is closer at every x, so it fits better.

A Both models increase every year, so rising data cannot decide between them. The comparison must use how close each prediction is to the data.

B This names the better model for an invalid reason. Many increasing data sets are linear; only comparing predictions with the data settles which fits here.

C The yearly increases are 11, 12, 14 and 17, which grow rather than stay about the same. Constant increases would favor a linear model.

Calculator option: Enter the table, then graph both models on the same screen; Model E's curve passes visibly closer to the four later points.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

15. Correct answer: C M-C2415

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Harder

C $y = 120(0.95)^x$

The values fall by a nearly constant factor, 93/120 ≈ 72/93 ≈ 0.77, for each 5-minute step, so an exponential decay model fits. A factor of 0.77 every 5 minutes means a factor b each minute with $b^5 \approx 0.77$, so $b \approx 0.95$. Check: $120(0.95)^{10} \approx 72$, matching the data.

1. Ratios of consecutive values: 93/120 ≈ 0.775 and 72/93 ≈ 0.774, so each 5 minutes multiplies y by about 0.77.
2. Per minute, $b^5 \approx 0.77$, so $b \approx 0.95$.
3. The initial value is 120, giving $y = 120(0.95)^x$.

A This linear model predicts 73.5 at x = 15, far above the data value of 56. The points curve, so the linear model fits poorly.

B 0.77 is the factor for each 5-minute step, not for each minute. With b = 0.77 per minute, the model predicts only about 33 at x = 5.

D A base greater than 1 produces exponential growth. The temperature difference is decreasing, so the base must be between 0 and 1.

Calculator option: Enter the points in a table and run an exponential regression of the form $y = ab^x$; it returns $a \approx 120$ and $b \approx 0.95$.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

CHAPTER 25

Probability, Sampling, Inference and Evaluating Claims

This chapter covers the last three Problem-Solving and Data Analysis skills: Probability and conditional probability; Inference from sample statistics and margin of error; and Evaluating statistical claims: observational studies and experiments. They need less arithmetic than most Math questions and more careful reading.

Two questions settle most of these items. For probability, ask: **which group am I choosing from?** That group's total is the denominator. For studies and surveys, ask: **how were the subjects chosen, and how were they assigned to groups?** The answers tell you which conclusions the data can support.

What the questions look like

- A probability from counts, such as "If one of these students is selected at random, what is the probability that…"
- A conditional probability from a **two-way table**, a table that sorts the same people or things by two categories at once, with row and column totals.
- An estimate for a whole population from a random sample, such as the number of students at a school who would answer yes.
- An interpretation of a **margin of error**, the amount added to and subtracted from a sample estimate to give a range of plausible values for the population.
- A question about how sample size affects the margin of error.
- A choice of the conclusion a study supports: cause or only association, and for which group of people.

How to recognize them

A **population** is the whole group a question is about, such as all students in a district. A **sample** is the part of the population that was actually measured. A **random sample** is chosen by a chance process from a defined population. Random sampling helps avoid selection bias, but a particular sample can still differ from the population.

Signal in the question	What it usually means
"selected at random" with counts or a table	Probability = favorable outcomes ÷ total outcomes
"given that," "if one of the … is selected," "of those who …"	Conditional probability. The denominator is the restricted group.
"random sample," "based on the sample," "best estimate"	Apply the sample proportion or mean to the population
"margin of error," "plausible," "most appropriate conclusion"	Estimate ± margin gives plausible population values
"randomly assigned," "treatment," "control group"	A randomized experiment can support a causal conclusion.

Signal in the question	What it usually means
"surveyed," "observed," "recorded," "compared people who …"	Observational study. Association only.

The method

Probability

1. **Name the group you are choosing from.** Read the words after "given that," "of the" or "if one of the." That group's total is the denominator.
2. **Count the favorable outcomes inside that group.** That count is the numerator.
3. **Simplify at the end** and compare with the choices. Fractions such as 12/50 often appear reduced, as 6/25.
4. For "A or B" when A and B cannot both happen, add their counts.

Inference from a sample

1. Check that the sample was random, and name the population it came from. Conclusions extend only to that population.
2. Compute the sample proportion or sample mean.
3. To estimate a count, multiply the sample proportion by the population size.
4. With a margin of error, write the plausible range: estimate − margin to estimate + margin. The range describes the population value, not the sample and not individual people.
5. With the same confidence level and sampling method and similar variability, larger random samples generally have smaller margins of error. A sample 4 times as large has a margin about half as large.

TRAP A large sample does not fix a biased one. A survey of 5,000 volunteers who chose to respond still may not represent the population.

Two kinds of biased samples come up often. A **convenience sample** uses whoever is easy to reach, such as students in one class or shoppers at one store. A **voluntary response sample** uses only people who choose to answer, such as an online poll. Both tend to overrepresent people with a particular trait or a strong opinion, so neither supports a conclusion about the whole population.

NOTE If a question mentions a confidence level, such as 95%, you usually do not need to calculate with it. Read the estimate plus or minus the margin of error as the range of plausible values for the population.

Evaluating a study: two questions

Ask how subjects were **selected** (chosen from a population) and how they were **assigned** (placed into treatment groups). Random selection supports generalizing to the population. Random assignment supports causal inference when a properly conducted experiment finds a difference that is unlikely to be due to chance alone, called a statistically significant difference.

How the study was designed	Random assignment to groups	No random assignment
Random selection from a population	Causal evidence that can extend to the population sampled	Association only, for the population sampled

How the study was designed	Random assignment to groups	No random assignment
No random selection	Causal evidence for those studied; wider use requires similar subjects and conditions	Association only, for the subjects studied

An **experiment** assigns treatments. An **observational study** only records what people already do, so other differences between the groups, called **confounding variables**, could explain any pattern.

Worked examples

Example 1: Probability from counts

A playlist has 30 songs: 12 pop, 10 rock and 8 jazz. If one song is chosen at random, what is the probability that it is not pop?

Thinking. You are choosing from all 30 songs, so the denominator is 30. Songs that are not pop: 10 + 8 = 18. The probability is 18/30 = 3/5.

Why the traps fail. The answer 2/3 treats the three genres as equally likely. The answer 2/5 finds the probability of pop itself, the opposite event.

Example 2: Three questions, three denominators

A hypothetical community center sorted 150 members by age group and favorite activity.

Age group	Swimming	Fitness classes	Total
Under 30	45	15	60
30 and over	35	55	90
Total	80	70	150

(a) If a member who prefers swimming is chosen at random, what is the probability that the member is under 30? The group is the 80 swimmers, so the answer is 45/80 = 9/16.

(b) If a member under 30 is chosen at random, what is the probability that the member prefers swimming? The group is the 60 members under 30, so the answer is 45/60 = 3/4.

(c) If any member is chosen at random, what is the probability that the member is under 30 and prefers swimming? The group is all 150 members, so the answer is 45/150 = 3/10.

The lesson. The numerator is 45 every time. Only the denominator changes, and the wording decides it.

Example 3: From a sample to a range of plausible values

A random sample of 580 residents of a hypothetical town of 12,000 found that 45% support building a skate park. The margin of error is 4 percentage points.

Thinking. The plausible range for the town is 45% − 4% = 41% to 45% + 4% = 49%. In residents, that is 0.41 × 12,000 = 4,920 to 0.49 × 12,000 = 5,880.

What you can say. It is plausible that between 41% and 49% of all residents support the park. Because every plausible value is below 50%, the estimate does not support a claim that a majority of residents favor the park.

What you cannot say. That exactly 45% of residents support it, or that every sample of 580 residents would land between 41% and 49%.

Example 4: What a larger sample changes

Two random samples are taken from the same population of students: one with 100 students and one with 900. Each estimates the mean number of hours students sleep on school nights.

Thinking. At the same confidence level, with the same sampling method and similar variability, the larger sample is expected to have the smaller margin of error. Because 900 is 9 times 100, its margin is roughly one third as large, since $\sqrt{9} = 3$. Neither estimate is guaranteed to equal the true mean; the larger sample simply narrows the plausible range.

What barely matters. The size of the population, as long as it is much larger than the sample.

Example 5: Experiment or observational study?

Study A. Eighty student volunteers were randomly assigned either to study vocabulary with flashcards or to reread a word list. The flashcard group scored higher on a later quiz, on average, and the difference was statistically significant.

Study B. A random sample of 300 students at a school was asked how they study. Students who said they use flashcards had higher average grades.

Thinking. Study A combined random assignment with a statistically significant difference, supporting a conclusion that flashcards increased quiz scores under the study conditions. The volunteers were not randomly selected, so the conclusion applies to students like these volunteers. Study B used random selection but no assignment, so it shows an association that extends to the school. Cause cannot be concluded: students who choose flashcards may also study longer or be more motivated.

Example 6: Harder-module example: percents instead of counts

At a hypothetical school, 60% of students ride the bus. Of the bus riders, 25% are in grade 9. Of the students who do not ride the bus, 40% are in grade 9. If a grade 9 student is selected at random, what is the probability that the student rides the bus?

Thinking. No counts are given, so invent a convenient total, such as 1,000 students. Then 600 ride the bus and 400 do not. Grade 9 bus riders: $0.25 \times 600 = 150$. Grade 9 non-riders: $0.40 \times 400 = 160$. There are $150 + 160 = 310$ grade 9 students in all.

Answer. The condition is "a grade 9 student," so the denominator is 310. The probability is $150/310 = 15/31$, or about 0.48.

Why the trap fails. The tempting answer 0.25 is the probability that a bus rider is in grade 9. It reverses the condition.

Common traps

Trap	How to spot it
Using the grand total for a conditional probability	The question restricts the group with "given that" or "of the."
Reversing the condition	Check which group follows "given that"; that group is the denominator.

Trap	How to spot it
Treating categories as equally likely	The categories have different counts, so 1/3 or 1/2 is rarely right.
Reading the sample value as the population value	The answer says "exactly" or ignores the margin of error.
Applying the margin of error to individuals	The range describes a mean or proportion, not single people or days.
Generalizing beyond the population sampled	The conclusion names a larger group, such as "all teenagers in the country."
Claiming cause without random assignment	The study only surveyed or observed.
Thinking volunteers rule out cause	Random assignment still supports cause for subjects like the volunteers.

Harder-module variations

- **An unknown in a two-way table.** If a cell is x and a conditional probability is given, write the probability with x in the numerator and in the denominator total, set it equal to the given value, and solve.
- **Two estimates with margins of error.** Write both plausible ranges and check which values they share. Overlap alone does not establish whether a difference is statistically significant; that requires a comparison of the difference itself.
- **Which change tends to shrink the margin of error?** A larger random sample from the same population, keeping the confidence level and method the same. Surveying a different group, rewording the question or reporting more decimal places does not make the estimate more precise.
- **The greatest or least plausible count.** Use the top or bottom of the plausible range, then multiply by the population size.
- **Studies with both features, or neither.** Use the two-question table: selection decides how far the result extends, and assignment decides whether cause can be claimed.
- **Expected counts.** If the probability of an outcome is p and there are n trials, about p × n outcomes are expected.

CHECK Before choosing a conclusion about a study, say aloud who was studied and how they were placed into groups. Most wrong choices claim too much on one of those two points.

Check yourself

1. In "the probability that a student is a senior, given that the student plays chess," which group's total is the denominator?
2. An estimate is 30% with a margin of error of 4 percentage points. What is the plausible range?
3. Which design feature allows a conclusion about cause?
4. Which design feature allows a result to be generalized to a larger population?
5. If you want a smaller margin of error, what should you change about a random sample?

6. Answers: (1) The students who play chess. (2) 26% to 34%. (3) Random assignment to treatments. (4) Random selection from that population. (5) Make the sample larger, keeping it random.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

Probability and Conditional Probability
skillbyskillprep.com/skills/probability

Sample Statistics and Margin of Error
skillbyskillprep.com/skills/sample-statistics-margin-of-error

Evaluating Statistical Claims
skillbyskillprep.com/skills/evaluating-statistical-claims

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2501

In a hypothetical survey, 50 students named their favorite sport to watch: 18 chose soccer, 12 chose basketball and 20 chose another sport. If one of these students is selected at random, what is the probability that the student chose basketball?

A 6/25 **B** 6/19 **C** 1/3 **D** 2/5

2 M-C2502

A hypothetical school has 2,400 students. A random sample of 150 of these students was asked whether they would use a proposed after-school study room, and 54 said yes. Based on the sample, which of the following is the best estimate of the number of students at the school who would say yes?

A 36 **B** 54 **C** 864 **D** 1,536

3 M-C2503

Preferred Study Time by Grade (hypothetical data)

Grade	Morning	Afternoon	Total
Grade 10	24	36	60
Grade 11	30	20	50
Total	54	56	110

The table shows the results of a hypothetical survey that asked 110 students whether they prefer to study in the morning or the afternoon. If one of the grade 11 students is selected at random, what is the probability that the student prefers to study in the morning?

A 3/11 **B** 27/55 **C** 5/9 **D** 3/5

4 M-C2504

A hypothetical weather station recorded the sky condition at noon on each of 40 days. It was sunny on 14 days, partly cloudy on 16 days and overcast on 10 days. If one of these days is selected at random, what is the probability that the sky was either sunny or overcast?

A 1/4 **B** 7/20 **C** 3/5 **D** 2/3

5 M-C2505

Rain Forecasts and Outcomes (hypothetical data)

Forecast	Rain occurred	No rain	Total
Rain forecast	42	18	60
No rain forecast	12	108	120
Total	54	126	180

The table summarizes a hypothetical weather station's forecasts and outcomes for 180 days. If one of the days on which rain occurred is selected at random, what is the probability that rain had been forecast for that day?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

6 M-C2506

Sport and Practice Time (hypothetical data)

Practice time	Track	Swimming	Tennis	Total
Morning	34	46	30	110
Evening	26	24	40	90
Total	60	70	70	200

The table shows the sport and the usual practice time for 200 athletes at a hypothetical sports club. If an athlete who practices in the evening is selected at random, what is the probability that the athlete is a swimmer?

A 3/25 **B** 4/15 **C** 12/35 **D** 7/20

7 M-C2507

A hypothetical survey of a random sample of 250 students from a large school district found that 62% of the sampled students get at least 8 hours of sleep on school nights. The margin of error for this estimate is 6 percentage points. Which conclusion is most appropriate?

A Exactly 62% of all students in the district get at least 8 hours of sleep on school nights.

B The margin of error means that the actual percentage among the 250 sampled students could be any value from 56% to 68%.

C Every random sample of 250 students from the district will show between 56% and 68% getting at least 8 hours of sleep.

D It is plausible that between 56% and 68% of all students in the district get at least 8 hours of sleep on school nights.

8 M-C2508

Two hypothetical surveys estimated the mean number of minutes per day that students at a large high school spend at sports practice. Survey 1 used a random sample of 80 students, and Survey 2 used a random sample of 600 students from the same school. Both used the same sampling method and confidence level. Which statement is most likely true?

A Survey 2's estimate has a smaller margin of error than Survey 1's estimate.

B Survey 1's estimate has a smaller margin of error, because a smaller sample has less variation.

C Both estimates have the same margin of error, because both samples came from the same school.

D Survey 2's estimate must equal the true mean for the school, because its sample is large.

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9 M-C2509

In a hypothetical study, 60 volunteers from a running club were randomly assigned to one of two warm-up routines before a 100-meter sprint. The dynamic-warm-up group had a faster mean sprint time than the static-stretching group, and the difference was statistically significant. Which conclusion is best supported by the study?

A The dynamic warm-up causes faster sprint times for all high school and adult runners.

B The dynamic warm-up likely causes faster sprint times for runners similar to the volunteers in the study.

C No conclusion about cause is possible, because the runners were volunteers rather than a random sample.

D The study shows only an association, because the researchers did not select the runners at random.

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10 M-C2510

A hypothetical school district has 15,000 students. A random sample of 400 of these students found that 38% read at least one book for pleasure each month, with a margin of error of 5 percentage points. Based on these results, what is the greatest plausible number of students in the district who read at least one book for pleasure each month?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

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11 M-C2511

To learn whether students at a hypothetical high school support adding a new sport, a student reporter surveyed 120 students as they arrived at a basketball game. Of those surveyed, 80% supported adding the new sport. Which statement about the survey is most appropriate?

A The results show that 80% of all students at the school support adding the new sport, because 120 students is a large sample.

B The results show that attending basketball games causes students to support adding new sports.

C The results can be generalized to all high school students in the state, because the survey asked a direct question.

D The results may not represent all students at the school, because the students surveyed were not randomly selected and may be more interested in sports.

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12 M-C2512

Instrument Survey (hypothetical data)

Grade	Plays an instrument	Does not play	Total
Grade 9	x	70 − x	70
Grade 12	27	63	90
Total	x + 27	133 − x	160

The table summarizes a hypothetical survey of 160 students in grades 9 and 12 about whether they play a musical instrument. If a student who plays an instrument is selected at random, the probability that the student is in grade 9 is 5/8. What is the value of x?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

13 M-C2513

At two hypothetical weather stations, random samples of days were used to estimate the mean daily wind speed. Station 1's estimate was 14.2 miles per hour with a margin of error of 1.5 miles per hour. Station 2's estimate was 12.9 miles per hour with a margin of error of 1.1 miles per hour. Which conclusion is best supported by these estimates?

A The plausible ranges overlap, and a value such as 13.5 miles per hour lies in both ranges.

B Station 1's mean daily wind speed is greater than Station 2's by exactly 1.3 miles per hour.

C Station 1's mean daily wind speed is greater than Station 2's, because 14.2 is greater than 12.9.

D Every day at Station 1 had a wind speed between 12.7 and 15.7 miles per hour.

14 M-C2514

A hypothetical school district selected 200 students at random from its 9,000 students, then randomly assigned them either to take a weekly recipe-based cooking class or to take no class. After 8 weeks, the class group reported eating more servings of vegetables per day, on average, and the difference was statistically significant. Which conclusion is most appropriate?

A The cooking class causes increased vegetable eating for all teenagers in the country.

B The cooking class is associated with increased vegetable eating, but cause cannot be determined, because only 200 of the 9,000 students took part.

C The cooking class likely causes increased reported vegetable eating among students in the district.

D The results apply only to the 200 students in the study, because the other 8,800 students did not take part.

15 M-C2515

A hypothetical study selected 600 adults at random from a city and asked whether they had played on a sports team in high school and how many hours they currently exercise each week. Adults who had played on a high school team reported exercising more hours per week, on average, than adults who had not. Which conclusion is best supported?

A Playing on a high school sports team causes adults in the city to exercise more.

B Among adults in the city, having played on a high school sports team is associated with exercising more hours per week.

C The association applies only to the 600 adults who were surveyed and cannot be extended to the city.

D No conclusion of any kind is possible, because the adults were not assigned to play on high school teams.

Practice Set Explanations

1. Correct answer: A M-C2501

Probability and conditional probability · Problem-Solving and Data Analysis · Easier

A *6/25*

Probability from counts is the number of favorable outcomes divided by the total number of equally likely outcomes. Twelve of the 50 students chose basketball, so the probability is 12/50, which reduces to 6/25, or 0.24.

1. Favorable outcomes: 12 basketball choices. Total outcomes: 50 students.

2. Probability: 12/50 = 6/25.

B 6/19 equals 12/38, which compares basketball fans with the 38 students who chose something else. The denominator must be all 50 students.

C 1/3 treats the three categories as equally likely. The categories have different numbers of students, so each probability depends on its count.

D 2/5 equals 20/50, the probability that a student chose another sport. It answers a question about a different category.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

2. Correct answer: C M-C2502

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Easier

C *864*

A random sample supports an estimate for the population it came from. The sample proportion is 54/150 = 0.36. Applying it to the whole school gives 0.36 × 2,400 = 864 students. Because the sample was random, 36% is a reasonable estimate for all 2,400 students.

1. Sample proportion: p = 54/150 = 0.36.

2. Estimate for the school: 0.36 × 2,400 = 864 students.

A 36 is the sample percent, 36%, written as a count. The estimate must apply that percent to the 2,400 students at the school.

B 54 is the number of yes answers in the sample only. The school is 16 times as large as the sample, so the estimate must be larger.

D 1,536 is 64% of 2,400, the estimated number of students who would say no. It answers the opposite question.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

3. Correct answer: D M-C2503

Probability and conditional probability · Problem-Solving and Data Analysis · Easier

D *3/5*

The phrase "one of the grade 11 students" sets the denominator: only the 50 grade 11 students can be selected. Of those 50, 30 prefer to study in the morning, so the probability is 30/50, which reduces to 3/5.

1. Condition: grade 11, so the denominator is 50.

2. Grade 11 students who prefer morning: 30.

3. Probability: 30/50 = 3/5.

A 3/11 equals 30/110, which divides by all 110 students. The condition limits the selection to the 50 grade 11 students.

B 27/55 equals 54/110, the probability that any surveyed student prefers morning. It ignores the condition about grade 11.

C 5/9 equals 30/54, the probability that a student who prefers morning is in grade 11. The condition and the outcome are reversed.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

4. Correct answer: C M-C2504

Probability and conditional probability · Problem-Solving and Data Analysis · Easier

C *3/5*

A day cannot be both sunny and overcast, so add the two counts: 14 + 10 = 24 favorable days out of 40. The probability is 24/40, which reduces to 3/5. Check: the other 16 days give 16/40 = 2/5, and 3/5 + 2/5 = 1.

1. Favorable days: 14 sunny + 10 overcast = 24.

2. Probability: 24/40 = 3/5.

A 1/4 equals 10/40, the probability of an overcast day alone. The question asks for sunny or overcast, so both counts belong in the numerator.

B 7/20 equals 14/40, the probability of a sunny day alone. The overcast days must be added to the favorable outcomes.

D 2/3 treats the three sky conditions as equally likely and picks 2 of 3. The counts differ, so use 24 of the 40 days.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

5. Correct answer: 7/9 or .7777 or .7778 or 0.777 (other equivalent forms accepted) M-C2505

Probability and conditional probability · Problem-Solving and Data Analysis · Medium

The condition "one of the days on which rain occurred" makes the denominator the 54 rainy days. Rain had been forecast on 42 of them, so the probability is 42/54 = 7/9. A common wrong answer is 7/10, which is 42/60: it divides by the days with a rain forecast and reverses the condition.

1. Condition: rain occurred, so the denominator is 54.

2. Rainy days that had a rain forecast: 42.

3. Probability: 42/54 = 7/9.

Calculator option: If you prefer a decimal, compute 42/54 = 0.7777... and enter .7777 or .7778 so that the entry fills the space; 7/9 is the simplest entry.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

6. Correct answer: B M-C2506

Probability and conditional probability · Problem-Solving and Data Analysis · Medium

B *4/15*

The condition "an athlete who practices in the evening" limits the denominator to the 90 evening athletes. Of those 90, 24 are swimmers, so the probability is 24/90, which reduces to 4/15, or about 0.27.

1. Condition: evening practice, so the denominator is 90.

2. Evening athletes who are swimmers: 24.

3. Probability: 24/90 = 4/15.

A 3/25 equals 24/200, which divides by every athlete in the club. The condition restricts the selection to the 90 evening athletes.

C 12/35 equals 24/70, the probability that a swimmer practices in the evening. The roles of the condition and the outcome are reversed.

D 7/20 equals 70/200, the probability that any athlete in the club is a swimmer. It ignores the condition about evening practice.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

7. Correct answer: D M-C2507

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Medium

D *It is plausible that between 56% and 68% of all students in the district get at least 8 hours of sleep on school nights.*

A margin of error turns a sample estimate into a range of plausible values for the population. Here 62% − 6% = 56% and 62% + 6% = 68%, so it is plausible that between 56% and 68% of all students in the district get at least 8 hours of sleep on school nights.

1. Lower end: 62% − 6% = 56%.

2. Upper end: 62% + 6% = 68%.

3. The range gives plausible values for all students in the district, the population that was sampled.

A A sample estimate is not exact for the population. The margin of error is reported because the district's true percent may differ from 62%.

B The sample's value is known exactly: 62% of the 250 students. The plausible range describes the whole district, not the sample.

C Different random samples vary, and some could fall outside the range. The margin of error describes the population value, not every future sample.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

8. Correct answer: A M-C2508

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Medium

A *Survey 2's estimate has a smaller margin of error than Survey 1's estimate.*

For random samples from the same population, using the same method and confidence level, a larger sample generally gives a more precise estimate. Survey 2 used 600 students instead of 80, so its estimate most likely has a smaller margin of error. This is an expected tendency, not a guarantee for every realized pair of samples.

1. Both samples are random and use the same population, sampling method and confidence level.

2. A larger sample generally gives a smaller margin of error, so Survey 2 (n = 600) is likely more precise than Survey 1 (n = 80).

B A smaller sample's estimate varies more from sample to sample, not less. That makes it less precise, so its margin of error tends to be larger.

C Margin of error depends on sample size, not only on which population was sampled. These two sample sizes differ greatly.

D A larger random sample narrows the margin of error, but no sample guarantees that its estimate equals the population mean exactly.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

9. Correct answer: B M-C2509

Evaluating statistical claims: observational studies and experiments · Problem-Solving and Data Analysis · Medium

B *The dynamic warm-up likely causes faster sprint times for runners similar to the volunteers in the study.*

Random assignment tends to balance other differences between the groups. The statistically significant difference therefore supports an effect of the warm-up under the study conditions. Because the participants were volunteers rather than a random sample of all runners, applying the result beyond them requires similar runners and conditions.

1. Random assignment and a statistically significant difference support a causal effect of the warm-up under the study conditions.

2. There was no random selection from all runners, so broader application requires runners and conditions similar to those studied.

A Without random selection from a larger population, the result cannot be generalized to all runners. The club volunteers may differ from runners in general.

C Random assignment, not random selection, is what supports a conclusion about cause. Using volunteers limits how widely the result applies, not whether cause can be inferred.

D Because treatments were randomly assigned, this is an experiment, not an observational study. A cause-and-effect conclusion is appropriate for runners like these.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

10. Correct answer: 6450 M-C2510

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Medium

The plausible range for the district's percent is 38% ± 5%, or 33% to 43%. The greatest plausible percent is 43%, and 0.43 × 15,000 = 6,450 students. Common wrong answers are 5,700, the point estimate 0.38 × 15,000, and 4,950, which uses the lower end of the range.

1. Upper end of the plausible range: 38% + 5% = 43%.
2. Apply it to the district: 0.43 × 15,000 = 6,450 students.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

11. Correct answer: D M-C2511

Evaluating statistical claims: observational studies and experiments · Problem-Solving and Data Analysis · Medium

D *The results may not represent all students at the school, because the students surveyed were not randomly selected and may be more interested in sports.*

To generalize from a sample to a population, the sample must be randomly selected from that population. Students arriving at a basketball game form a convenience sample and probably care more about sports than students in general, so the 80% may overstate support across the school.

1. Ask how the sample was chosen: students arriving at a basketball game, not a random sample of the school.
2. Without random selection, the 80% cannot be generalized to all students at the school.

A A large sample does not fix a biased selection method. Without random selection, even 120 responses may not reflect the whole school.

B A survey that assigns no treatment cannot show cause. The results describe the opinions of the students surveyed, not an effect of attending games.

C The sample was not randomly chosen even from this one school, so the results cannot extend to the state, a much larger population.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

12. Correct answer: 45 M-C2512

Probability and conditional probability · Problem-Solving and Data Analysis · Harder

The condition "a student who plays an instrument" makes the denominator x + 27, the total number of players. So x/(x + 27) = 5/8. Cross-multiplying gives 8x = 5x + 135, so 3x = 135 and x = 45. Check: 45/72 = 5/8. A common wrong answer is 43.75, which takes 5/8 of the 70 grade 9 students and uses the wrong denominator.

1. Students who play an instrument: x + 27, of whom x are in grade 9.
2. Set up the conditional probability: x/(x + 27) = 5/8.
3. Cross-multiply: 8x = 5x + 135, so 3x = 135 and x = 45.

Calculator option: Graph y = x/(x + 27) and y = 5/8; the graphs intersect at x = 45.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

13. Correct answer: A M-C2513

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Harder

A *The plausible ranges overlap, and a value such as 13.5 miles per hour lies in both ranges.*

Station 1's plausible range is 14.2 ± 1.5, or 12.7 to 15.7 miles per hour. Station 2's is 12.9 ± 1.1, or 11.8 to 14.0. They overlap from 12.7 to 14.0, so 13.5 is in both ranges. This establishes overlap, not equality of the means or the outcome of a formal significance test.

1. Station 1: from 14.2 − 1.5 = 12.7 to 14.2 + 1.5 = 15.7.
2. Station 2: from 12.9 − 1.1 = 11.8 to 12.9 + 1.1 = 14.0.
3. The intersection is 12.7 to 14.0, which contains 13.5; choice A describes this overlap.

B 1.3 is the difference between the two sample estimates. Each population mean is uncertain, so the true difference is not known exactly.

C The sample estimates alone do not establish the population ordering. A comparison must account for uncertainty; a higher point estimate is not a guarantee.

D A margin of error describes plausible values of the mean, not the range of individual days. Single days can be far above or below the mean.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

14. Correct answer: C M-C2514

Evaluating statistical claims: observational studies and experiments · Problem-Solving and Data Analysis · Harder

C *The cooking class likely causes increased reported vegetable eating among students in the district.*

Random assignment, together with the statistically significant difference, supports a causal effect on reported vegetable eating under the study conditions. Random selection from the district lets the result extend to its students. The conclusion concerns reported eating and this district, not all teenagers or independently measured consumption.

1. Random selection from the district supports generalization to the district's students.
2. Random assignment and a statistically significant difference support a causal effect under the study conditions.
3. The measured outcome is reported vegetable eating; choice C preserves that limit.

A The sample came from one district, so the conclusion extends only to that district's students, not to all teenagers in the country.

B Random assignment, not the share of students who took part, determines whether cause can be inferred. A random sample of 200 can represent the district.

D Random selection from all 9,000 students lets the results represent the district. The conclusion need not be limited to the participants.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

15. Correct answer: B M-C2515

Evaluating statistical claims: observational studies and experiments · Problem-Solving and Data Analysis · Harder

B *Among adults in the city, having played on a high school sports team is associated with exercising more hours per week.*

The adults were randomly selected but not assigned to play on teams, so this is an observational study. Random selection lets the association extend to adults in the city. Without random assignment, other factors, such as a lifelong enjoyment of sports, could explain the difference, so cause cannot be concluded.

1. Random selection from the city: the association can be generalized to the city's adults.
2. No random assignment: the study is observational, so cause cannot be concluded.

A No one was randomly assigned to play on a team, so other differences between the groups could explain the result. Cause cannot be concluded.

C The 600 adults were selected at random from the city, so the association found in the sample can be generalized to adults in the city.

D Without random assignment, cause cannot be shown, but an association can still be described and, because of random selection, generalized to the city.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

CHAPTER 26

Area, Volume, Lines, Angles and Triangles

Geometry questions on the digital SAT reward a small set of facts used precisely. This chapter covers composite areas, volumes, scaling, parallel lines cut by a transversal, triangle angle facts, and similar and congruent triangles. Most of these questions take one or two steps once you find the right relationship, so the method below is mainly about finding it quickly.

What the question looks like

Questions in this chapter usually take one of these forms:

- A figure with a few labeled lengths or angles and a request for one missing value, such as an angle marked x° or the area of an L-shaped region.
- A word problem about a physical object, such as a box, a can, a planter or a ramp, that asks for an area, a volume or a length.
- A scaling question: two similar figures, one k times as long as the other, and a question about perimeter, surface area or volume.
- A figure with parallel lines in which angles are written as expressions such as (3x + 10)°, and you solve for x or for an angle.

Some questions are multiple choice and some are student-produced response. Figures are drawn to scale unless the question says otherwise. When a figure is marked as not drawn to scale, trust the labels and stated facts, never the picture.

How to recognize it

Signal in the question	What it points to
An L-shape, a border, a path around a shape, or "the remaining region"	Composite area: add or subtract simple areas
"Holds," "capacity," "fill," or cubic units	Volume, usually with a reference-sheet formula
"Similar," "scale factor," or "k times as long"	Scaling: lengths by k, areas by k^2, volumes by k^3
Two lines stated to be parallel and crossed by a third line	Transversal angle pairs
One side of a triangle extended past a vertex	Exterior angle of a triangle
Two equal sides, such as "PQ = PR"	Isosceles triangle: the angles opposite those sides are equal
A segment inside a triangle drawn parallel to one side	Two similar triangles in one figure

The method

1. **Read the labels and the stated facts before computing.** Note what is given and what is asked. If the figure is not drawn to scale, do not estimate from it.
2. **Name the relationship.** Decide which fact connects the given values to the unknown: a formula, an angle pair, an angle sum, or a ratio between similar figures.
3. **Write one equation.** Set equal angles equal, set supplementary angles to a sum of 180, or write a proportion with matching sides in the same order.
4. **Solve, then answer the question that was asked.** Many questions want an angle after you find x, or a total after you find one part.
5. **Check units and size.** Area is in square units and volume is in cubic units. In a figure drawn to scale, your angle should look about right.

Area and volume

The Bluebook reference sheet gives every formula in this table. Your job is to choose the right one and combine shapes correctly.

Shape	Formula	Watch for
Rectangle	$A = \ell w$	Units: square units
Triangle	$A = (1/2)bh$	The height is perpendicular to the base
Circle	$A = \pi r^2$	Use the radius, not the diameter
Rectangular solid	$V = \ell wh$	Cubic units
Cylinder	$V = \pi r^2 h$	A "can" is a cylinder
Right circular cone and rectangular-base pyramid	$V = (1/3)\pi r^2 h$ and $V = (1/3)\ell wh$	Pointed solids need the 1/3
Sphere	$V = (4/3)\pi r^3$	A hemisphere is half of a sphere

For a composite figure, split it into pieces you know, then add the pieces or subtract a cutout from a larger shape. For a perimeter, trace only the outside edge.

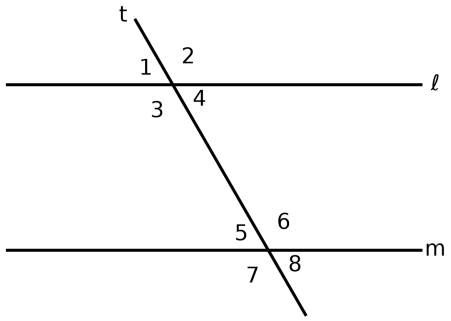
Scaling

If every length of a figure is multiplied by k, perimeters are multiplied by k, areas and surface areas by k^2, and volumes by k^3. Weight and capacity follow volume when the material is the same. You can also work backward: if two similar figures have areas in the ratio 4 : 9, their lengths are in the ratio 2 : 3.

Angles in triangles and polygons

- Angles that form a straight line add to 180°, and vertical angles are equal.
- The angles of a triangle add to 180°. This fact is on the reference sheet. A polygon with n sides splits into $n - 2$ triangles, so its angles add to $(n - 2) \times 180°$.
- An exterior angle of a triangle equals the sum of the two interior angles that are not next to it.
- In an isosceles triangle, the angles opposite the equal sides are equal. An equilateral triangle has three 60° angles.

Parallel lines and a transversal



A transversal crossing two parallel lines makes eight angles with two different measures, unless it is perpendicular to the lines, in which case all eight angles are 90°. In the figure, angles 1, 4, 5 and 8 are acute and equal, and angles 2, 3, 6 and 7 are obtuse and equal.

Pair	Example in the figure	Relationship
Corresponding	1 and 5; 2 and 6	Equal
Alternate interior	3 and 6; 4 and 5	Equal
Alternate exterior	1 and 8; 2 and 7	Equal
Same-side interior	3 and 5; 4 and 6	Add to 180°
Vertical	1 and 4; 2 and 3	Equal (true for any two crossing lines)

TIP Under time pressure you do not need the names. Two angles that are both acute or both obtuse are equal; an acute angle and an obtuse angle add to 180°. Use this shortcut for angles made by the same transversal crossing parallel lines. If the transversal is perpendicular to them, all the angles are 90°.

Similar and congruent triangles

Two triangles are similar when their angles match. Then every pair of matching sides has the same ratio. Two pairs of equal angles are enough, because the third angles must then match as well. A common setup is a segment drawn parallel to one side of a triangle, which cuts off a smaller triangle similar to the whole one.

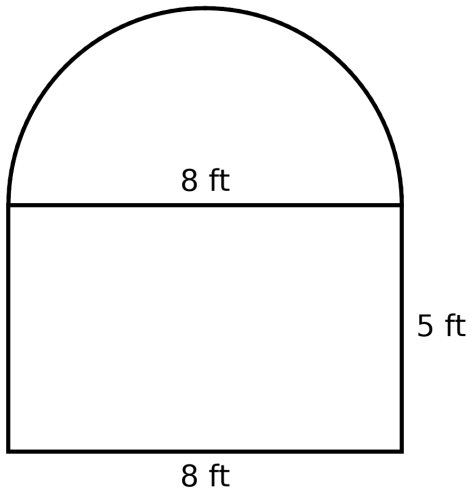
Congruent triangles have the same shape and the same size. These tests prove congruence:

Test	What must match
SSS	All three pairs of sides
SAS	Two sides and the angle between them
ASA	Two angles and the side between them
AAS	Two angles and a side that is not between them
HL	In right triangles, the hypotenuse and one leg

Matching angles alone (AAA) prove similarity but not congruence. Two sides and an angle that is not between them (SSA) do not prove congruence in general.

Worked examples

Example 1: A window with a rounded top



A window is shaped like a rectangle 8 feet wide and 5 feet tall with a semicircle on top, as shown. To the nearest square foot, what is the area of the glass?

Think: The shape splits into a rectangle and half of a circle. The semicircle's diameter is the top of the rectangle, 8 feet, so its radius is 4 feet.

Work: Rectangle: $8 \times 5 = 40$. Semicircle: $(1/2)\pi(4^2) = 8\pi \approx 25.13$. Total: $40 + 8\pi \approx 65.13$, or about 65 square feet.

Why the traps fail: Using 8 as the radius makes the half circle alone about 100.5 square feet. Forgetting the 1/2 adds about 25 extra square feet. Trim around the outside would be a perimeter question: $8 + 5 + 5 + 4\pi \approx 30.6$ feet, which counts the bottom, the two sides and half the circumference, but not the 8-foot diameter inside the window.

Example 2: A cone-shaped cup

A packaging supplier makes paper cups shaped like right circular cones. Each cup has a radius of 5 centimeters at the rim and a height of 12 centimeters. What is the volume of one cup, in cubic centimeters?

Think: The reference sheet gives $V = (1/3)\pi r^2 h$ for a cone.

Work: $V = (1/3)\pi(5^2)(12) = (1/3)(300\pi) = 100\pi$, or about 314.2 cubic centimeters.

Why the traps fail: Leaving out the 1/3 gives 300π, the volume of a cylinder with the same base and height. Using the diameter, 10, gives $(1/3)\pi(100)(12) = 400\pi$.

Example 3: Scaling a garden statue

Two solid garden statues made of the same stone are similar. The small one is 2 feet tall and the large one is 5 feet tall. The small statue needs 3 quarts of sealant to cover its surface and weighs 12 pounds. How much sealant does the large statue need, and how much does it weigh?

Think: The scale factor is $k = 5/2 = 2.5$. Sealant covers surface area, which scales by k^2. Weight follows volume, which scales by k^3.

Work: $k^2 = 6.25$, so the sealant is $3 \times 6.25 = 18.75$ quarts. $k^3 = 15.625$, so the weight is $12 \times 15.625 = 187.5$ pounds.

Why the traps fail: Multiplying both values by 2.5 gives 7.5 quarts and 30 pounds, which treats area and volume like length. Using k^3 for the sealant confuses a surface with a volume.

Example 4: Parallel lines with algebra

In the transversal figure above, lines ℓ and m are parallel, angle 4 measures $(2x + 4)°$, and angle 6 measures $(5x − 20)°$. What is the measure of angle 6?

Think: Angles 4 and 6 lie between the parallel lines on the same side of the transversal, so they are same-side interior angles. One is acute and one is obtuse, so they add to 180°.

Work: $(2x + 4) + (5x − 20) = 180$, so $7x − 16 = 180$, $7x = 196$ and $x = 28$. Angle 6 measures $5(28) − 20 = 120°$. Check: angle 4 measures $2(28) + 4 = 60°$, and $60 + 120 = 180$. The figure, drawn to scale, agrees.

Why the traps fail: Setting the expressions equal gives $x = 8$ and two 20° angles, which cannot be an acute angle and an obtuse angle. Stopping at $x = 28$ answers a question that was not asked.

Example 5: An isosceles triangle with an exterior angle

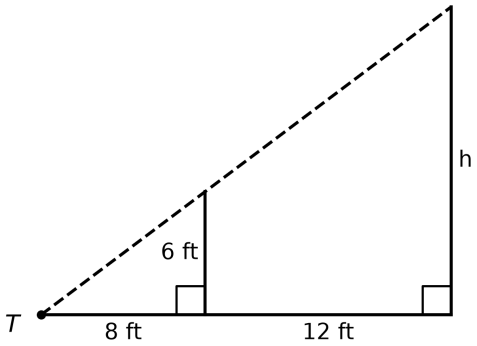
In triangle PQR, PQ = PR. Side QR is extended past R to point S, and angle PRS measures 110°. What is the measure of angle P?

Think: Angles PRQ and PRS form a straight line. Because PQ = PR, the angles opposite those sides, angle R and angle Q, are equal. The equal sides meet at P, so P is the vertex angle.

Work: Angle $PRQ = 180 − 110 = 70°$. Angle Q is also 70°, so angle $P = 180 − 70 − 70 = 40°$. The exterior angle fact confirms it: $110 = 40 + 70$.

Why the traps fail: Treating P as one of the equal angles gives 70°. Using 110° as an interior angle of the triangle makes two angles total 220°, which is impossible.

Example 6: A lamppost and a shadow



A person 6 feet tall stands on level ground 12 feet from the base of a lamppost. The person's shadow is 8 feet long and ends at point T. How tall is the lamppost?

Think: The light ray from the top of the lamppost passes the top of the person's head and reaches the ground at T. The small triangle (person and shadow) and the large triangle (lamppost and ground) share the angle at T, and each has a right angle, so they are similar.

Work: The large triangle's base runs from T to the lamppost: 8 + 12 = 20 feet. Matching sides give h/20 = 6/8, so h = 20 × 0.75 = 15 feet.

Why the traps fail: Using 12 as the large base gives h = 9, which measures the lamppost's base from the person instead of from T. Measure both bases from the shared vertex.

Example 7 (harder module): Side lengths from areas

Triangles ABC and DEF are similar. The area of triangle ABC is 20 square units, and the area of triangle DEF is 45 square units. The shortest side of triangle ABC is 8 units long. How long is the shortest side of triangle DEF?

Think: Areas scale by k^2, so first find k^2, then take its square root.

Work: $k^2 = 45/20 = 9/4$, so $k = 3/2$. The shortest side of triangle DEF is $8 \times 3/2 = 12$ units.

Why the traps fail: Applying the area ratio to a length gives 8 × 9/4 = 18. Inverting the ratio gives about 5.33, which would make the larger triangle's side shorter than the smaller one's.

Common traps

Trap	How to spot it
Using the diameter as the radius	The problem gives a distance across a circle; halve it first
Leaving out 1/3 for a cone or pyramid	The solid comes to a point
Scaling area or volume by k	The question asks about a surface, paint, capacity or weight
Setting a supplementary pair equal	One angle is acute and the other is obtuse
Answering with x instead of the angle	The last sentence asks for "the measure of angle …"

Trap	How to spot it
Counting an inside edge in a perimeter	The segment lies inside the combined figure
Mixing up matching sides	Write similar triangles with matching vertices in the same order
Estimating from a figure not drawn to scale	A note under the figure says it is not to scale

Harder-module variations

- **Angle chains.** Several angles depend on x, and you need two facts in a row, such as a vertical pair and then a same-side interior pair. Fill in every angle you can before writing an equation.
- **A bend between parallel lines.** Draw a third parallel line through the bend and use alternate interior angles to label its parts. Then add or subtract the relevant angles to find the angle the question marks; do not assume every orientation uses the same sum.
- **Working back to k.** Given an area ratio, take a square root; given a volume ratio, take a cube root. Then apply k^2 or k^3 to the quantity the question wants.
- **Composite solids.** A tank may be a cylinder with a hemisphere or cone on top. Compute each part with its own formula and add. Check whether a stated height belongs to one part or the whole tank.
- **Answers with π.** A student-produced response cannot include π, so a question may ask for k when the answer is written as kπ. Keep π as a symbol, then divide it out at the end.

Check yourself

1. Which angle pairs formed by parallel lines and a transversal add to 180°?
2. A cylinder's radius is tripled and its height stays the same. By what factor does its volume change?
3. What does an exterior angle of a triangle equal?
4. Two similar solids have volumes in the ratio 8 : 27. What is the ratio of their surface areas?
5. Which test shows that two right triangles are congruent using only two sides?
6. Answers: (1) same-side interior angles, and in general any acute angle with any obtuse angle; (2) 9, because only the radius is squared; (3) the sum of the two interior angles not next to it; (4) 4 : 9, because k = 2/3; (5) HL, the hypotenuse and one leg.

ONLINE (OPTIONAL) The website has a page for each skill in this chapter, with two video lessons and a link to practice the skill with a digital study coach.

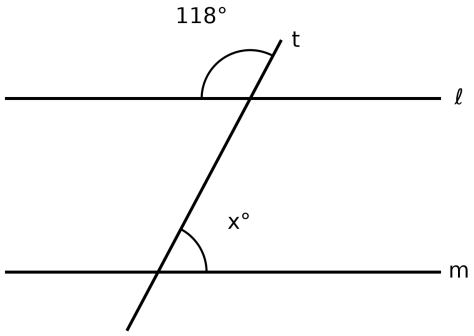
Area and Volume
skillbyskillprep.com/skills/area-and-volume

Lines, Angles, and Triangles
skillbyskillprep.com/skills/lines-angles-triangles

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

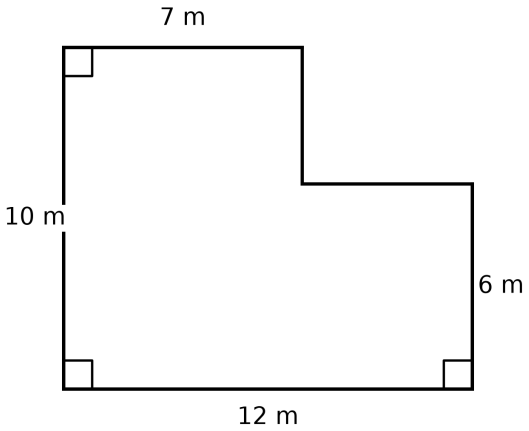
1 M-C2601



In the figure, line ℓ is parallel to line m, and line t intersects both lines. What is the value of x?

A 28 **B** 62 **C** 118 **D** 242

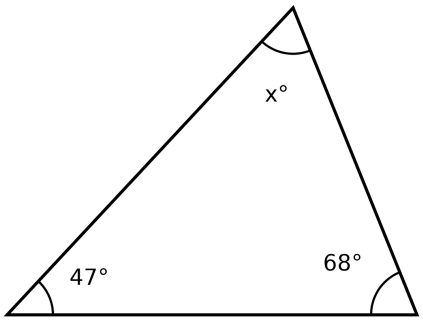
2 M-C2602



The figure shows the shape of a garden bed in which all angles are right angles. What is the area of the garden bed, in square meters?

A 44 **B** 72 **C** 78 **D** 100

3 M-C2603



In the triangle shown, what is the value of x?

A 65 **B** 112 **C** 115 **D** 245

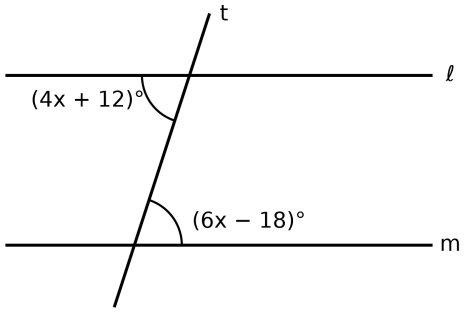
4 M-C2604

A shipping box is a rectangular prism with interior dimensions 30 centimeters by 20 centimeters by 15 centimeters. What is the interior volume of the box, in cubic centimeters?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

5 M-C2605



In the figure, line ℓ is parallel to line m, and line t intersects both lines. What is the measure, in degrees, of the angle labeled (4x + 12)°?

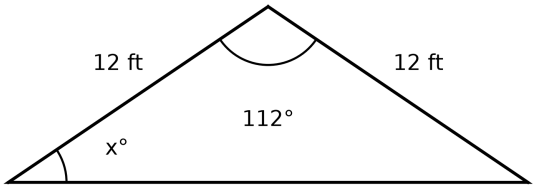
A 15 **B** 30 **C** 72 **D** 108

6 M-C2606

A company packages soup in cylindrical cans. Each can has an interior diameter of 8 centimeters and an interior height of 15 centimeters. What is the interior volume of one can, in cubic centimeters?

A 60π **B** 80π **C** 240π **D** 960π

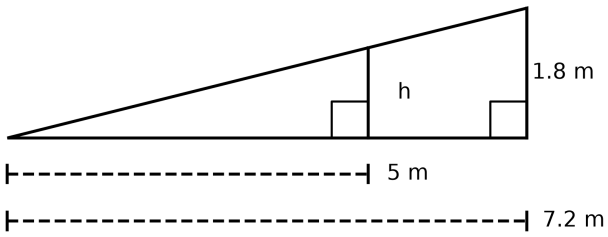
7 M-C2607



The cross section of a shed roof is an isosceles triangle. The two sloped rafters each have length 12 feet and meet at the peak to form a 112° angle, as shown. Each rafter meets the horizontal ceiling joist at an angle of x°. What is the value of x?

A 34 **B** 56 **C** 68 **D** 112

8 M-C2608

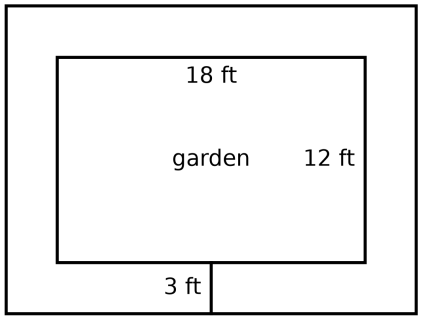


A straight ramp rises 1.8 meters over a horizontal distance of 7.2 meters. A vertical post supports the ramp at a point that is 5 meters, measured horizontally, from the lower end of the ramp, as shown. What is the height h, in meters, of the post?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

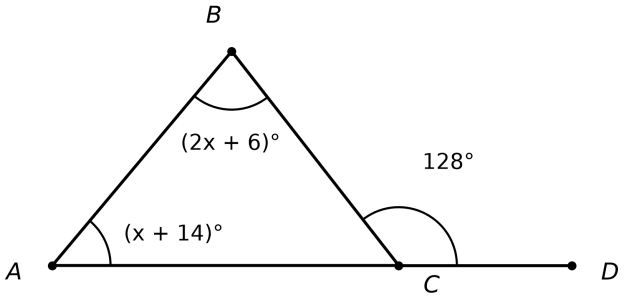
9 M-C2609



A rectangular garden measures 18 feet by 12 feet. A gravel path 3 feet wide surrounds the garden, forming a larger rectangle, as shown. What is the area of the path, in square feet?

A 180 **B** 216 **C** 252 **D** 432

10 M-C2610



In the figure, points A, C, and D lie on a line. What is the measure, in degrees, of angle ABC?

A 36 **B** 50 **C** 52 **D** 78

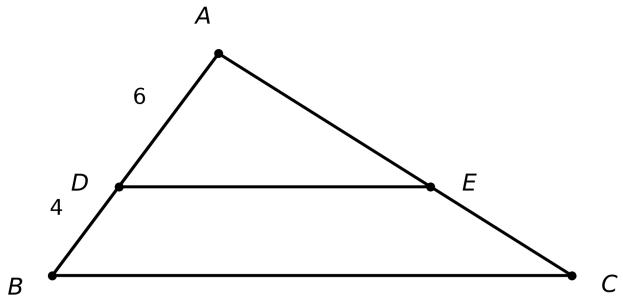
11 M-C2611

A wooden planter shaped like a rectangular prism holds 8 cubic feet of soil when full. A larger planter is built with the same shape, and each of its length, width, and height is 1.5 times the corresponding dimension of the original planter. How many cubic feet of soil does the larger planter hold when full?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

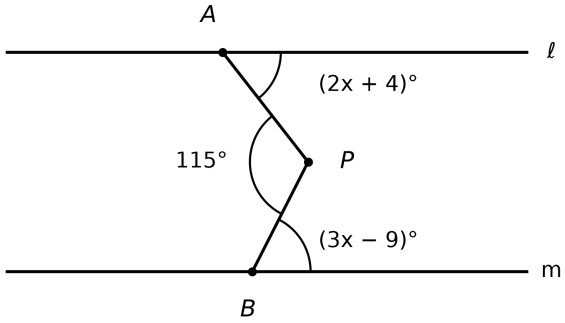
12 M-C2612



In triangle ABC, point D lies on side AB, point E lies on side AC, and segment DE is parallel to side BC. The length of AD is 6, the length of DB is 4, and the area of triangle ADE is 27. What is the area of quadrilateral DBCE?

A 12 **B** 18 **C** 48 **D** 75

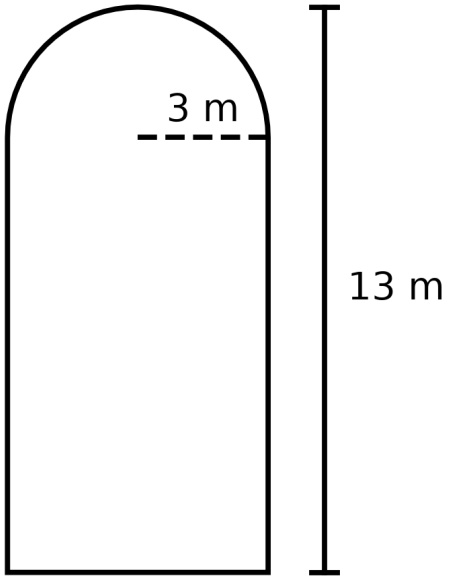
13 M-C2613



In the figure, line ℓ is parallel to line m. Point A lies on line ℓ, point B lies on line m, and point P lies between the two lines. What is the value of x?

A 24 **B** 37 **C** 50 **D** 52

14 M-C2614



A storage tank at a construction site is shaped like a right circular cylinder topped by a hemisphere with the same radius, as shown. The radius is 3 meters, and the total height of the tank, from the ground to the top of the hemisphere, is 13 meters. The volume of the tank is $k\pi$ cubic meters. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

15 M-C2615

Two packaging boxes are similar rectangular prisms. The smaller box has a surface area of 150 square inches and a volume of 125 cubic inches. The larger box has a surface area of 600 square inches. What is the volume, in cubic inches, of the larger box?

A 500 **B** 1,000 **C** 2,000 **D** 8,000

Practice Set Explanations

1. Correct answer: B M-C2601

Lines, angles, and triangles · Geometry and Trigonometry · Easier

B *62*

Corresponding angles formed by a transversal crossing parallel lines are equal, so the angle above line m and to the left of t also measures 118°. Angle x° and that angle together form a straight line, so they are supplementary: x = 180 − 118 = 62.

1. The angle above line m and to the left of line t corresponds to the 118° angle, so it measures 118°.
2. That angle and the angle marked x° form a straight line, so 118 + x = 180.
3. Solve: x = 62.

A This subtracts 90 from 118, treating the angles as related to a right angle. Angles that form a straight line add to 180°, not 90°.

C This assumes x° corresponds to the 118° angle. The angle that corresponds to 118° is the one beside x°, and x° is its supplement.

D This subtracts 118 from 360, as if the two angles filled a full turn. Two angles that form a straight line total only 180°.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

2. Correct answer: D M-C2602

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D *100*

Split the bed into two rectangles. The left part is 7 m wide and 10 m tall, with area 70 square meters. The right part is 12 − 7 = 5 m wide and 6 m tall, with area 30 square meters. The total is 70 + 30 = 100 square meters. Check: 12 × 10 − 5 × 4 = 100.

1. Missing lengths: the notch is 12 − 7 = 5 m wide and 10 − 6 = 4 m tall.
2. Left rectangle: 7 × 10 = 70 square meters.
3. Right rectangle: 5 × 6 = 30 square meters.
4. Total area: 70 + 30 = 100 square meters.

A 44 is the perimeter, 12 + 6 + 5 + 4 + 7 + 10 = 44 meters. The question asks for area, the space inside the boundary.

B 72 is the area of only the bottom 12-by-6 strip. The 7-by-4 rectangle above that strip is also part of the garden bed.

C This subtracts a 7-by-6 rectangle from 12 × 10. The missing corner is actually 5 m by 4 m, so 20, not 42, should be subtracted.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

3. Correct answer: A M-C2603

Lines, angles, and triangles · Geometry and Trigonometry · Easier

A *65*

The interior angles of a triangle add to 180°. So 47 + 68 + x = 180, which gives 115 + x = 180 and x = 65. This fact is printed on the reference sheet, so you can confirm it there during the test.

1. Angle sum of a triangle: 47 + 68 + x = 180.
2. Combine: 115 + x = 180.
3. Solve: x = 65.

B This subtracts only the 68° angle from 180. All three angles share the 180° total, so the 47° angle must be subtracted too.

C 115 is the sum of the two given angles. That sum equals the exterior angle at the third vertex, not the interior angle x°.

D This subtracts 115 from 360, as if a triangle's angles totaled 360°. That total belongs to quadrilaterals; a triangle's angles total 180°.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: 9000 M-C2604

Area and volume · Geometry and Trigonometry · Easier

Use V = ℓwh from the reference sheet: V = 30 × 20 × 15 = 9,000 cubic centimeters. Enter 9000 with no comma. A result of 2,700 comes from finding the surface area instead, and 65 comes from adding the three dimensions.

1. Volume of a rectangular prism: V = ℓwh.
2. Substitute: V = 30 × 20 × 15.
3. V = 9,000 cubic centimeters, entered as 9000.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

5. Correct answer: C M-C2605

Lines, angles, and triangles · Geometry and Trigonometry · Medium

C *72*

The two labeled angles lie between the parallel lines on opposite sides of line t, so they are alternate interior angles and are equal: 4x + 12 = 6x − 18. Then 30 = 2x and x = 15. The angle measures 4(15) + 12 = 72 degrees, and 6(15) − 18 = 72 confirms it.

1. Alternate interior angles are equal: 4x + 12 = 6x − 18.
2. Subtract 4x from both sides and add 18: 30 = 2x, so x = 15.
3. Angle measure: 4(15) + 12 = 72 degrees.

A 15 is the value of x, not the angle measure. Substitute x = 15 into 4x + 12 to finish the problem.

B This stops at $2x = 30$ and reports $2x$. Divide by 2 to get $x = 15$, then evaluate $4x + 12$.

D 108 is the supplement of the correct angle, the angle next to it along line ℓ. Alternate interior angles are equal, not supplementary.

Calculator option: Graph $y = 4x + 12$ and $y = 6x - 18$. They intersect at (15, 72): the x-coordinate is x, and the y-coordinate is the angle measure.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

6. Correct answer: C M-C2606

Area and volume · Geometry and Trigonometry · Medium

C *240π*

The radius is half the diameter, so $r = 4$ centimeters. Using $V = \pi r^2 h$ from the reference sheet, $V = \pi(4^2)(15) = \pi(16)(15) = 240\pi$ cubic centimeters, which is about 754 cubic centimeters.

1. Radius: $r = 8 \div 2 = 4$ centimeters.
2. Cylinder volume: $V = \pi r^2 h = \pi(4^2)(15)$.
3. $V = 240\pi$ cubic centimeters.

A This multiplies π by r and h without squaring the radius: $\pi(4)(15)$. The cylinder formula uses r^2.

B This includes the factor 1/3 from the cone formula. A cylinder's volume is $\pi r^2 h$, with no factor of 1/3.

D This uses the diameter, 8, as the radius: $\pi(8^2)(15)$. The radius is half the diameter, 4 centimeters.

Calculator option: Evaluate $\pi \times 4^2 \times 15 \approx 754$ and compare it with each choice in decimal form; only $240\pi \approx 754$ matches.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

7. Correct answer: A M-C2607

Lines, angles, and triangles · Geometry and Trigonometry · Medium

A *34*

Because the rafters are equal in length, the triangle is isosceles, and the angles opposite the equal sides (the base angles at the joist) are equal. The three angles total 180°, so $112 + 2x = 180$. Then $2x = 68$ and $x = 34$.

1. Equal rafters make the triangle isosceles, so both base angles measure $x°$.
2. Angle sum: $112 + x + x = 180$.
3. $2x = 68$, so $x = 34$.

B 56 is half of 112, the angle between a rafter and the vertical line through the peak. The angle with the horizontal joist is $90 - 56 = 34$.

C 68 is $180 - 112$, the total of both base angles. That total must be split equally between the two base angles.

D 112° is the peak angle itself. The base angles must be smaller, because together with the 112° angle they total only 180°.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

8. Correct answer: 1.25 or 5/4 M-C2608

Lines, angles, and triangles · Geometry and Trigonometry · Medium

The post, the ground and the ramp form a smaller right triangle that shares the angle at the lower end with the full ramp triangle, so the triangles are similar. Then $h/5 = 1.8/7.2 = 1/4$, so $h = 1.25$ meters. Measuring the 5 m from the high end instead gives 0.55, and the inverted ratio $h/5 = 7.2/1.8$ gives 20.

1. Let h be the post's height. The post triangle and the ramp triangle share the angle at the lower end and each has a right angle, so they are similar.
2. Match corresponding sides: $h/5 = 1.8/7.2$.
3. $1.8/7.2 = 0.25$, so $h = 5 \times 0.25 = 1.25$ meters.

Calculator option: Model the ramp as the line $y = (1.8/7.2)x$ with the lower end at the origin. Its y-value at $x = 5$ is 1.25, the post height.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

9. Correct answer: B M-C2609

Area and volume · Geometry and Trigonometry · Medium

B *216*

The outer rectangle adds 3 feet on each side, so it measures $(18 + 6)$ by $(12 + 6)$, or 24 feet by 18 feet, with area 432 square feet. The garden's area is $18 \times 12 = 216$ square feet. The path is the difference: $432 - 216 = 216$ square feet.

1. Outer dimensions: $18 + 2(3) = 24$ feet and $12 + 2(3) = 18$ feet.
2. Outer area: $24 \times 18 = 432$ square feet.
3. Garden area: $18 \times 12 = 216$ square feet.
4. Path area: $432 - 216 = 216$ square feet.

A This multiplies the garden's perimeter, 60 feet, by the 3-foot width. That misses the four 3-by-3 corner squares, 36 square feet in all.

C This adds four strips that each run the full outer length. The four 3-by-3 corners are then counted twice, adding an extra 36 square feet.

D 432 is the area of the outer rectangle, garden included. Subtract the garden's 216 square feet to leave only the path.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

10. Correct answer: D M-C2610

Lines, angles, and triangles · Geometry and Trigonometry · Medium

D *78*

An exterior angle of a triangle equals the sum of the two interior angles that are not next to it. So (x + 14) + (2x + 6) = 128, which gives 3x + 20 = 128 and x = 36. Angle ABC measures 2(36) + 6 = 78 degrees. Check: 50°, 78° and 52° total 180°.

1. Exterior angle theorem: (x + 14) + (2x + 6) = 128.
2. 3x + 20 = 128, so 3x = 108 and x = 36.
3. Angle ABC = 2(36) + 6 = 78 degrees.

A 36 is the value of x. The question asks for angle ABC, which measures 2x + 6 = 78 degrees.

B 50 is the measure of angle BAC, found from x + 14. The question asks for the angle at vertex B.

C 52 is 180 − 128, the interior angle at vertex C beside the exterior angle, not the angle at vertex B.

Calculator option: Graph y = (x + 14) + (2x + 6) and y = 128; the intersection at x = 36 confirms the solution before you evaluate 2x + 6.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

11. Correct answer: 27 M-C2611

Area and volume · Geometry and Trigonometry · Medium

When every length is multiplied by k, volume is multiplied by k^3. Here k = 1.5, so the volume is multiplied by $1.5^3 = 3.375$, and 8 × 3.375 = 27 cubic feet. Multiplying by 1.5 gives 12, and multiplying by $1.5^2 = 2.25$ gives 18; those factors apply to lengths and areas, not volumes.

1. Scale factor for lengths: k = 1.5.
2. Volume scales by $k^3 = 1.5^3 = 3.375$.
3. New volume: 8 × 3.375 = 27 cubic feet.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

12. Correct answer: C M-C2612

Lines, angles, and triangles · Geometry and Trigonometry · Harder

C *48*

Because DE is parallel to BC, triangle ADE is similar to triangle ABC with scale factor AB/AD = 10/6 = 5/3. Areas scale by the square of that factor, $(5/3)^2 = 25/9$, so the area of triangle ABC is 27 × 25/9 = 75. The quadrilateral is the rest of the triangle: 75 − 27 = 48.

1. DE is parallel to BC, so triangle ADE is similar to triangle ABC, and AB = 6 + 4 = 10.
2. Length ratio of ABC to ADE: 10/6 = 5/3. Area ratio: $(5/3)^2 = 25/9$.
3. Area of triangle ABC: 27 × 25/9 = 75.
4. Area of DBCE: 75 − 27 = 48.

A This multiplies 27 by $(4/6)^2$, as if quadrilateral DBCE were similar to triangle ADE. A quadrilateral cannot be similar to a triangle.

B This scales the area by the length ratio 10/6, giving 45 for triangle ABC, and then subtracts 27. Areas scale by the square of the length ratio.

D 75 is the area of the whole triangle ABC. Subtract the 27 square units of triangle ADE to get the quadrilateral's area.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

13. Correct answer: A M-C2613

Lines, angles, and triangles · Geometry and Trigonometry · Harder

A *24*

Draw a line through P parallel to ℓ and m. It splits angle APB into two parts that are alternate interior angles with the marked angles at A and B. So (2x + 4) + (3x − 9) = 115. Then 5x − 5 = 115, 5x = 120, and x = 24. Check: 52° + 63° = 115°.

1. Draw a line through P parallel to ℓ and m; it splits angle APB into two angles.
2. By alternate interior angles, those parts measure (2x + 4)° and (3x − 9)°, so (2x + 4) + (3x − 9) = 115.
3. 5x − 5 = 115, so 5x = 120 and x = 24.

B This sets the two marked angles' sum equal to 180, as if they were same-side interior angles. Together they equal the 115° angle at P.

C This sets the sum equal to 245, the reflex angle at P (360 − 115). The two marked angles add up to the 115° angle itself.

D 52 is the measure of the angle at A, 2(24) + 4. The question asks for the value of x, which is 24.

Calculator option: Graph y = (2x + 4) + (3x − 9) and y = 115; they intersect at x = 24. Writing the equation from the figure is the real work, and the graph only confirms it.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

14. Correct answer: 108 M-C2614

Area and volume · Geometry and Trigonometry · Harder

The hemisphere rises one radius, 3 m, above the cylinder, so the cylinder is 13 − 3 = 10 m tall. Cylinder: $\pi(3^2)(10) = 90\pi$. Hemisphere: half of $(4/3)\pi(3^3)$, or 18π. Total: 108π, so k = 108. Using 13 m as the cylinder's height gives 135, and using a full sphere gives 126.

1. Cylinder height: 13 − 3 = 10 meters, because the dome rises one radius above the cylinder.
2. Cylinder volume: $\pi(3^2)(10) = 90\pi$ cubic meters.
3. Hemisphere volume: $(1/2)(4/3)\pi(3^3) = 18\pi$ cubic meters.
4. Total: $90\pi + 18\pi = 108\pi$, so k = 108.

Calculator option: Check by computing $\pi \times 3^2 \times 10 + (2/3) \times \pi \times 3^3$ and dividing the result by π; the calculator shows 108.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

15. Correct answer: B M-C2615

Area and volume · Geometry and Trigonometry · Harder

B *1,000*

Surface areas scale by k^2, where k is the length scale factor. Here $k^2 = 600/150 = 4$, so $k = 2$. Volumes scale by $k^3 = 8$, so the larger box's volume is $125 \times 8 = 1{,}000$ cubic inches.

1. Area ratio: $k^2 = 600 \div 150 = 4$, so the length factor is $k = 2$.
2. Volume ratio: $k^3 = 2^3 = 8$.
3. Larger volume: $125 \times 8 = 1{,}000$ cubic inches.

A This multiplies the volume by 4, the surface-area ratio. Volume scales by the cube of the length factor, $2^3 = 8$.

C This multiplies by $4^2 = 16$, squaring the area ratio. First take the square root of 4 to get the length factor 2, then cube it.

D This multiplies by $4^3 = 64$, treating the area ratio 4 as the length factor. The length factor is $\sqrt{4} = 2$.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

CHAPTER 27

Right Triangles and Trigonometry

This chapter covers the Pythagorean theorem, the two special right triangles on the reference sheet, and the three trigonometric ratios. Every question here rests on one picture: a right triangle whose sides are named from the point of view of one angle. Once you can label that picture quickly, most of these questions become a single equation.

What the question looks like

- Find a missing side when two sides are known: a ladder against a wall, a diagonal brace, a ramp, or the distance between two points.
- Use a 30°-60°-90° or 45°-45°-90° triangle, often hidden inside a square, an equilateral triangle or a rectangle.
- Find a ratio such as sin A or tan B from side lengths, or find one ratio from another.
- Find a side or an angle in a real setting, such as the rise of a ramp or the height reached by a crane.
- Use the complementary-angle relationship: sin x° = cos y°, and you need x + y or one of the values.
- Compare similar right triangles, whose matching angles have equal trigonometric ratios.

Answer choices often contain radicals, such as $6\sqrt{3}$, or exact fractions, such as 20/29.

How to recognize it

Signal in the question	Tool to use
A right angle and two known sides	Pythagorean theorem
A 30°, 45° or 60° angle, a square's diagonal, or an equilateral triangle's height	Special right triangles
sin, cos or tan in the stem, or an angle of elevation or incline	SOH-CAH-TOA
Sine on one side of an equation and cosine on the other	Complementary angles
"Triangle DEF is similar to triangle ABC" plus a ratio	Ratios carry over to matching angles
Two points in the xy-plane and the word "distance"	Pythagorean theorem on the horizontal and vertical gaps

The method

1. **Mark the right angle and the angle the question is about.**
2. **Label the sides from that angle.** The hypotenuse is across from the right angle. The opposite side is across from your angle. The adjacent side touches your angle and is not the hypotenuse.
3. **Choose the tool** from the table above: the Pythagorean theorem, the special-triangle ratios or SOH-CAH-TOA.
4. **Write one equation and solve.** Keep radicals and fractions exact if the choices are exact.

5. **Check the result.** The hypotenuse must be the longest side. The sine and cosine of an acute angle are between 0 and 1.

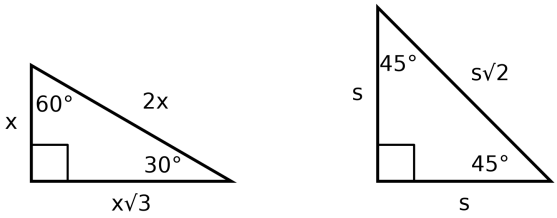
The Pythagorean theorem

For legs a and b and hypotenuse c, $a^2 + b^2 = c^2$. When the unknown is a leg, subtract: $a^2 = c^2 - b^2$. Common whole-number triples save time, and so do their multiples.

Triple	Multiples you may see
3, 4, 5	6, 8, 10; 9, 12, 15; 15, 20, 25
5, 12, 13	15, 36, 39
8, 15, 17	16, 30, 34
7, 24, 25	14, 48, 50
20, 21, 29	40, 42, 58

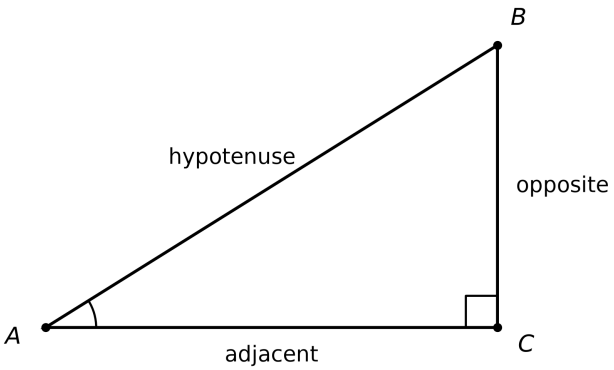
The distance between two points in the xy-plane is a hypotenuse whose legs are the horizontal and vertical gaps.

Special right triangles



Both triangles are on the reference sheet. In a 30°-60°-90° triangle, the short leg x is opposite the 30° angle, the long leg $x\sqrt{3}$ is opposite the 60° angle, and the hypotenuse is 2x. In a 45°-45°-90° triangle, the legs are equal (s) and the hypotenuse is $s\sqrt{2}$. To go from a hypotenuse h back to a leg, divide by $\sqrt{2}$: $s = h/\sqrt{2} = h\sqrt{2}/2$.

SOH-CAH-TOA



For an acute angle A in a right triangle:

Ratio	Definition	Memory aid
sin A	opposite/hypotenuse	SOH
cos A	adjacent/hypotenuse	CAH
tan A	opposite/adjacent	TOA

These ratios depend only on the angle, not on the size of the triangle. That is why similar right triangles share the same sine, cosine and tangent for matching angles.

Complementary angles

The two acute angles of a right triangle add to 90°. The side opposite one of them is adjacent to the other, so sin A = cos B whenever A + B = 90°. In general, sin x° = cos(90° − x°). If sin a° = cos b° for acute angles a and b, then a + b = 90.

NOTE Use the calculator for decimal values of sine, cosine and tangent, and use the inverse functions ($\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$) to find an angle from a ratio. Check whether the calculator is in degree or radian mode before you trust a result. Degree measures in a question carry the ° symbol.

Worked examples

Example 1: A diagonal path

A rectangular community garden is 45 meters wide and 60 meters long. A straight path runs from one corner to the opposite corner. How long is the path?

Think: The path is the hypotenuse of a right triangle with legs 45 and 60. Both legs are multiples of 15: 45 = 3 × 15 and 60 = 4 × 15.

Work: The 3, 4, 5 triple scaled by 15 gives a hypotenuse of 75 meters. Check: $45^2 + 60^2 = 2{,}025 + 3{,}600 = 5{,}625 = 75^2$.

Why the traps fail: Adding the legs gives 105 meters, the walk along two edges, not the diagonal. Stopping before the square root leaves 5,625.

Example 2: A missing leg

A 29-foot support cable runs from the top of a vertical pole to a stake in level ground 20 feet from the base of the pole. How tall is the pole?

Think: The cable is the hypotenuse, and the pole and the ground are the legs. The unknown is a leg, so subtract.

Work: $h^2 + 20^2 = 29^2$, so $h^2 = 841 - 400 = 441$ and h = 21 feet.

Why the traps fail: Adding the squares gives $\sqrt{1{,}241} \approx 35.2$ feet, a pole taller than its own cable, which is impossible.

Example 3: A 30°-60°-90° triangle from the long leg

In right triangle JKL, angle K is the right angle, angle J measures 30°, and JK = 9. Find KL and JL.

Think: JK touches angle J, so it is opposite angle L, the 60° angle. That makes JK the long leg, $x\sqrt{3}$.

Work: $x\sqrt{3} = 9$, so $x = 9/\sqrt{3} = 9\sqrt{3}/3 = 3\sqrt{3}$. The short leg KL is $3\sqrt{3}$, about 5.2, and the hypotenuse JL is $2x = 6\sqrt{3}$, about 10.4.

Why the traps fail: Treating 9 as the short leg gives KL = $9\sqrt{3}$, which would put the longer leg across from the smaller angle. The side opposite the smaller angle is always the shorter side.

Example 4: A square's diagonal

A square garden plot has a diagonal path 10 meters long. What is the area of the plot?

Think: The diagonal splits the square into two 45°-45°-90° triangles, and the diagonal is their hypotenuse.

Work: $s\sqrt{2} = 10$, so $s = 10/\sqrt{2} = 5\sqrt{2}$. The area is $s^2 = (5\sqrt{2})^2 = 25 \times 2 = 50$ square meters. A shortcut: the area of a square is half the square of its diagonal, 100/2 = 50.

Why the traps fail: Treating 10 as the side gives 100. Dividing 10 by 2 instead of $\sqrt{2}$ gives a side of 5 and an area of 25.

Example 5: All three ratios

In right triangle PQR, angle R is the right angle, PR = 40, QR = 9, and PQ = 41. Find sin P, cos P and tan P.

Think: From angle P, the opposite side is QR = 9, the adjacent side is PR = 40, and the hypotenuse is PQ = 41.

Work: sin P = 9/41, cos P = 40/41 and tan P = 9/40. Notice that sin Q = 40/41 = cos P, as the complementary-angle rule predicts. Check the triangle: $9^2 + 40^2 = 81 + 1{,}600 = 1{,}681 = 41^2$.

Why the traps fail: Labeling sides from Q instead of from P swaps opposite and adjacent, giving sin P = 40/41. Using 41 in the denominator of a tangent mixes up TOA with SOH.

Example 6: Finding a side and an angle

A crane's boom is 50 feet long and is raised to a 35° angle above horizontal. About how high is the tip of the boom above its pivot? A separate access ramp rises 2 feet over a horizontal run of 24 feet. What angle does the ramp make with the ground?

Think: The height is opposite the 35° angle and the boom is the hypotenuse, so use sine. For the ramp, the rise and run are the opposite and adjacent sides, so use tangent and then the inverse tangent.

Work: Height = 50 sin 35° ≈ 50(0.574) ≈ 28.7 feet. For the ramp, $\tan\theta = 2/24 = 1/12$, so $\theta = \tan^{-1}(1/12) \approx 4.8°$.

Why the traps fail: Using cosine gives 50 cos 35° ≈ 41.0 feet, the horizontal reach, not the height. In radian mode, 50 sin(35) ≈ −21.4, a negative height that signals the wrong mode.

Example 7 (harder module): Complementary angles with algebra

For acute angles, $\cos(4a + 2)° = \sin(3a + 4)°$. What is the value of a?

Think: A cosine equals the sine of the complementary angle, so the two angle expressions add to 90.

Work: $(4a + 2) + (3a + 4) = 90$, so $7a + 6 = 90$ and $a = 12$. Check: the angles are $4(12) + 2 = 50$ and $3(12) + 4 = 40$, and cos 50° = sin 40°.

Why the traps fail: Setting the expressions equal gives a = 2 and the statement cos 10° = sin 10°, which is false. For acute angles, sine and cosine of the same angle are equal only at 45°; setting the arguments equal is not a general method for a sine-cosine equation.

Example 8 (harder module): Ratios in similar triangles

In right triangle ABC, angle C is the right angle and sin A = 3/5. Triangle DEF is similar to triangle ABC, with D, E and F matching A, B and C. If DE = 35, what are EF and tan D?

Think: Matching angles have equal ratios, so sin D = 3/5. DE is the hypotenuse of triangle DEF, and EF is opposite angle D.

Work: EF = 35 × 3/5 = 21. The remaining leg is DF = $\sqrt{(35^2 - 21^2)} = \sqrt{(1{,}225 - 441)} = \sqrt{784} = 28$. So tan D = 21/28 = 3/4.

Why the traps fail: Treating EF as the leg adjacent to D gives EF = 35 × 4/5 = 28, but EF matches BC, which is opposite angle A. Writing tangent as adjacent over opposite gives 4/3, the reciprocal.

Common traps

Trap	How to spot it
Using the wrong side as the hypotenuse	The hypotenuse is across from the right angle and is the longest side
Swapping opposite and adjacent	Label the sides from the angle named in the question
Mixing up the 30°-60°-90° legs	The short leg is opposite the 30° angle
Dividing by 2 instead of √2	You are going from a 45°-45°-90° hypotenuse to a leg
A calculator in radian mode	A negative or strange value for an acute angle
Setting the angles in sin a° = cos b° equal	For acute angles, equal sine and cosine values mean the angles add to 90°
Adding squares to find a leg	The unknown side is not the hypotenuse, so subtract

Harder-module variations

- **Hidden right triangles.** An altitude splits an equilateral triangle into two 30°-60°-90° triangles. A rectangle's diagonal makes two right triangles. A box's interior diagonal uses the theorem twice: $d = \sqrt{(\ell^2 + w^2 + h^2)}$.
- **One ratio from another.** Build a triangle from the given ratio and find the third side. If cos θ = 0.8 = 4/5, the sides are 4 and 5, the third side is 3, and tan θ = 3/4 = 0.75.
- **Exact answers in student-produced responses.** A question may ask for k when a length equals $k\sqrt{3}$. Solve exactly, then enter only k.
- **Conditions on complementary angles.** If x° and y° are acute, sin x° = cos y° and x = 2y, then 3y = 90, so y = 30 and x = 60.

Check yourself

1. A 30°-60°-90° triangle has a hypotenuse of 14. How long are its legs?
2. Which side is adjacent to angle A in a right triangle?
3. If sin A = cos B and both angles are acute, what is A + B?
4. A square has sides of length 7. How long is its diagonal?

5. Your calculator reports sin 25 as about −0.13. What went wrong?
6. Answers: (1) 7 and $7\sqrt{3}$; (2) the leg that touches angle A, not the hypotenuse; (3) 90°; (4) $7\sqrt{2}$; (5) the calculator is in radian mode, so switch it to degrees.

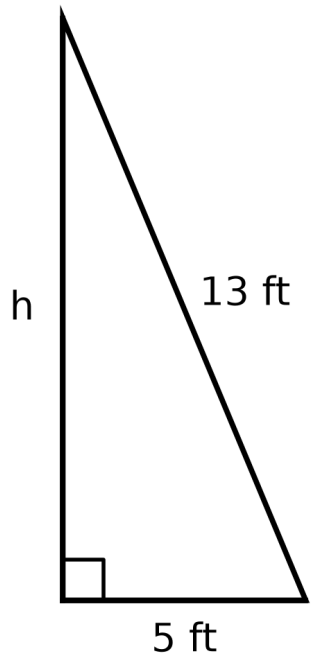
ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/
right-triangles-trigonometry

Practice Set

15 questions, arranged from easier to harder. Answer all of them before reading the explanations.

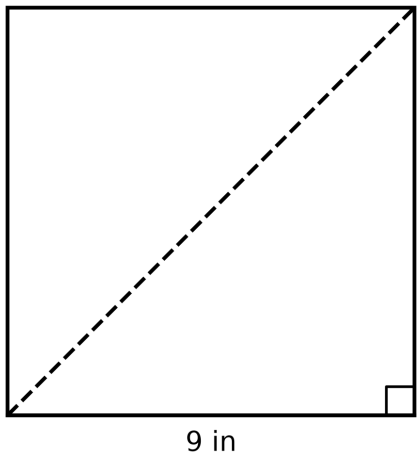
1 M-C2701



A 13-foot ladder leans against a vertical wall, as shown. The bottom of the ladder is 5 feet from the base of the wall on level ground. How high up the wall, in feet, does the top of the ladder reach?

A 12 **B** $\sqrt{194}$ **C** 18 **D** 144

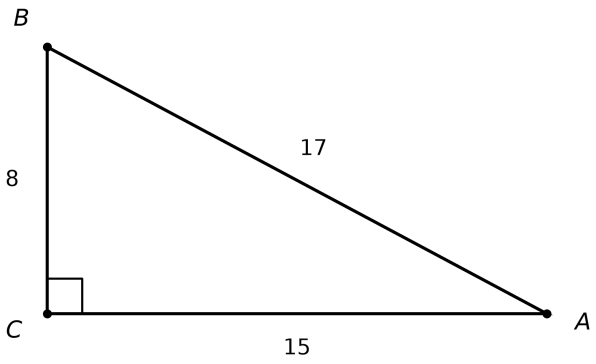
2 M-C2702



A square floor tile has a side length of 9 inches, as shown. What is the length, in inches, of a diagonal of the tile?

A $(9\sqrt{2})/2$ **B** 9 **C** $9\sqrt{2}$ **D** 18

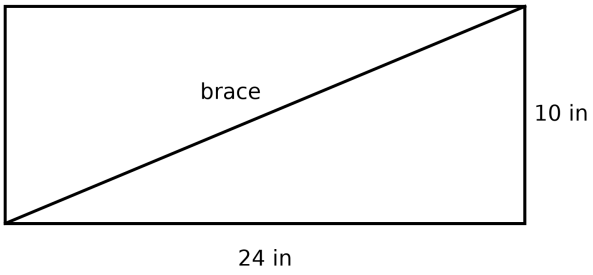
3 M-C2703



In right triangle ABC shown, what is the value of tan A?

A 8/17 **B** 8/15 **C** 15/17 **D** 15/8

4 M-C2704

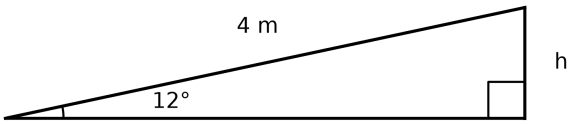


A rectangular wall frame is 24 inches wide and 10 inches tall. A straight brace runs from one corner of the frame to the opposite corner, as shown. What is the length of the brace, in inches?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

5 M-C2705



A straight ramp is 4 meters long and makes a 12° angle with the level ground, as shown. Which expression gives the height h, in meters, of the top of the ramp above the ground?

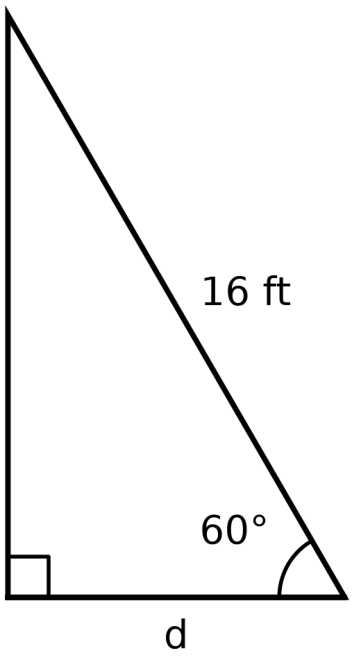
A 4/sin 12° **B** 4 cos 12° **C** 4 tan 12° **D** 4 sin 12°

6 M-C2706

In right triangle ABC, angle C is the right angle, and sin A = 24/25. What is the value of cos B?

A 7/25 **B** 7/24 **C** 24/25 **D** 25/24

7 M-C2707



A 16-foot ladder leans against a vertical wall and makes a 60° angle with the level ground, as shown. What is the distance d, in feet, from the bottom of the ladder to the base of the wall?

A 8 **B** $8\sqrt{2}$ **C** $8\sqrt{3}$ **D** $16\sqrt{3}$

8 M-C2708

In right triangle PQR, angle R is a right angle, and $\tan P = 5/12$. What is the value of $\cos P$?

A 5/13 **B** 12/13 **C** 13/12 **D** 12/5

9 M-C2709

On a garden map in the xy-plane, each unit represents 1 meter. Two sprinkler heads are located at the points (−2, 3) and (6, −3). What is the distance, in meters, between the sprinkler heads?

A $2\sqrt{7}$ **B** $2\sqrt{13}$ **C** 10 **D** 14

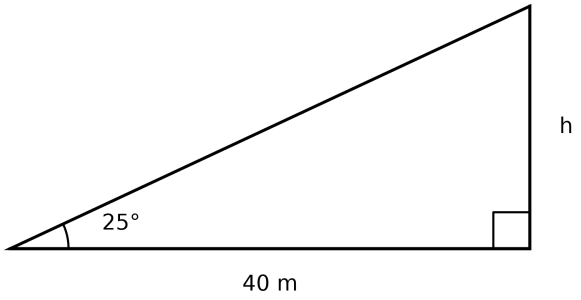
10 M-C2710

In the equation $\sin(x + 20)° = \cos 46°$, the value of x is between 0 and 70. What is the value of x?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

11 M-C2711



A straight section of a roller-coaster track rises at a constant angle of 25° above the horizontal, as shown. Over a horizontal distance of 40 meters, the track rises h meters. To the nearest meter, what is the value of h?

A 17 **B** 19 **C** 36 **D** 44

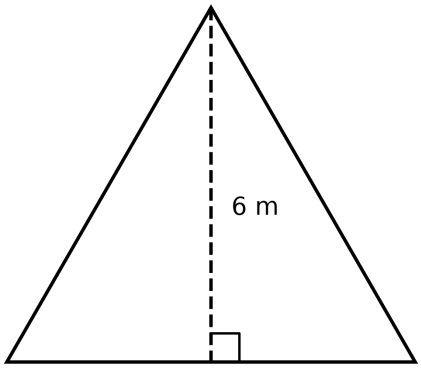
12 M-C2712

A packaging box is a rectangular prism with interior dimensions 12 centimeters by 4 centimeters by 3 centimeters. What is the length, in centimeters, of the longest straight rod that fits completely inside the box?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

13 M-C2713



A garden bed is shaped like an equilateral triangle with a height of 6 meters, as shown. The area of the garden bed is $k\sqrt{3}$ square meters. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

14 M-C2714

Right triangles ABC and DEF are similar, with vertices A, B, and C corresponding to vertices D, E, and F, respectively. Angles C and F are right angles. In triangle ABC, AB = 25 and AC = 24. In triangle DEF, EF = 21. What is the value of tan E?

A 7/25 **B** 7/24 **C** 24/25 **D** 24/7

15 M-C2715

In the equation $\sin(3k - 4)° = \cos(2k + 14)°$, both $(3k - 4)°$ and $(2k + 14)°$ are acute angles. What is the value of k?

A 16 **B** 18 **C** 34 **D** 44

Practice Set Explanations

1. Correct answer: A M-C2701

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A *12*

The wall, the ground and the ladder form a right triangle with the ladder as the hypotenuse. By the Pythagorean theorem, $h^2 + 5^2 = 13^2$, so $h^2 = 169 - 25 = 144$ and h = 12 feet. The sides 5, 12 and 13 form a common Pythagorean triple.

1. Let h be the height reached on the wall. The ladder is the hypotenuse of a right triangle.
2. Pythagorean theorem: $h^2 + 5^2 = 13^2$.
3. $h^2 = 169 - 25 = 144$, so h = 12 feet.

B This adds the squares, $13^2 + 5^2 = 194$, as if the ladder were a leg. The ladder is the hypotenuse, so subtract instead.

C This adds the lengths 13 and 5. The Pythagorean theorem relates the squares of the sides, not the sides themselves.

D 144 is the value of h^2, not h. Take the square root to finish: $\sqrt{144} = 12$ feet.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

2. Correct answer: C M-C2702

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C *$9\sqrt{2}$*

A diagonal cuts the square into two 45°-45°-90° triangles whose legs are sides of the square. In such a triangle the hypotenuse is the leg times $\sqrt{2}$, so the diagonal is $9\sqrt{2}$ inches, about 12.7 inches. The reference sheet lists these side ratios as s, s and $s\sqrt{2}$.

1. The diagonal splits the square into two 45°-45°-90° triangles with legs of 9 inches.
2. In a 45°-45°-90° triangle, hypotenuse = leg × $\sqrt{2}$.
3. Diagonal = $9\sqrt{2}$ inches.

A This divides 9 by $\sqrt{2}$, which goes from a diagonal back to a side. Here the side is known, so multiply by $\sqrt{2}$ instead.

B 9 is the side length. The diagonal is the hypotenuse of a right triangle, so it must be longer than either side.

D This adds two sides of the square. The diagonal is a straight path, so it is shorter than walking along two sides.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

3. Correct answer: B M-C2703

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B *8/15*

Tangent is opposite over adjacent. From angle A, the opposite side is BC = 8 and the adjacent leg is AC = 15, so tan A = 8/15. The hypotenuse, 17, is not part of the tangent ratio.

1. From angle A: opposite side BC = 8, adjacent side AC = 15, hypotenuse AB = 17.
2. tan A = opposite/adjacent = 8/15.

A 8/17 is opposite over hypotenuse, which is sin A. Tangent compares the two legs, opposite over adjacent.

C 15/17 is adjacent over hypotenuse, which is cos A. Tangent uses the opposite leg over the adjacent leg.

D This flips the ratio to adjacent over opposite. 15/8 is tan B, measured from the other acute angle.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

4. Correct answer: 26 M-C2704

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The brace is the hypotenuse of a right triangle with legs 24 and 10. So $b^2 = 24^2 + 10^2 = 576 + 100 = 676$ and b = 26 inches. Spotting the triple 5, 12, 13 scaled by 2 gets there faster. Adding the two sides gives 34, a common slip.

1. Let b be the brace length; it is the hypotenuse of a right triangle with legs 24 and 10 inches.
2. $b^2 = 24^2 + 10^2 = 576 + 100 = 676$.
3. b = $\sqrt{676}$ = 26 inches.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

5. Correct answer: D M-C2705

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D *$4 \sin 12°$*

The ramp is the hypotenuse, and h is the side opposite the 12° angle. Sine is opposite over hypotenuse, so $\sin 12° = h/4$ and $h = 4 \sin 12°$. In degree mode this is about 0.83 meter.

1. Let h be the height. The 4 m ramp is the hypotenuse, and h is opposite the 12° angle.
2. $\sin 12° = h/4$.
3. Multiply both sides by 4: $h = 4 \sin 12°$.

A This solves $\sin 12° = 4/h$, which puts the ramp opposite the angle. The result, about 19 meters, is longer than the ramp itself.

B Cosine uses the adjacent side, so $4 \cos 12°$ is the horizontal distance the ramp covers along the ground, not its height.

C Tangent is opposite over adjacent, and the adjacent side is unknown here. Multiplying the hypotenuse by $\tan 12°$ mixes up the sides.

Calculator option: In degree mode, 4 sin 12° ≈ 0.83, a sensible height for a 4-meter ramp, while 4/sin 12° ≈ 19.2 is longer than the ramp.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

6. Correct answer: C M-C2706

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C *24/25*

Angles A and B are complementary, and the side opposite angle A is the side adjacent to angle B. So cos B = sin A = 24/25. In general, sin x° = cos(90° − x°), and here B = 90° − A.

1. In right triangle ABC, A + B = 90°, so B = 90° − A.
2. sin A = cos(90° − A) = cos B.
3. cos B = 24/25.

A 7/25 is cos A, found from the missing leg 7. The question asks for cos B, which equals sin A.

B 7/24 is tan B, the opposite leg over the adjacent leg from angle B. The question asks for a cosine.

D 25/24 flips the correct ratio. Sine and cosine of an acute angle are always less than 1, because the hypotenuse is the longest side.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

7. Correct answer: A M-C2707

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A *8*

The wall, ground and ladder form a 30°-60°-90° triangle with hypotenuse 16. The angle at the top of the ladder is 30°, and d is the side opposite it, so d is the shortest side: half the hypotenuse, 16/2 = 8 feet. Check: 16 cos 60° = 8.

1. The angles are 60° at the ground, 90° at the wall and 30° at the top: a 30°-60°-90° triangle.
2. Hypotenuse 2x = 16, so x = 8.
3. d is opposite the 30° angle, so d = x = 8 feet.

B This uses the 45°-45°-90° ratios. The angles here are 30°, 60° and 90°, so the sides are x, x√3 and 2x.

C 8√3 is the height the ladder reaches on the wall, the side opposite the 60° angle. The distance d is opposite the 30° angle.

D This multiplies the hypotenuse by √3. In a 30°-60°-90° triangle the hypotenuse is 2x, so x = 8, and the longer leg is only 8√3.

Calculator option: In degree mode, 16 cos 60° = 8 confirms the answer.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

8. Correct answer: B M-C2708

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B *12/13*

tan P = 5/12 means the leg opposite P is 5k and the leg adjacent to P is 12k for some k > 0. The hypotenuse is $\sqrt{(25k^2 + 144k^2)} = 13k$. So cos P = adjacent/hypotenuse = 12k/13k = 12/13.

1. Let the leg opposite P be 5 and the leg adjacent to P be 12 (any common multiple gives the same ratios).
2. Hypotenuse: $\sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$.
3. cos P = adjacent/hypotenuse = 12/13.

A 5/13 is opposite over hypotenuse, which is sin P. Cosine uses the adjacent leg, 12, over the hypotenuse.

C This is hypotenuse over adjacent, the reciprocal of cos P. The cosine of an acute angle is always less than 1.

D 12/5 is adjacent over opposite, the reciprocal of tan P. It cannot be a cosine, which is at most 1.

Calculator option: In degree mode, the inverse tangent of 5/12 is about 22.6°; the cosine of that angle is about 0.923, which matches 12/13.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

9. Correct answer: C M-C2709

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C *10*

The horizontal change is 6 − (−2) = 8 and the vertical change is −3 − 3 = −6. These are the legs of a right triangle, so the distance is $\sqrt{(8^2 + 6^2)} = \sqrt{100} = 10$ meters. The distance formula is the Pythagorean theorem written in coordinates.

1. Horizontal change: 6 − (−2) = 8. Vertical change: −3 − 3 = −6.
2. Distance = $\sqrt{(8^2 + (-6)^2)} = \sqrt{(64 + 36)}$.
3. Distance = $\sqrt{100} = 10$ meters.

A This subtracts the squares: $\sqrt{(64 - 36)} = \sqrt{28} = 2\sqrt{7}$. The distance is a hypotenuse, so the squares of the legs are added.

B This finds the horizontal change as 6 − 2 = 4, dropping the negative sign of −2. Then $\sqrt{(16 + 36)}$ gives $2\sqrt{13}$.

D This adds the legs, 8 + 6 = 14, which is the length of a path along the grid lines. The straight-line distance uses the Pythagorean theorem.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

10. Correct answer: 24 M-C2710

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The sine of an angle equals the cosine of its complement, so $\sin(x + 20)° = \cos 46°$ when $(x + 20) + 46 = 90$. Then $x + 66 = 90$ and $x = 24$. Setting $x + 20$ equal to 46 gives 26, and forgetting the 20 gives 44.

1. $\sin \theta° = \cos(90° - \theta°)$, so the two angles must be complementary.
2. $(x + 20) + 46 = 90$.
3. $x + 66 = 90$, so $x = 24$.

Calculator option: In degree mode, graph $y = \sin(x + 20)$ and $y = \cos(46)$; in the window $0 < x < 70$ they intersect at $x = 24$.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

11. Correct answer: B M-C2711

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B *19*

The horizontal distance is adjacent to the 25° angle and the rise h is opposite it, so $\tan 25° = h/40$. Then $h = 40 \tan 25° \approx 40(0.4663) \approx 18.65$, which rounds to 19 meters.

1. The 40 m horizontal distance is adjacent to the 25° angle; the rise h is opposite it.
2. $\tan 25° = h/40$, so $h = 40 \tan 25°$.
3. $h \approx 18.65$, which rounds to 19 meters.

A This computes $40 \sin 25° \approx 16.9$, treating 40 as the length of the track. The 40 meters is the horizontal distance, adjacent to the angle.

C This computes $40 \cos 25° \approx 36.3$. Cosine relates the adjacent side to the hypotenuse, but the rise is the opposite side.

D This computes $40/\cos 25° \approx 44.1$, the length of the sloped track itself. The question asks for the vertical rise.

Calculator option: Make sure the calculator is in degree mode: $40 \tan(25) \approx 18.65$. In radian mode the same entry gives about −5.3, a sign that the mode is wrong.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

12. Correct answer: 13 M-C2712

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The longest rod runs from a bottom corner to the opposite top corner. The bottom diagonal d satisfies $d^2 = 12^2 + 4^2 = 160$. That diagonal and the 3 cm height are the legs of a second right triangle, so the rod length is $\sqrt{(160 + 9)} = \sqrt{169} = 13$. Stopping at $\sqrt{160} \approx 12.6$, or adding the edges to get 19, are common errors.

1. Let d be the diagonal of the 12-by-4 base: $d^2 = 12^2 + 4^2 = 160$.
2. The rod is the hypotenuse of a right triangle with legs d and 3.
3. Rod length $= \sqrt{(160 + 3^2)} = \sqrt{169} = 13$ centimeters.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

13. Correct answer: 12 M-C2713

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The height splits the triangle into two 30°-60°-90° triangles, and the height is the longer leg, $x\sqrt{3} = 6$. So $x = 6/\sqrt{3} = 2\sqrt{3}$, and each side is $2x = 4\sqrt{3}$. Area $= (1/2)(4\sqrt{3})(6) = 12\sqrt{3}$, so $k = 12$. Treating 6 as the side length gives 9, and using half the base, $2\sqrt{3}$, as the whole base gives 6.

1. The height divides the triangle into two 30°-60°-90° triangles; the height is the longer leg, $x\sqrt{3}$.
2. $x\sqrt{3} = 6$, so $x = 6/\sqrt{3} = 2\sqrt{3}$, and the side length is $2x = 4\sqrt{3}$ meters.
3. Area $= (1/2)(4\sqrt{3})(6) = 12\sqrt{3}$ square meters.
4. So $k = 12$.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

14. Correct answer: D M-C2714

Right triangles and trigonometry · Geometry and Trigonometry · Harder

D *24/7*

In triangle ABC, $BC = \sqrt{(25^2 - 24^2)} = \sqrt{49} = 7$. Corresponding angles of similar triangles are equal, so $\tan E = \tan B$. From angle B, the opposite leg is AC = 24 and the adjacent leg is BC = 7, so $\tan E = 24/7$. Check with DEF: the scale factor is $21/7 = 3$, so DF = 72, and $72/21 = 24/7$.

1. Missing leg in triangle ABC: $BC = \sqrt{(25^2 - 24^2)} = 7$.
2. Similar triangles have equal corresponding angles, so angle E = angle B.
3. $\tan B$ = opposite/adjacent = AC/BC = 24/7, so $\tan E = 24/7$.

A 7/25 is cos E, adjacent over hypotenuse. Tangent compares the two legs, opposite over adjacent.

B 7/24 is tan D, measured from the other acute angle. From angle E, the opposite leg is DF, which corresponds to AC = 24.

C 24/25 is sin E, opposite over hypotenuse. Tangent uses the adjacent leg, which corresponds to BC = 7, instead of the hypotenuse.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

15. Correct answer: A M-C2715

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A *16*

For acute angles, $\sin a° = \cos b°$ exactly when $a + b = 90$. So $(3k - 4) + (2k + 14) = 90$, which gives $5k + 10 = 90$ and k

= 16. Check: the two angles are 44° and 46°, which are complementary.

1. For acute angles, sin a° = cos b° when a + b = 90.
2. (3k − 4) + (2k + 14) = 90, so 5k + 10 = 90.
3. 5k = 80, so k = 16.

B This sets 3k − 4 equal to 2k + 14, as if sine and cosine of the same angle were equal. That happens only at 45°.

C This sets the sum equal to 180, treating the angles as supplementary. Sine equals the cosine of the complement, so the sum is 90.

D 44 is the measure of the angle 3k − 4 when k = 16. The question asks for the value of k itself.

Calculator option: In degree mode, graph y = sin(3x − 4) and y = cos(2x + 14) for 0 < x < 30; the graphs cross only at x = 16. Writing the complement equation is still faster.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

CHAPTER 28

Circles

Circle questions on the digital SAT draw on a short list of ideas: area and circumference, arcs and sectors as fractions of a whole circle, radian measure, the equation of a circle in the xy-plane, and two angle facts about tangents and inscribed angles. Most errors come from mixing up radius and diameter, or arc and sector, rather than from hard algebra, so the method below keeps those apart.

What the question looks like

- Find an area or a circumference from a radius or a diameter, often for a wheel, a track or a garden bed.
- Find an arc length or a sector area from a central angle given in degrees or radians.
- Convert an angle between degrees and radians.
- Read the center and radius from an equation, write an equation from a center and a point, or complete the square to put an equation in standard form.
- Use an angle fact: an inscribed angle, an angle inscribed in a semicircle, or a tangent line meeting a radius.

Student-produced responses cannot include π, so a question may say that a length or area "is kπ" and ask for k.

How to recognize it

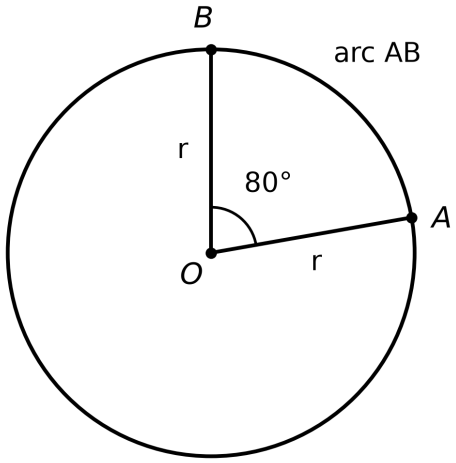
Signal in the question	Tool to use
"Radius," "diameter," "around," or "one full revolution"	$C = 2\pi r$ and $A = \pi r^2$
A central angle and "arc," or the distance along part of a circle	Arc length as a fraction of the circumference
A central angle and a region, slice or watered area	Sector area as a fraction of the circle's area
π in an angle measure, or the word "radians"	π radians = 180°
An equation with equal nonzero coefficients on x^2 and y^2, and no xy term	Complete the square; a positive radius squared gives a circle
"Tangent"	The radius to the point of tangency is perpendicular to the tangent
An angle whose vertex is on the circle	Inscribed angle = half its intercepted arc

The method

1. **Decide what kind of answer you need:** a length (circumference or arc), an area (circle or sector), an angle, or an equation.
2. **Find the radius.** Halve a diameter. In an equation, write it in standard form first, then take the square root of the right side.

3. **If only part of the circle is involved, find the fraction.** Use θ/360 for an angle in degrees or θ/(2π) for an angle in radians.
4. **Multiply the fraction by the whole.** Use the full circumference for an arc and the full area for a sector.
5. **Match the form of the choices.** Keep π as a symbol when the choices or the prompt use it, and round only at the end.

Arcs and sectors



A central angle has its vertex at the center of the circle. The arc and the sector it cuts off are the same fraction of the whole circle as the angle is of a full turn.

Measure	Central angle in degrees	Central angle θ in radians
Arc length	$(\theta/360) \times 2\pi r$	$r\theta$
Sector area	$(\theta/360) \times \pi r^2$	$(1/2)r^2\theta$

Radians

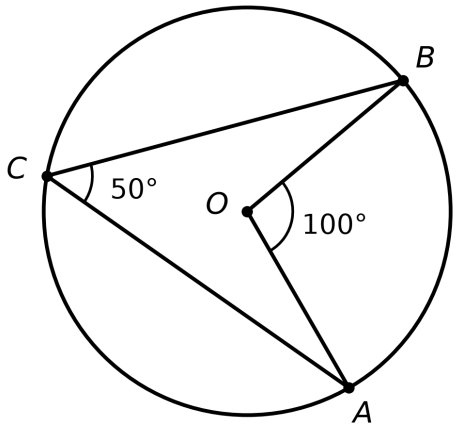
A full circle measures 360°, or 2π radians; both facts are on the reference sheet. So π radians = 180°. To change degrees to radians, multiply by π/180. To change radians to degrees, multiply by 180/π.

Degrees	30°	45°	60°	90°	180°	270°	360°
Radians	π/6	π/4	π/3	π/2	π	3π/2	2π

The circle equation

A circle with center (h, k) and radius r has the equation $(x - h)^2 + (y - k)^2 = r^2$. Two details cause most errors. First, the signs inside the parentheses are the opposite of the center's coordinates, so $(x + 3)^2$ means h = −3. Second, the right side is r^2, not r. When an equation is expanded, as in $x^2 + y^2 + Dx + Ey + F = 0$, complete the square in x and in y to return it to standard form. Half of each x and y coefficient, squared, is the number you add to both sides. The resulting right side must be positive for a circle: zero gives a single point, and a negative value gives no real points.

Angle facts



- A central angle equals the measure of the arc it intercepts.
- An inscribed angle has its vertex on the circle and equals half of the arc it intercepts. In the figure, central angle AOB and inscribed angle ACB open toward the same arc AB, so the inscribed angle is 100/2 = 50 degrees.
- An angle inscribed in a semicircle, with its sides running to the two ends of a diameter, is a right angle.
- A tangent line touches the circle at exactly one point and is perpendicular to the radius drawn to that point.

Worked examples

Example 1: The length of a curved path

A curved garden path follows an arc of a circle with radius 9 meters, and the arc's central angle is 80°, as in the arc figure above. How long is the path?

Think: The path is an arc, so it is a fraction of the circumference.

Work: The fraction is 80/360 = 2/9. The circumference is $2\pi(9) = 18\pi$. The path is $(2/9)(18\pi) = 4\pi$, about 12.6 meters.

Why the traps fail: Using πr in place of $2\pi r$ gives 2π, half the true length. Using the full area, 81π, gives 18π, which is the sector's area, not the length of the path.

Example 2: The area of a sector

A garden bed is shaped like a sector of a circle with radius 10 feet and a central angle of 72°. What is the area of the bed?

Think: A sector is a fraction of the circle's area.

Work: The fraction is 72/360 = 1/5. The circle's area is $\pi(10^2) = 100\pi$. The bed's area is $(1/5)(100\pi) = 20\pi$, about 62.8 square feet.

Why the traps fail: Dividing by 180 instead of 360 gives 40π. Taking the fraction of the circumference gives 4π, which is the length of the curved edge, not an area.

Example 3: Radians on a Ferris wheel

A Ferris wheel has a radius of 18 meters. A car moves through an angle of $5\pi/6$ radians. What is this angle in degrees, and how far does the car travel?

Think: Convert with π radians = 180°. For the distance, use $s = r\theta$ with θ in radians.

Work: $(5\pi/6)(180/\pi) = 900/6 = 150°$. The distance is $s = 18(5\pi/6) = 15\pi$, about 47.1 meters. The degree method agrees: $(150/360)(2\pi \times 18) = (5/12)(36\pi) = 15\pi$.

Why the traps fail: The formula $s = r\theta$ works only in radians. Putting 150 in for θ gives 2,700 meters, far more than a full revolution, about 113 meters.

Example 4: Completing the square

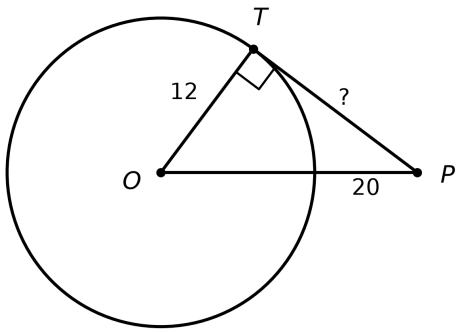
The equation $x^2 + y^2 - 6x + 10y + 9 = 0$ defines a circle. What are its center and radius?

Think: Group the x terms and the y terms, move the constant, and complete each square.

Work: $(x^2 - 6x) + (y^2 + 10y) = -9$. Add $(-6/2)^2 = 9$ and $(10/2)^2 = 25$ to both sides: $(x - 3)^2 + (y + 5)^2 = -9 + 9 + 25 = 25$. The center is $(3, -5)$ and the radius is 5.

Why the traps fail: Adding 9 and 25 on the left only leaves $r^2 = -9$, which is impossible. Reading the signs directly gives the center $(-3, 5)$. Reporting 25 as the radius skips the square root.

Example 5: A tangent segment



A circle with center O has radius 12. Point P is 20 units from O, and segment PT is tangent to the circle at T. How long is PT?

Think: A tangent is perpendicular to the radius at the point of tangency, so triangle OTP has a right angle at T. Its hypotenuse is OP.

Work: $PT^2 = 20^2 - 12^2 = 400 - 144 = 256$, so PT = 16. The sides 12, 16 and 20 are the 3, 4, 5 triple scaled by 4.

Why the traps fail: Treating PT as the hypotenuse gives $\sqrt{(400 + 144)} \approx 23.3$, longer than OP, which cannot happen in a right triangle with hypotenuse OP.

Example 6: An angle in a semicircle

Triangle ABC is inscribed in a circle, and AB is a diameter. If AC = 6 and BC = 8, what is the area of the circle?

Think: Angle C is inscribed in a semicircle, so it is a right angle. AB is the hypotenuse of right triangle ABC.

Work: AB = $\sqrt{(6^2 + 8^2)} = \sqrt{100} = 10$. The radius is 5, so the area is $\pi(5^2) = 25\pi$, about 78.5 square units.

Why the traps fail: Using the diameter, 10, as the radius gives 100π. Without the right-angle fact, the triangle seems to give too little information to find AB.

Example 7 (harder module): A circle tangent to an axis

A circle in the xy-plane has center (−3, 4) and is tangent to the x-axis. What is an equation of the circle?

Think: The circle touches the x-axis at one point directly below the center. The radius is the vertical distance from the center to the axis.

Work: The center is 4 units above the x-axis, so r = 4 and $r^2 = 16$. The equation is $(x + 3)^2 + (y - 4)^2 = 16$.

Why the traps fail: Using the distance to the y-axis, 3, gives $r^2 = 9$; that circle would be tangent to the y-axis instead. Writing 4 on the right side describes a circle with radius 2.

Example 8 (harder module): A coefficient on the squared terms

The equation $2x^2 + 2y^2 - 8x + 12y = 6$ defines a circle. What is its radius?

Think: Standard form needs coefficients of 1 on x^2 and y^2, so divide every term by 2 before completing the square.

Work: $x^2 + y^2 - 4x + 6y = 3$. Add 4 and 9 to both sides: $(x - 2)^2 + (y + 3)^2 = 3 + 4 + 9 = 16$. The center is (2, −3) and the radius is 4.

Why the traps fail: Completing the square without dividing first adds the wrong constants. Forgetting to divide the 6 by 2 gives $r^2 = 19$, a radius of $\sqrt{19}$.

Common traps

Trap	How to spot it
Using the diameter as the radius	The problem gives a width across the circle
Mixing up arc length and sector area	A length is asked for (path, edge, distance), or an area (region, bed, watered)
Dividing the angle by 180 instead of 360	You are finding a fraction of the whole circle
Using $s = r\theta$ with degrees	The angle has a ° symbol, so convert it or use $\theta/360$
Copying the center's signs from the equation	$(x - h)$ means the x-coordinate is h, so $(x + 2)$ means −2
Reporting r^2 as the radius	The right side of standard form is the radius squared
Making an inscribed angle equal the arc	The vertex is on the circle, so take half
Treating a tangent segment as the hypotenuse	The hypotenuse is the segment from the center to the outside point

Harder-module variations

- **Equation from a diameter.** If the endpoints of a diameter are given, the center is their midpoint and the radius is half the distance between them.

- **Inside, on or outside.** Substitute a point into the left side of the standard form. A value less than r^2 means inside, equal means on the circle, and greater means outside.
- **Finding the angle from an arc.** Rearrange $s = r\theta$ to $\theta = s/r$. An arc as long as the radius corresponds to 1 radian.
- **Tangent line equations.** A nonhorizontal, nonvertical radius has a tangent whose slope is its negative reciprocal. A horizontal radius has a vertical tangent; a vertical radius has a horizontal tangent. In every case, the tangent passes through the point of tangency.
- **Unknown constants.** If an expanded equation contains a constant c and the radius is given, complete the square, set the right side equal to r^2, and solve for c.

Check yourself

1. How many radians equal 120°?
2. What are the center and radius of $(x + 1)^2 + (y - 6)^2 = 49$?
3. An inscribed angle intercepts an arc of 140°. What does the angle measure?
4. What fraction of a circle's area is a sector with a 45° central angle?
5. How does a tangent line meet the radius drawn to the point of tangency?
6. Answers: (1) $2\pi/3$; (2) center $(-1, 6)$, radius 7; (3) 70°; (4) 1/8; (5) at a right angle.

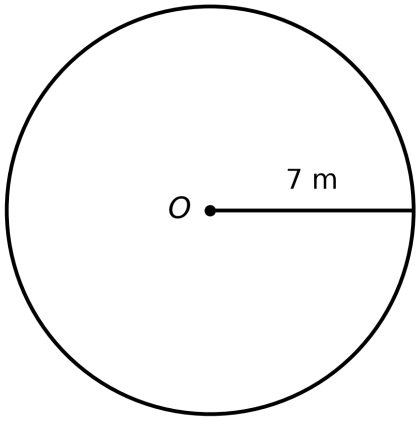
ONLINE (OPTIONAL) This skill's page on the website has two video lessons and a link to practice the skill with a digital study coach.

skillbyskillprep.com/skills/circles

Practice Set

12 questions, arranged from easier to harder. Answer all of them before reading the explanations.

1 M-C2801



A circular garden has a radius of 7 meters, as shown. What is the area of the garden, in square meters?

A 7π **B** 14π **C** 49π **D** 196π

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2 M-C2802

In the xy-plane, the graph of $(x - 4)^2 + (y + 1)^2 = 25$ is a circle. What are the coordinates of the center of the circle and the length of its radius?

A Center $(-4, 1)$, radius 5
B Center $(4, 1)$, radius 5
C Center $(4, -1)$, radius 25
D Center $(4, -1)$, radius 5

---

3 M-C2803

A Ferris wheel has a diameter of 50 meters. To the nearest meter, how far does a seat on the edge of the wheel travel during one complete revolution?

A 79 **B** 157 **C** 314 **D** 1,963

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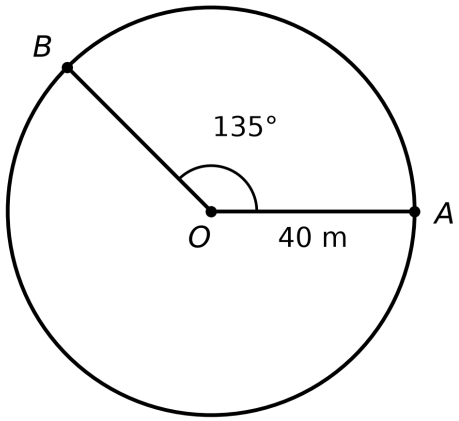
4 M-C2804

A central angle of a circle measures $3\pi/4$ radians. What is the measure of this angle in degrees?

A 2.36 **B** 67.5 **C** 135 **D** 270

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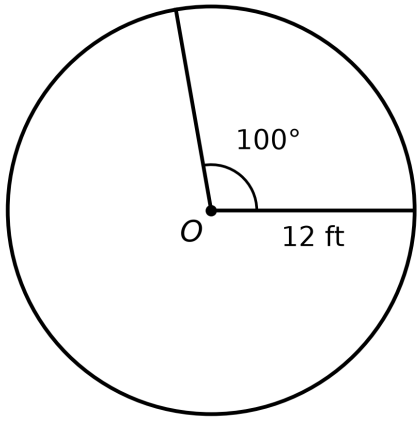
5 M-C2805



Points A and B lie on a circular track with center O and radius 40 meters, and angle AOB measures 135°, as shown. A runner travels along the track from A to B on the shorter arc. What is the length of that arc, in meters?

A 15π **B** 30π **C** 60π **D** 600π

6 M-C2806

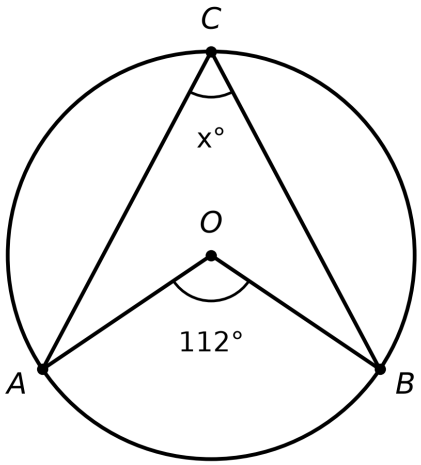


A garden sprinkler at point O rotates through a 100° angle and waters a sector of a circle with radius 12 feet, as shown. The area of the watered sector is kπ square feet. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

7 M-C2807



Points A, B, and C lie on the circle with center O shown. Angle AOB measures 112°. What is the value of x?

A 56 **B** 68 **C** 112 **D** 124

8 M-C2808

A model train runs on a circular track with a radius of 24 inches. The train travels along the track through a central angle of $7\pi/6$ radians. The distance the train travels is $k\pi$ inches. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

9 M-C2809

On a garden plan drawn in the xy-plane, a sprinkler is located at the point (−2, 3). The sprinkler waters a circular region whose edge passes through the point (1, 7). Which equation represents the edge of the watered region?

A $(x - 2)^2 + (y + 3)^2 = 25$ **B** $(x + 2)^2 + (y - 3)^2 = 5$

C $(x - 1)^2 + (y - 7)^2 = 25$ **D** $(x + 2)^2 + (y - 3)^2 = 25$

10 M-C2810

The equation $x^2 + y^2 + 8x - 6y = 11$ defines a circle in the xy-plane. What is the radius of the circle?

Student-produced response. Write your answer on the answer sheet or below.

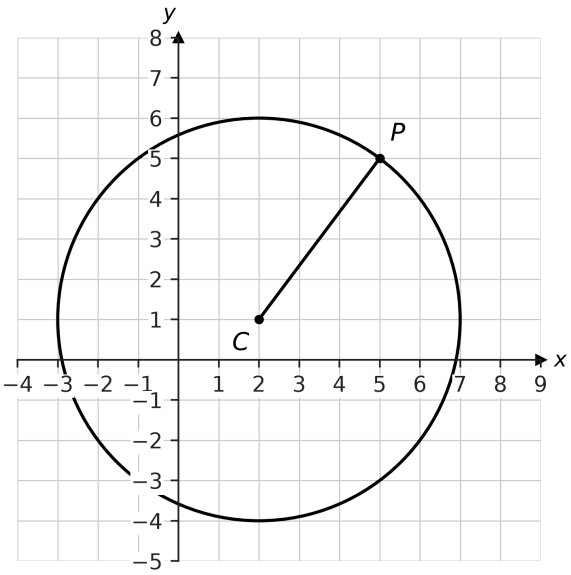
Answer: _____

11 M-C2811

The equation $x^2 + y^2 - 10x + 4y + c = 0$, where c is a constant, defines a circle in the xy-plane with radius 7. What is the value of c?

A −20 **B** 20 **C** 22 **D** 78

12 M-C2812



In the xy-plane shown, a circle has center C(2, 1), and point P(5, 5) lies on the circle. Line t is tangent to the circle at P. Which equation defines line t?

A $4x - 3y = 5$ **B** $3x - 4y = -5$ **C** $3x + 4y = 35$ **D** $3x + 4y = 10$

Practice Set Explanations

1. Correct answer: C M-C2801

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C *49π*

The area of a circle is $A = \pi r^2$, which is on the reference sheet. With $r = 7$, $A = \pi(7^2) = 49\pi$ square meters, or about 153.9 square meters. Area is measured in square units because it counts the space inside the circle.

1. Area of a circle: $A = \pi r^2$.
2. Substitute $r = 7$: $A = \pi(7^2) = 49\pi$ square meters.

A This multiplies π by the radius without squaring it. Area uses r^2, so the radius must be squared first.

B 14π is the circumference, $2\pi r$. Circumference measures the distance around the circle, not the space inside it.

D This squares the diameter, 14, instead of the radius. Using the diameter makes the area four times too large.

Review: Chapter 28, Circles (page 405)

2. Correct answer: D M-C2802

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D *Center (4, −1), radius 5*

The standard form $(x - h)^2 + (y - k)^2 = r^2$ has center (h, k) and radius r. Here $x - 4$ gives $h = 4$, and $y + 1 = y - (-1)$ gives $k = -1$. Since $r^2 = 25$, the radius is 5.

1. Compare with $(x - h)^2 + (y - k)^2 = r^2$.
2. $x - 4$ gives $h = 4$; $y + 1 = y - (-1)$ gives $k = -1$.
3. $r^2 = 25$, so $r = 5$.

A This flips both signs. In $(x - h)^2$, the center's x-coordinate is h, so $x - 4$ means $h = 4$, not −4.

B This gets h right but reads $y + 1$ as $k = 1$. Write $y + 1$ as $y - (-1)$, so $k = -1$.

C The center is right, but 25 is r^2, not r. The radius is the square root, $\sqrt{25} = 5$.

Calculator option: Graph the equation. The circle's leftmost and rightmost points are (−1, −1) and (9, −1); their midpoint, (4, −1), is the center, and they are 10 units apart, so the radius is 5.

Review: Chapter 28, Circles (page 405)

3. Correct answer: B M-C2803

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B *157*

One complete revolution carries a seat once around the circle, a distance equal to the circumference. $C = 2\pi r = \pi d$, so with $d = 50$ meters, $C = 50\pi \approx 157.08$, which rounds to 157 meters.

1. The distance in one revolution equals the circumference.
2. $C = \pi d = \pi(50) = 50\pi$ meters.
3. $50\pi \approx 157.08$, so about 157 meters.

A This computes $\pi \times 25$, using the radius in the formula $C = \pi d$. Use either $2\pi r$ with $r = 25$ or πd with $d = 50$.

C This computes $2\pi \times 50$, treating the diameter as the radius in $C = 2\pi r$. The radius is half of 50, or 25 meters.

D This computes the area, $\pi(25^2) \approx 1{,}963$ square meters. The distance around the wheel is a length, the circumference.

Review: Chapter 28, Circles (page 405)

4. Correct answer: C M-C2804

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C *135*

Since π radians equals 180°, multiply the radian measure by 180/π: $(3\pi/4)(180/\pi) = 3(180)/4 = 135$ degrees. A quick check: $3\pi/4$ is three-quarters of π, and three-quarters of 180° is 135°.

1. π radians = 180°, so multiply the radian measure by 180/π.
2. $(3\pi/4) \times (180/\pi) = 540/4$.
3. $540/4 = 135$ degrees.

A 2.36 is the decimal value of $3\pi/4$, which is still the radian measure. Multiply by 180/π to convert to degrees.

B This multiplies by 90/π instead of 180/π. A half turn is π radians, which equals 180°, not 90°.

D This multiplies by 360/π. A full turn is 2π radians, so π radians is 180°, not 360°.

Review: Chapter 28, Circles (page 405)

5. Correct answer: B M-C2805

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B *30π*

An arc is the same fraction of the circumference as its central angle is of 360°. The fraction is $135/360 = 3/8$, and the circumference is $2\pi(40) = 80\pi$. So the arc length is $(3/8)(80\pi) = 30\pi$ meters, about 94 meters.

1. Fraction of the circle: $135/360 = 3/8$.
2. Circumference: $2\pi(40) = 80\pi$ meters.
3. Arc length: $(3/8)(80\pi) = 30\pi$ meters.

A This takes 3/8 of 40π, using πr instead of the full circumference, $2\pi r$. The circumference is 80π meters.

C This uses 135/180 as the fraction, comparing the angle with a half turn. The central angle should be compared with 360°.

D 600π is the area of the sector, $(3/8)\pi(40^2)$. The question asks for a distance along the track, an arc length.

Review: Chapter 28, Circles (page 405)

6. Correct answer: 40 M-C2806

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A sector's area is the same fraction of the circle's area as its central angle is of 360°. The circle's area is $\pi(12^2) = 144\pi$, and the fraction is 100/360 = 5/18. So the sector's area is $(5/18)(144\pi) = 40\pi$, and k = 40. Dividing by 180 instead of 360 gives 80, and finding the arc length instead gives 20/3.

1. Fraction of the circle: 100/360 = 5/18.
2. Area of the full circle: $\pi(12^2) = 144\pi$ square feet.
3. Sector area: $(5/18)(144\pi) = 40\pi$, so k = 40.

Review: Chapter 28, Circles (page 405)

7. Correct answer: A M-C2807

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A *56*

Angle ACB is an inscribed angle and angle AOB is a central angle, and both intercept the same arc AB. That arc measures 112°, the same as the central angle, and an inscribed angle measures half of its intercepted arc. So x = 112/2 = 56.

1. Angle AOB is a central angle, so arc AB measures 112°.
2. Angle ACB is inscribed and intercepts arc AB.
3. Inscribed angle = half the arc: x = 112/2 = 56.

B This treats the inscribed and central angles as supplementary, 180 − 112. The inscribed angle is half the central angle that intercepts the same arc.

C This makes the inscribed angle equal to the central angle. An inscribed angle is only half of the arc it intercepts.

D 124 is half of the major arc, (360 − 112)/2. Angle ACB opens toward the minor arc AB, which measures 112°.

Review: Chapter 28, Circles (page 405)

8. Correct answer: 28 M-C2808

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When a central angle θ is measured in radians, arc length is $s = r\theta$. So $s = 24(7\pi/6) = 28\pi$ inches, and k = 28. Converting first gives the same result: 7π/6 radians is 210°, and $(210/360)(48\pi) = 28\pi$. Using the diameter, 48, in $s = r\theta$ gives 56 instead.

1. Arc length with θ in radians: $s = r\theta$.
2. $s = 24 \times (7\pi/6) = (168/6)\pi = 28\pi$ inches.
3. So k = 28.

Review: Chapter 28, Circles (page 405)

9. Correct answer: D M-C2809

Circles · Geometry and Trigonometry · Medium

D $(x + 2)^2 + (y - 3)^2 = 25$

The center is (−2, 3), and the radius is the distance from the center to (1, 7): $\sqrt{((1 + 2)^2 + (7 - 3)^2)} = \sqrt{(9 + 16)} = 5$. In the form $(x - h)^2 + (y - k)^2 = r^2$, that gives $(x + 2)^2 + (y - 3)^2 = 25$.

1. Center (h, k) = (−2, 3).
2. Radius: $\sqrt{((1 - (-2))^2 + (7 - 3)^2)} = \sqrt{(9 + 16)} = 5$.
3. Equation: $(x + 2)^2 + (y - 3)^2 = 25$.

A This reverses the signs of the center. The center (−2, 3) gives x − (−2) = x + 2 and y − 3.

B The center is right, but the right side must be $r^2 = 25$. Writing 5 describes a circle with radius $\sqrt{5}$.

C This uses the point on the edge, (1, 7), as the center. The sprinkler's location, (−2, 3), is the center.

Calculator option: Substitute (1, 7) into each choice; only choice D gives a true statement, 9 + 16 = 25. Graphing the choices shows the same result.

Review: Chapter 28, Circles (page 405)

10. Correct answer: 6 M-C2810

Circles · Geometry and Trigonometry · Harder

Complete the square in each variable. Add 16 to complete $x^2 + 8x$ and 9 to complete $y^2 - 6y$, adding both to each side: $(x + 4)^2 + (y - 3)^2 = 11 + 16 + 9 = 36$. So $r^2 = 36$ and r = 6. Answers of 36 (forgetting the square root) or $\sqrt{11}$ (skipping the completion) are common errors.

1. Group terms: $(x^2 + 8x) + (y^2 - 6y) = 11$.
2. Complete each square by adding $(8/2)^2 = 16$ and $(-6/2)^2 = 9$ to both sides.
3. $(x + 4)^2 + (y - 3)^2 = 36$.
4. $r^2 = 36$, so r = 6.

Calculator option: Graph the equation; the circle's leftmost and rightmost points are (−10, 3) and (2, 3), which are 12 units apart, so the radius is 6.

Review: Chapter 28, Circles (page 405)

11. Correct answer: A M-C2811

Circles · Geometry and Trigonometry · Harder

A *−20*

Complete the square: $(x^2 - 10x + 25) + (y^2 + 4y + 4) = -c + 25 + 4$, so $(x - 5)^2 + (y + 2)^2 = 29 - c$. The right side equals $r^2 = 49$, so 29 − c = 49 and c = −20.

1. Complete the squares: $(x^2 - 10x + 25) + (y^2 + 4y + 4) = -c + 25 + 4$.
2. $(x - 5)^2 + (y + 2)^2 = 29 - c$.
3. Radius 7 means 29 − c = 49.
4. c = 29 − 49 = −20.

B This moves c to the right side without changing its sign, giving 29 + c = 49. Moving c across the equals sign makes it −c.

C This sets 29 − c equal to 7, the radius. The right side of the standard form is the radius squared, 49.

D This writes the right side as c − 29 instead of 29 − c, moving the completed constants across the equals sign without changing their signs.

Calculator option: Add a slider for c and graph the equation. At c = −20 the circle passes through (12, −2), which is 7 units to the right of the center (5, −2).

Review: Chapter 28, Circles (page 405)

12. Correct answer: C M-C2812

Circles · Geometry and Trigonometry · Harder

C $3x + 4y = 35$

A tangent line is perpendicular to the radius at the point of tangency. Radius CP has slope $(5 - 1)/(5 - 2) = 4/3$, so line t has slope $-3/4$. Through (5, 5): $y - 5 = -(3/4)(x - 5)$, which becomes $3x + 4y = 35$. Check: $3(5) + 4(5) = 35$.

1. Slope of radius CP: $(5 - 1)/(5 - 2) = 4/3$.
2. A tangent is perpendicular to the radius, so the slope of line t is $-3/4$.
3. Through P(5, 5): $y - 5 = -(3/4)(x - 5)$.
4. Multiply by 4 and rearrange: $3x + 4y = 35$.

A $4x - 3y = 5$ has slope 4/3 and passes through both C and P. It is the line containing the radius, not the tangent line.

B $3x - 4y = -5$ has slope 3/4, the reciprocal of 4/3 without the sign change. Perpendicular slopes are negative reciprocals.

D $3x + 4y = 10$ has the correct slope, $-3/4$, but it passes through the center (2, 1) instead of the point of tangency (5, 5).

Calculator option: Graph $(x - 2)^2 + (y - 1)^2 = 25$ together with each choice; only $3x + 4y = 35$ touches the circle at exactly one point, (5, 5).

Review: Chapter 28, Circles (page 405)

PART IV

Full-Length Practice and Final Preparation

Practice Test 1

Practice Test 1: Answers, Scoring and Review

Practice Test 2

Practice Test 2: Answers, Scoring and Review

The Final Two Weeks and Beyond

CHAPTER 29

Practice Test 1

This chapter holds a full-length practice test with the structure and timing of the digital SAT: 98 questions in four modules, in the usual test-day order. After these instructions you will find an answer sheet, the two Reading and Writing modules, the Math reference sheet and the two Math modules. Take the test when your study plan from Chapter 2 schedules it, usually after you have worked through most of the skill chapters.

What the test contains

Part	Questions	Time	Average pace
Reading and Writing, Module 1	27	32 minutes	about 71 seconds per question
Reading and Writing, Module 2	27	32 minutes	about 71 seconds per question
Break	—	10 minutes	—
Math, Module 1	22	35 minutes	about 95 seconds per question
Math, Module 2	22	35 minutes	about 95 seconds per question
Total	98	134 minutes of testing, plus the break	—

Set up test conditions

A practice test teaches you the most when it feels like the real thing. Plan for about two and a half hours in one sitting, at the time of day you expect to test.

1. Work in a quiet room. Put your phone in another room and use a separate timer or a watch without an alarm.
2. Have pencils, scratch paper and a calculator ready, ideally a graphing calculator. On test day, the Desmos graphing calculator is built into the Bluebook app, and you may also bring an approved handheld calculator; confirm your model on the College Board website.
3. Use the Math reference sheet printed before Math Module 1. On test day it is available throughout both Math modules.
4. Work on one module at a time. Do not look ahead, and do not return to a module after its time ends; on the digital SAT, a closed module cannot be reopened.
5. Stop when time is called, even in the middle of a question. A STOP line marks the end of each module.
6. Take the full 10-minute break between Reading and Writing and Math. Stand up, drink some water and eat a snack. Do not use the break to check answers.

An honest note about Module 2

On the digital SAT, the second module of each section adapts to your performance on the first: it is, on average, either harder or easier than Module 1. A printed test cannot do that. In this book, each Module 2 is a fixed set of questions that is somewhat harder than Module 1, similar in spirit to the higher-difficulty path. Take it whatever your Module 1 result. If Module 2 feels harder, that is by design, not a sign that something went wrong.

Using the answer sheet

The answer sheet that follows has a row of bubbles for each multiple-choice question and a labeled answer line for each student-produced response. Mark answers on the sheet as you go, so that a module never closes with answers still in the margins.

For student-produced responses, write your number, fraction or decimal on the labeled line. To rehearse digital entry, follow the rules from Chapter 15 even though the printed line has more room:

- A positive answer can use up to 5 characters; a negative answer can use up to 6, including the minus sign.
- You may enter a fraction or a decimal. If a fraction is too long to fit, enter its decimal form.
- Write a mixed number as an improper fraction or a decimal: 7/2 or 3.5, never 3 1/2.
- Do not write a percent sign, dollar sign, comma, units or symbols such as π.
- Reduce fractions. If a decimal is too long to fit, as a repeating decimal is, cut it off or round it so that it fills every available space.

Pacing and guessing

- Use the two-pass method and the checkpoints from Chapter 3, counted by time remaining: 22, 12 and 3 minutes left in a Reading and Writing module; 28, 17 and 3 minutes left in a Math module.
- On the first pass, answer what you can and circle the number of any question you want to revisit.
- Never leave a question blank. The SAT has no penalty for wrong answers, so when about a minute remains, fill in an answer for every question you have not finished.
- Write a small "g" next to each guess. You will treat guesses as misses when you review.

When you finish

Note how many questions you left unfinished in each module when time ran out. Then turn to Chapter 30, which walks you through the answer key, the score worksheet and the explanations. Score the whole test at once, after all four modules; checking answers between modules would change how you take the rest of the test.

Practice Test 1 Answer Sheet

Choose one bubble for multiple-choice questions. On rows labeled Answer, write a number, fraction or decimal. Follow the entry rules in Chapter 15.

Reading and Writing, Module 1

1 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ

Reading and Writing, Module 2

1 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ

Practice Test 1 Answer Sheet (continued)

Choose one bubble for multiple-choice questions. On rows labeled Answer, write a number, fraction or decimal. Follow the entry rules in Chapter 15.

Math, Module 1

1 Ⓐ Ⓑ Ⓒ Ⓓ	**9** Answer: _____	**17** Answer: _____
2 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**18** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Ⓐ Ⓑ Ⓒ Ⓓ	**19** Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
5 Answer: _____	**13** Answer: _____	**21** Answer: _____
6 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Ⓐ Ⓑ Ⓒ Ⓓ	
8 Ⓐ Ⓑ Ⓒ Ⓓ	**16** Ⓐ Ⓑ Ⓒ Ⓓ	

Math, Module 2

1 Ⓐ Ⓑ Ⓒ Ⓓ	**9** Ⓐ Ⓑ Ⓒ Ⓓ	**17** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**18** Answer: _____
3 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Answer: _____	**19** Ⓐ Ⓑ Ⓒ Ⓓ
4 Answer: _____	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	**13** Ⓐ Ⓑ Ⓒ Ⓓ	**21** Answer: _____
6 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Answer: _____	
8 Answer: _____	**16** Ⓐ Ⓑ Ⓒ Ⓓ	

Practice Test 1: Reading and Writing, Module 1

27 questions. 32 minutes.

1 RW-T1M1-01

> Every evening, Inés carried her grandmother's folding chair to the end of the pier and set it facing the harbor. Neither of them ever discussed whether they would go; the walk was simply part of the day, as _____ as the tide. When the fishing boats finally rounded the breakwater, the two of them were always there, a thermos of tea balanced between their feet.

Which choice completes the text with the most logical and precise word or phrase?

A unusual **B** regular **C** dangerous **D** tiring

---

2 RW-T1M1-02

> Unlike whales and seals, sea otters have no thick layer of blubber to hold in body heat. Instead, they stay warm with a dense coat of fur and a high metabolism, which burns energy quickly. An adult sea otter must eat 20 to 30 percent of its body mass in food each day just to meet its energy needs. Wildlife officials therefore ask kayakers to keep their distance, since an otter that is repeatedly disturbed while resting cannot _____ energy.

Which choice completes the text with the most logical and precise word or phrase?

A conserve **B** display **C** measure **D** waste

---

3 RW-T1M1-03

> In many flamenco performances, the dancer does not simply follow music that has been fixed in advance. Instead, the dancer and the guitarist _____ each other throughout the piece. A sudden burst of footwork can signal the guitarist to shift the rhythm, and a change in the guitar's pattern can, in turn, lead the dancer into a new sequence of steps. As a result, the same dance may unfold somewhat differently from one night to the next.

Which choice completes the text with the most logical and precise word or phrase?

A ignore **B** imitate **C** compete with **D** respond to

---

4 RW-T1M1-04

> Before the printing press came into use in Europe in the fifteenth century, books there were copied by hand, often by scribes working in monasteries. Even careful copyists sometimes skipped a line or misread an unfamiliar word. Because later scribes usually worked from whatever copy was nearby rather than from the original, they tended to _____ such slips, and a single error could appear in dozens of manuscripts made decades apart.

Which choice completes the text with the most logical and precise word or phrase?

A correct **B** conceal **C** predict **D** reproduce

---

5 RW-T1M1-05

> The following text is from George Eliot's 1861 novel *Silas Marner*.
>
> No one knew where wandering men had their homes or their origin; and how was a man to be explained unless you at least knew somebody who knew his father and mother? To the peasants of old times, the world outside their own direct experience was a region of vagueness and mystery: to their untravelled thought a state of wandering was a conception as dim as the winter life of the swallows that came back with the spring; and even a settler, if he came from distant parts, hardly ever ceased to be viewed with a remnant of _____, which would have prevented any surprise if a long course of inoffensive conduct on his part had ended in the commission of a crime; especially if he had any reputation for knowledge, or showed any skill in handicraft.

Which choice completes the text with the most logical and precise word or phrase?

A admiration **B** curiosity **C** distrust **D** envy

6 RW-T1M1-06

> The following text is from Christina Rossetti's 1872 poetry collection *Sing-Song: A Nursery Rhyme Book*.
>
> Who has seen the wind?
> Neither I nor you:
> But when the leaves hang trembling
> The wind is passing thro'.
>
> Who has seen the wind?
> Neither you nor I:
> But when the trees bow down their heads
> The wind is passing by.

Which choice best states the main purpose of the text?

A To suggest that something invisible can be recognized by its effects on the things around it

B To describe how trees and leaves change from one season to the next

C To argue that people should spend more time observing nature

D To compare the strength of the wind in a forest with its strength in an open field

7 RW-T1M1-07

> Snow-covered sea ice reflects most of the sunlight that reaches it, while open ocean water absorbs most of the sunlight that reaches it. <u>When a patch of ice melts and exposes the dark water beneath, that area begins to absorb far more of the Sun's energy than it did before.</u> The extra heat warms the surrounding water, which speeds the melting of nearby ice and exposes still more open water. Climate scientists call a cycle like this, in which a change produces effects that strengthen the original change, a positive feedback.

Which choice best describes the function of the underlined sentence in the text as a whole?

A It presents a finding that challenges the definition of positive feedback given at the end of the text.

B It describes the first step in the self-reinforcing process that the rest of the text explains.

C It offers an example of how sea ice can protect ocean water from warming.

D It explains why snow-covered ice reflects more sunlight than open water does.

8 RW-T1M1-08

> **Text 1**
>
> A hypothetical experiment asked volunteers to sort a deck of colored cards into piles as quickly as they could, either alone or while two observers watched. Volunteers who were watched finished faster, on average, than those who worked alone. The researchers concluded that the presence of an audience improves people's performance.
>
> **Text 2**
>
> Psychologists who study social facilitation have found that being observed does not affect all tasks in the same way. An audience tends to raise a person's level of arousal, which makes the person's most practiced, automatic responses more likely. On a simple or well-rehearsed task, those responses are usually correct, so performance improves. On a complex or newly learned task, however, the most likely responses are often errors, and performance tends to suffer.

Based on the texts, how would the author of Text 2 most likely respond to the researchers' conclusion in Text 1?

A By arguing that the volunteers who were watched probably sorted the cards less accurately than those who worked alone

B By rejecting the conclusion, since an audience raises arousal and higher arousal generally makes performance worse

C By suggesting that the conclusion may hold for a simple task such as card sorting but is too broad to apply to all tasks

D By accepting the conclusion, since an audience makes a person's correct responses more likely regardless of the task

---

9 RW-T1M1-09

> For three weeks, Ama had practiced the same violin piece every afternoon, and she was sure she had made no progress at all. The fast passage in the middle still tripped her up, and her teacher kept saying, “Again.” Then, while looking for a photo on her phone, she found a recording she had made on the first day. She pressed play and winced at the scratchy, halting notes. Was that really how she had sounded? She picked up her violin and played the piece straight through.

Which choice best states the main idea of the text?

A Hearing an old recording helps Ama realize that she has improved more than she thought.

B Ama decides that her teacher's demands are unfair and stops practicing the piece.

C Ama discovers that she prefers listening to music over playing it.

D A photo on Ama's phone reminds her why she first wanted to learn the violin.

---

10 RW-T1M1-10

> Biologists in a hypothetical study recorded the alarm calls that a population of ground squirrels made when a hawk flew overhead and when a coyote approached on foot. The two calls sounded different: the hawk call was a single high whistle, while the coyote call was a series of short chirps. After a whistle, squirrels dove into the nearest burrow; after chirps, they stood upright near their burrows and watched. When the biologists played recordings of each call with no predator in sight, the squirrels reacted just as they had to the real predators.

Which choice best states the main idea of the text?

A Ground squirrels are more frightened of hawks than of coyotes.

B Ground squirrels cannot tell the difference between a recorded call and a call made by another squirrel.

C The squirrels' alarm calls appear to communicate what kind of predator is near, not merely that danger is present.

D Playing recorded calls is the most reliable way for biologists to study how squirrels behave.

11 RW-T1M1-11

> A marine science class examined survey data on the percentage of the seafloor covered by live coral at four reef sites. The students noted that live coral cover declined at three of the sites between 2016 and 2024 but increased slightly at _____

Live Coral Cover at Four Reef Sites (hypothetical data)

Reef site	Coral cover in 2016 (%)	Coral cover in 2024 (%)
North Point	42	38
Turtle Bay	35	21
Long Shoal	28	30
Coral Gardens	51	44

Which choice most effectively uses data from the table to complete the statement?

A North Point. **B** Turtle Bay. **C** Coral Gardens. **D** Long Shoal.

12 RW-T1M1-12

> In Sarah Orne Jewett's 1896 book *The Country of the Pointed Firs*, the narrator spends a summer lodging with Mrs. Almira Todd, a widow who grows herbs and sells homemade remedies to her neighbors. The narrator suggests that Mrs. Todd brings a sense of ceremony to some of these sales, treating the handoff of certain remedies as a solemn and secret occasion.

Which quotation from *The Country of the Pointed Firs* most effectively illustrates the claim?

A "Being a very large person, her full skirts brushed and bent almost every slender stalk that her feet missed."

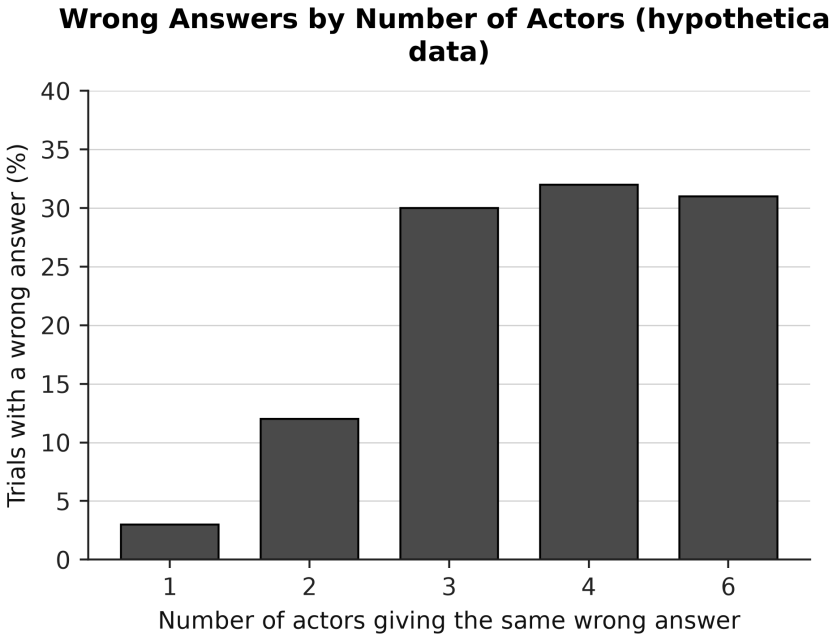
B "with certain vials she gave cautions, standing in the doorway, and there were other doses which had to be accompanied on their healing way as far as the gate, while she muttered long chapters of directions, and kept up an air of secrecy and importance to the last."

C "The village doctor and this learned herbalist were upon the best of terms."

D "This cooling and refreshing drink had been brought to wonderful perfection through a long series of experiments"

13 RW-T1M1-13

> Psychologists running a hypothetical study asked participants to judge which of three lines matched the length of a reference line. Before each participant answered, a group of actors working with the researchers all gave the same wrong answer. The researchers tested groups of one, two, three, four, or six actors. They concluded that adding actors increased participants' tendency to go along with the group only up to a point, after which additional actors made little difference.



Which choice best describes data from the graph that support the researchers' conclusion?

A Participants gave a wrong answer on 3 percent of trials with one actor and on 31 percent of trials with six actors.

B The rate of wrong answers rose from 3 percent with one actor to 30 percent with three actors but stayed between 30 and 32 percent with three, four, or six actors.

C Wrong answers were most common when there were four actors, occurring on 32 percent of trials.

D The rate of wrong answers rose steadily with each actor added, reaching its highest level with six actors.

14 RW-T1M1-14

> Wildlife biologists working in a hypothetical savanna reserve placed motion-activated cameras near several water holes. Over one dry season, zebras visited the water holes mostly around midday, while lions, which hunt zebras, visited mostly after dark. The biologists hypothesized that the zebras were timing their visits to avoid lions rather than for some other reason, such as the midday heat. If this hypothesis is correct, then in a nearby reserve where lions have been absent for many years, zebras would most likely _____

Which choice most logically completes the text?

A visit water holes at a wider range of times, including after dark.

B visit water holes mostly around midday, as the zebras in the first reserve did.

C stop visiting water holes during the dry season.

D be seen near water holes less often than other grazing animals are.

15 RW-T1M1-15

> Some early photographs of city streets show broad avenues that appear nearly deserted, even though those streets were busy when the pictures were taken. The explanation lies in the exposure times of early photographic processes, which often lasted many seconds or even minutes. Buildings, lampposts, and anyone who happened to stand still for that long were recorded clearly, but walkers and carriages moved through the scene too quickly to leave a lasting image. A historian using such photographs as evidence about city life should therefore treat them as reliable records of _____

Which choice most logically completes the text?

A how many people lived in each city at the time.

B which streets were most crowded at particular times of day.

C the stationary features of streets rather than the amount of traffic on them.

D the activities that residents considered most worth recording.

16 RW-T1M1-16

> Earth's oceans store and move enormous amounts of heat. Ocean currents act like a giant conveyor belt, carrying heat from the equator toward the _____ water flowing north in the Atlantic helps keep parts of northwestern Europe milder than other places at similar latitudes.

Which choice completes the text so that it conforms to the conventions of Standard English?

A poles, warm **B** poles warm **C** poles; warm **D** poles, and, warm

17 RW-T1M1-17

> Historians who study trade in the medieval Mediterranean often rely on letters written by merchants. A single bundle of these letters, sent between business partners in different ports, _____ details about prices, shipping delays, and trustworthy agents that official records rarely mention.

Which choice completes the text so that it conforms to the conventions of Standard English?

A reveals **B** reveal **C** have revealed **D** are revealing

18 RW-T1M1-18

> Photographers who work outdoors at night, when little natural light is available, often use exposures lasting several seconds so that their cameras can gather enough light. A long exposure brightens a dim _____ it also records any movement, so a passing car becomes a streak of light and swaying branches turn into a soft blur.

Which choice completes the text so that it conforms to the conventions of Standard English?

A scene, however, **B** scene however **C** scene; however; **D** scene; however,

19 RW-T1M1-19

> A bat's winter hibernation is broken up by short bursts of activity. Every few weeks, a hibernating bat briefly _____ its body temperature back to normal levels, a process that uses up a large share of the fat it stored in the fall. For this reason, bats that are disturbed during winter may run out of energy before insects return.

Which choice completes the text so that it conforms to the conventions of Standard English?

A raising **B** to raise **C** raises **D** having raised

20 RW-T1M1-20

> Choreographers sometimes build an entire dance from one ordinary movement. In a piece imagined by a student choreographer, a single gesture—the motion of tying a _____ becomes the source of every phrase the dancers perform, appearing at different speeds, in reverse, and in mirror image.

Which choice completes the text so that it conforms to the conventions of Standard English?

A shoe, **B** shoe— **C** shoe; **D** shoe

21 RW-T1M1-21

> Trade across the Indian Ocean linked East Africa, Arabia, India, and Southeast Asia for many centuries, carrying goods such as textiles, spices, and ceramics between distant ports. Long before European ships began arriving in the late fifteenth century, sailors who crossed this ocean _____ to time their voyages by the monsoon, a wind system that reverses direction with the seasons.

Which choice completes the text so that it conforms to the conventions of Standard English?

A learn **B** will learn **C** have learned **D** had learned

22 RW-T1M1-22

> For a hypothetical study of memory for everyday objects, adults were asked to draw a familiar coin without looking at one. The results were _____ participants had handled the coin thousands of times, most of them left out or misplaced basic features, such as the direction a portrait faced and the position of the date.

Which choice completes the text so that it conforms to the conventions of Standard English?

A striking: although **B** striking, although **C** striking although **D** striking, although,

23 RW-T1M1-23

> In classical ballet, dancers often try to make difficult movements look effortless, rising onto the tips of their toes and leaping as if gravity had little hold on them. _____ many modern dance choreographers of the twentieth century emphasized weight and effort, asking dancers to fall, roll, and press their bodies into the floor.

Which choice completes the text with the most logical transition?

A Similarly, **B** For example, **C** By contrast, **D** As a result,

24 RW-T1M1-24

> Cold water can hold more dissolved oxygen than warm water can. As the upper layer of the ocean warms, it also becomes less dense than the colder water beneath it, so the two layers mix less readily, and mixing is one of the main ways oxygen from the surface reaches deeper water. _____ many ocean regions have seen the amount of oxygen below the surface decline in recent decades, a trend that can crowd fish and other animals into a thinner layer of oxygen-rich water near the top.

Which choice completes the text with the most logical transition?

A In contrast, **B** Consequently, **C** Nevertheless, **D** Instead,

25 RW-T1M1-25

> While researching a topic, a student has taken the following notes:
>
> • Mansa Musa ruled the West African empire of Mali in the early fourteenth century.
>
> • In 1324, he made a pilgrimage from Mali to Mecca.
>
> • Historical accounts report that his caravan included some 60,000 people.
>
> • He stopped in Cairo, where his spending and gift giving were reportedly so lavish that they lowered the value of gold there.
>
> • He helped make the city of Timbuktu a major center of trade and Islamic learning.

The student wants to emphasize the size of Mansa Musa's caravan. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A When Mansa Musa traveled to Mecca in 1324, his caravan reportedly included some 60,000 people.

B Mansa Musa, who ruled the West African empire of Mali, made a pilgrimage to Mecca in 1324.

C During his 1324 pilgrimage, Mansa Musa stopped in Cairo, where his lavish spending reportedly lowered the value of gold.

D Mansa Musa helped make the city of Timbuktu a major center of trade and Islamic learning.

26 RW-T1M1-26

While researching a topic, a student has taken the following notes:

- Anna Atkins (1799–1871) was an English botanist and photographer.
- She used the cyanotype, a process devised by the scientist John Herschel in 1842.
- To make a cyanotype, an object is laid on chemically treated paper and exposed to sunlight; after the paper is washed in water, the uncovered areas turn deep blue.
- Atkins used the process to record specimens of algae found in the British Isles.
- The first part of her work *Photographs of British Algae: Cyanotype Impressions* appeared in 1843.

The student wants to introduce *Photographs of British Algae: Cyanotype Impressions* to an audience unfamiliar with Anna Atkins. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A *Photographs of British Algae: Cyanotype Impressions* first appeared in 1843, the year after the cyanotype process was devised.

B Anna Atkins used a process devised by the scientist John Herschel to record specimens of algae.

C In 1843, Atkins published the first part of *Photographs of British Algae: Cyanotype Impressions*, which recorded algae specimens.

D English botanist and photographer Anna Atkins created *Photographs of British Algae: Cyanotype Impressions*, which first appeared in 1843, by laying algae specimens on chemically treated paper and exposing them to sunlight.

27 RW-T1M1-27

While researching a topic, a student has taken the following notes:

- Biologists in a hypothetical study asked whether squirrels find the nuts they bury by remembering the locations or by smell alone.
- Each of 12 squirrels was tested separately and buried 10 hazelnuts in its own outdoor enclosure.
- Researchers then buried 10 hazelnuts in other spots in each enclosure.
- If squirrels relied on smell alone, they would be expected to find the researchers' nuts as often as their own.
- When released into their enclosures two days later, the squirrels dug up their own nuts far more often than the researchers' nuts.

The student wants to explain how the study's design allowed the biologists to test between the two possible explanations. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A The squirrels dug up their own nuts far more often than they dug up the nuts buried by the researchers.

B Because the enclosure held both a squirrel's own nuts and an equal number buried by the researchers, a squirrel relying on smell alone would be expected to find the researchers' nuts as often as its own.

C Each of the 12 squirrels buried 10 hazelnuts in an outdoor enclosure and was released back into it two days later.

D The biologists concluded that memory, not smell alone, guided the squirrels' searches for their buried nuts.

STOP. If you finish before time is called, check your work in this module only.

Practice Test 1: Reading and Writing, Module 2

27 questions. 32 minutes.

1 RW-T1M2-01

> Kelp forests grow in cool, shallow coastal waters, where the tall seaweed provides food and shelter for fish, sea urchins, and many other animals. When winter storms tear large patches of kelp from the seafloor, the animals that depend on it may have to _____ to other parts of the coast until the kelp grows back.

Which choice completes the text with the most logical and precise word or phrase?

A return **B** contribute **C** relocate **D** surrender

---

2 RW-T1M2-02

> The following text is from Elizabeth Gaskell's 1853 novel *Cranford*. The narrator is describing the residents of a small English town.
>
> I imagine that a few of the gentlefolks of Cranford were poor, and had some difficulty in making both ends meet; but they were like the Spartans, and _____ their smart under a smiling face. We none of us spoke of money, because that subject savoured of commerce and trade, and though some might be poor, we were all aristocratic.

Which choice completes the text with the most logical and precise word or phrase?

A confessed **B** multiplied **C** forgot **D** concealed

---

3 RW-T1M2-03

> Psychologists use the term "spotlight effect" for people's tendency to overestimate how much others notice their appearance and actions. In a hypothetical study, students who had been asked to wear an oversized, brightly colored hat into a crowded lecture hall later guessed that about half of their classmates had noticed it; in fact, fewer than a quarter had. The researchers proposed that the error arises because people are so _____ their own perspective that they struggle to judge how little attention others actually pay them.

Which choice completes the text with the most logical and precise word or phrase?

A absorbed in **B** skeptical of **C** detached from **D** dissatisfied with

---

4 RW-T1M2-04

> The following text is from Nathaniel Hawthorne's 1851 novel *The House of the Seven Gables*. Hepzibah Pyncheon has entered a room that holds a portrait of her ancestor Colonel Pyncheon.
>
> Face to face with this picture, on entering the apartment, Miss Hepzibah Pyncheon came to a pause; regarding it with a singular scowl, a strange contortion of the brow, which, by people who did not know her, would probably have been interpreted as an expression of bitter anger and ill-will. But it was no such thing. She, in fact, felt a reverence for the pictured visage, of which only a far-descended and time-stricken virgin could be susceptible; and this forbidding scowl was the _____ result of her near-sightedness, and an effort so to concentrate her powers of vision as to substitute a firm outline of the object instead of a vague one.

Which choice completes the text with the most logical and precise word or phrase?

A calculated **B** innocent **C** spiteful **D** theatrical

5 RW-T1M2-05

> Kofi had spent the whole summer building a small sailboat in his uncle's garage, and on launch day half the neighborhood came down to the lake to watch. He pushed off from the dock, raised the sail, and waited for the wind to carry him out. The boat turned in a slow, stubborn circle and drifted back toward the dock, where his little sister was already laughing. Kofi's face burned. Then his uncle waded in up to his knees, reached under the stern, and calmly tightened the rudder pin that Kofi had forgotten to secure.

Which choice best describes the function of the underlined sentence in the text as a whole?

A It explains why Kofi decided to build a sailboat.

B It presents a setback that leads to Kofi's embarrassment and to his uncle's help.

C It suggests that the neighbors expected the boat to fail.

D It shows the sailing skill that Kofi gained during the summer.

6 RW-T1M2-06

> Insect-eating bats are best known for hunting by echolocation: they emit high-pitched calls and locate prey from the returning echoes. During a hypothetical field study, however, a population of such bats kept catching insects at nearly their usual rate on moonlit nights even when researchers played recorded noise that drowned out the echoes. On dark nights, the same noise sharply reduced the bats' success. Further tests showed that in bright moonlight the bats could spot insects outlined against the sky. The researchers propose that these bats shift between senses as conditions change, relying more on sight when there is enough light to see.

Which choice best describes the overall structure of the text?

A It explains how echolocation works, then argues that the method researchers used to test it was flawed.

B It describes a well-known behavior, presents evidence that the behavior is rare, and concludes that vision matters more to bats than echolocation does.

C It compares the hunting success of bats on two kinds of nights, then recommends that future studies take place only on dark nights.

D It notes how a group of animals is known to hunt, reports findings that complicate that picture, and offers an explanation that accounts for those findings.

7 RW-T1M2-07

> **Text 1**
>
> A photograph used as evidence, whether in a newspaper or a history book, earns trust by recording a scene without interference. Once a photographer asks subjects to repeat a gesture or moves an object into the frame, the image becomes a kind of illustration. Such pictures may be striking, but they should not be presented as documents of what happened.
>
> **Text 2**
>
> Purists sometimes argue that a documentary photograph loses its value as a record the moment a photographer intervenes. Yet every photograph involves intervention: someone chose where to stand, what to leave out of the frame, and which instant to capture. These choices shape a viewer's understanding as surely as a staged gesture does. What matters is whether the image, and the caption beside it, honestly reflect the event.

Based on the texts, how would the author of Text 2 most likely respond to the argument in Text 1?

A By contending that Text 1 draws too sharp a line, since even photographs taken without staging reflect choices that shape what viewers understand

B By agreeing that staged photographs should never appear in newspapers or history books

C By arguing that staged photographs are more truthful than unstaged ones because they show what the photographer intended

D By claiming that viewers rarely notice whether a photograph has been staged, so the distinction does not matter

---

8 RW-T1M2-08

> When the letter from the music conservatory arrived, Mrs. Okafor read it twice at the kitchen table before handing it to her son. Chidi scanned it, grinned, and began talking at once about the city, the orchestra, and the practice rooms that stayed open all night. His mother nodded and smiled and asked sensible questions about train fares and winter coats. Only after he had gone upstairs to call his friends did she sit down again, smooth the letter flat with both hands, and look for a long time at the empty chair across from her.

Based on the text, what is true about Mrs. Okafor?

A She doubts that Chidi is talented enough to succeed at the conservatory.

B She is angry that Chidi applied to the conservatory without asking her.

C She supports Chidi outwardly while privately feeling the loss that his departure will bring.

D She plans to move to the city so that she can stay near Chidi.

---

9 RW-T1M2-09

> A wildlife biologist fitted four red foxes in a hypothetical study area with radio collars and recorded how far each fox traveled per night, on average, in summer and in winter. Every fox traveled farther per night in winter than in summer. The fox with the greatest average distance per night in both seasons was _____

Average Distance Traveled per Night by Four Red Foxes (hypothetical data)

Fox	Summer (km per night)	Winter (km per night)
Juniper	6.2	8.9
Cedar	7.8	10.4
Willow	5.1	7.3
Aspen	6.9	9.6

Which choice most effectively uses data from the table to complete the statement?

A Juniper. **B** Cedar. **C** Willow. **D** Aspen.

10 RW-T1M2-10

> Anne Brontë's 1847 novel *Agnes Grey* is narrated by Agnes, a young woman whose family is struggling with money troubles. When Agnes announces that she would like to work as a governess, her mother and sister doubt that she is ready. Agnes insists that her family underestimates her because they have never seen her abilities put to the test.

Which quotation from *Agnes Grey* most effectively illustrates the claim?

A "I wish *I* could do something," said I.

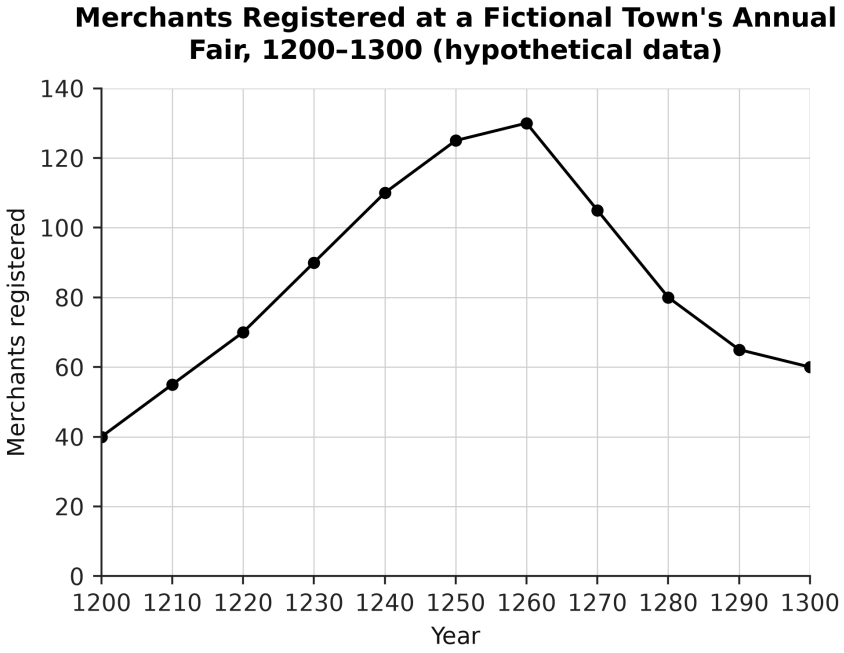
B "I do not pretend to be able to instruct great girls; but surely I could teach little ones: and I should like it so much: I am so fond of children."

C "Mary got her drawing materials, and steadily set to work. I got mine too; but while I drew, I thought of other things."

D "You do not know half the wisdom and prudence I possess, because I have never been tried."

11 RW-T1M2-11

> Historians studying a fictional medieval town have used its records to track how many merchants registered each year to sell goods at the town's annual fair. One historian argues that the fair thrived until a new road, completed in 1265, let traders bypass the town and travel directly to a larger market, after which the fair steadily lost merchants.



Which choice best describes data from the graph that support the historian's argument?

A The number of registered merchants was higher in 1300 than in 1200.

B The largest number of merchants registered in any year shown was 130.

C The number of registered merchants rose at every point shown from 1200 to 1260 and fell at every point shown from 1270 to 1300.

D The number of registered merchants fell between 1200 and 1260 and then rose after 1265.

12 RW-T1M2-12

> Many reef-building corals host one of several types of symbiotic algae, which live inside the coral's tissue and supply it with food. Marine biologists studying a hypothetical reef observed that after an unusually warm summer, colonies of one coral species that hosted mainly algae of type D bleached less than colonies of the same species that hosted mainly type C. The biologists hypothesize that type D algae make corals more tolerant of heat; an alternative explanation is that the type D colonies simply happened to grow in cooler parts of the reef.

Which finding, if true, would most directly support the biologists' hypothesis?

A When colonies hosting mainly type D algae and colonies hosting mainly type C algae were grown side by side in tanks at the same high temperature, the type D colonies bleached less.

B On the reef, colonies hosting mainly type D algae were found mostly in deeper areas, where the water stays cooler during summer.

C Both type C and type D algae supply corals with food that the algae produce through photosynthesis.

D After the warm summer, some colonies hosting mainly type C algae recovered their normal color within several months.

---

13 RW-T1M2-13

> Along some coastlines, winds push surface water away from shore, and cold water from the deep ocean rises to replace it. This process, called upwelling, carries nutrients from the depths toward the surface, where they feed tiny algae that in turn support fish. In a hypothetical study, researchers compared two years along one such coast: in the first year, the winds that drive upwelling blew steadily all spring, while in the second year they were unusually weak. Compared with the first year, the second year most likely had _____

Which choice most logically completes the text?

A fewer algae near the surface, because fewer nutrients rose from the deep ocean.

B colder surface water near the shore, because more deep water reached the surface.

C more fish near the shore, because the weaker winds made the water calmer.

D the same amount of algae, because algae depend on sunlight rather than on nutrients.

---

14 RW-T1M2-14

> A great deal of what historians know about farming in the ancient Roman countryside comes from agricultural handbooks written by wealthy landowners. These handbooks offer detailed advice on managing estates, from planting vines to organizing workers, and they are invaluable for understanding how such estates were meant to run. Yet their authors wrote from the perspective of owners seeking profit, and few comparable writings survive from the tenants, hired laborers, and enslaved workers who did most of the farming. A historian relying mainly on these handbooks should therefore recognize that they are likely to reveal more about _____

Which choice most logically completes the text?

A the techniques that ordinary farmers developed independently of landowners.

B how rural life actually felt to the people who performed most of the labor.

C the reasons that so few writings by rural laborers have survived.

D landowners' goals and expectations for their estates than about workers' own experiences.

---

15 RW-T1M2-15

> A camera lens with an adjustable diaphragm controls the amount of light that reaches the film or sensor through _____ aperture, an opening that can be widened or narrowed. At the same focal length and focus distance, a wider aperture lets in more light and reduces depth of field, which can make the background look softer and more blurred.

Which choice completes the text so that it conforms to the conventions of Standard English?

A it's **B** its **C** its' **D** their

16 RW-T1M2-16

> A hypothetical survey of mammals living along a mountain stream recorded three species at different elevations: water shrews, which hunted aquatic insects near the stream's _____ river otters, which caught fish in deeper pools farther downhill; and beavers, which built dams in the slow, wide stretches near the valley floor.

Which choice completes the text so that it conforms to the conventions of Standard English?

A source, **B** source **C** source; **D** source:

17 RW-T1M2-17

> Caravanserais, roadside inns built around open courtyards, once stood along the trade routes that crossed Central Asia. Because _____ offered water, shelter, and a gate that could be closed at night, merchants traveling with camels and valuable goods often planned their daily routes so that they could reach one before dark.

Which choice completes the text so that it conforms to the conventions of Standard English?

A it **B** this **C** that **D** they

18 RW-T1M2-18

> Dance teachers at a fictional community arts center asked new students which styles interested them most. The style that the largest number of students said they hoped to _____ hip-hop, followed closely by tap and contemporary dance.

Which choice completes the text so that it conforms to the conventions of Standard English?

A learn was **B** learn, was **C** learn: was **D** learn; was

19 RW-T1M2-19

> In the far North Atlantic, surface water that has grown cold and salty near Greenland sinks toward the seafloor and begins to flow slowly south. This water, which may not return to the surface for centuries, _____ part of a vast system of ocean currents that moves heat, salt, and dissolved gases around the globe and helps shape climates far from where the water first sank.

Which choice completes the text so that it conforms to the conventions of Standard English?

A forming **B** having formed **C** forms **D** to form

20 RW-T1M2-20

> Reconstructing the government of the Byzantine Empire often means piecing together scattered evidence. Byzantine officials commonly sealed letters and documents with small lead disks stamped with their names and titles. These seals, many of which have survived even though the documents they once closed were _____ give historians a detailed record of who held which offices in different parts of the empire and when.

Which choice completes the text so that it conforms to the conventions of Standard English?

A lost— **B** lost, **C** lost; **D** lost

21 RW-T1M2-21

> A photographer documenting a remote mountain valley chose a large-format camera and a heavy tripod to keep it steady during long exposures. The camera exposed a single large sheet of film at a time and recorded remarkable detail, but it also demanded patience. Setting up the camera again at each new site along the valley, _____

Which choice completes the text so that it conforms to the conventions of Standard English?

A an hour or more often passed before the light changed enough for a single picture.

B the light often took an hour or more to change before a single picture was taken.

C a single picture often required waiting an hour or more for the light to change.

D she often waited an hour or more for the light to change before taking a single picture.

22 RW-T1M2-22

> At a hypothetical wildlife rehabilitation center, orphaned harbor seal pups pass through several stages of care before being released. First, staff members feed each pup a nutrient-rich formula until it reaches a healthy weight. _____ the pups are moved to a shallow pool, where they learn to catch live fish on their own. Finally, after passing a health check, they are released into the ocean near where they were found.

Which choice completes the text with the most logical transition?

A Nevertheless, **B** For instance, **C** Next, **D** In contrast,

23 RW-T1M2-23

> Social psychologists have found that people are often less likely to help someone in need when other bystanders are present. _____ in one hypothetical experiment, participants who heard a staged call for help while alone went to assist about 80 percent of the time, while those who heard the same call alongside two silent strangers did so less than half the time.

Which choice completes the text with the most logical transition?

A For example, **B** However, **C** Similarly, **D** As a result,

24 RW-T1M2-24

> Choreographers who create dances for film must work around the camera: a close-up can hide a dancer's footwork, and a quick cut can break a phrase that was meant to flow without interruption. _____ photographers who document live dance performances must decide which single instant will stand for a movement that unfolds over several seconds, knowing that whatever falls outside the frame or between shutter clicks will be lost.

Which choice completes the text with the most logical transition?

A Instead, **B** Likewise, **C** For this reason, **D** Even so,

---

25 RW-T1M2-25

> While researching a topic, a student has taken the following notes:
>
> • In a hypothetical study, biologists observed two groups of meerkats living in the same desert reserve.
>
> • Meerkat sentries watch for predators while other members of the group dig for food.
>
> • The North group had 18 members; the South group had 9.
>
> • In the North group, a sentry stood guard on about 90 percent of foraging days.
>
> • In the South group, a sentry stood guard on about 60 percent of foraging days.

The student wants to emphasize a difference in sentry behavior between the two groups. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Both groups of meerkats lived in the same desert reserve, where biologists observed them.

B Meerkat sentries watch for predators while other members of the group dig for food.

C The North group had 18 members, twice as many as the South group's 9.

D A sentry stood guard on about 90 percent of foraging days in the North group but on only about 60 percent in the South group.

---

26 RW-T1M2-26

> While researching a topic, a student has taken the following notes:
>
> - Psychologists use the term "mere exposure effect" for the tendency of people to prefer things simply because they have encountered them before.
> - In one hypothetical study, participants viewed unfamiliar abstract shapes; some shapes were shown once and others ten times.
> - Participants rated the shapes they had seen ten times as more pleasing.
> - In a second hypothetical study, participants heard short melodies in an unfamiliar musical style; some melodies were played once and others eight times.
> - Participants rated the melodies they had heard eight times as more enjoyable.

The student wants to emphasize a similarity between the findings of the two studies. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A One study used abstract shapes, while the other used melodies in an unfamiliar musical style.

B The mere exposure effect is the tendency of people to prefer things simply because they have encountered them before.

C In both studies, participants preferred the items they had encountered more often, whether those items were shapes or melodies.

D Participants in the first study saw some shapes ten times, while participants in the second study heard some melodies eight times.

27 RW-T1M2-27

While researching a topic, a student has taken the following notes:

- Labanotation is a system for recording human movement in written symbols.
- It was developed by the Hungarian-born dance theorist Rudolf Laban, who published a version of the system in 1928.
- A vertical three-line staff represents the dancer's body; symbols on the staff show the direction, level, and duration of a movement and which body part moves.
- The Dance Notation Bureau, founded in New York City in 1940, used the system to notate choreographic works so that dance would have a lasting written record.
- Labanotation became part of dance education at colleges and universities, including Ohio State University.

The student wants to explain the significance of labanotation. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Labanotation, which Rudolf Laban presented in 1928, uses a vertical three-line staff to represent the dancer's body.

B By allowing dances to be recorded in written symbols, labanotation gave choreographic works a lasting written record and became part of dance education at colleges and universities.

C In labanotation, symbols placed on a staff show the direction, level, and duration of a movement and which part of the body moves.

D Rudolf Laban was a Hungarian-born dance theorist who published a version of labanotation in 1928.

STOP. If you finish before time is called, check your work in this module only.

Math Reference Sheet

Use these facts during both Math modules. Your test-day reference sheet may present them differently.

Topic	Formula or fact
Circle	$A = \pi r^2$ $C = 2\pi r$
Rectangle	$A = \ell w$
Triangle	$A = (1/2)bh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Special right triangles	30°-60°-90°: sides x, $x\sqrt{3}$, 2x 45°-45°-90°: sides s, s, $s\sqrt{2}$
Rectangular prism	$V = \ell wh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = (4/3)\pi r^3$
Cone	$V = (1/3)\pi r^2 h$
Pyramid	$V = (1/3)\ell wh$
Angles and arcs	A circle has 360° of arc, or 2π radians. The angles of a triangle sum to 180°.

Practice Test 1: Math, Module 1

22 questions. 35 minutes. A calculator is allowed on every question. Use the reference sheet before Module 1.

1 M-T1M1-01

What value of x is the solution to the equation 5x + 14 = 2x + 35?

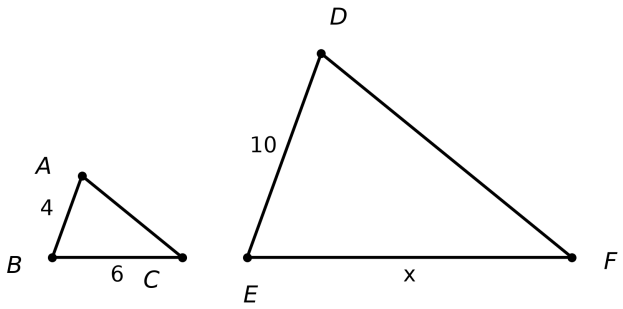
A 3 **B** 7 **C** 49/3 **D** 21

2 M-T1M1-02

A car rental company charged $56 per day to rent a midsize car last year. This year the daily rate is 15% greater than last year's rate. What is this year's daily rate?

A $8.40 **B** $47.60 **C** $56.15 **D** $64.40

3 M-T1M1-03



In the figure, triangle ABC is similar to triangle DEF, where vertices A, B, and C correspond to vertices D, E, and F, respectively. What is the value of x?

A 20/3 **B** 12 **C** 15 **D** 25

4 M-T1M1-04

x	f(x)
1	8
3	14
5	20

The table shows three values of x and their corresponding values of f(x) for the linear function f. Which equation defines f?

A f(x) = 3x + 5 **B** f(x) = 3x + 8 **C** f(x) = 6x + 2 **D** f(x) = 8x

5 M-T1M1-05

What value of x is the solution to the equation (2/3)(x − 4) = 10?

Student-produced response. Write your answer on the answer sheet or below.

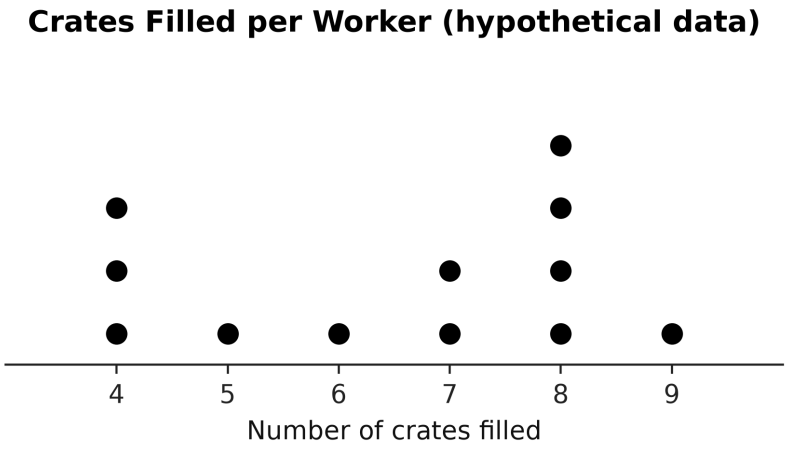
Answer: _____

6 M-T1M1-06

Which expression is equivalent to $25x^2 - 36$?

A $(5x - 6)(5x + 6)$ **B** $(5x - 6)^2$ **C** $(25x - 6)(x + 6)$ **D** $(5x - 18)(5x + 18)$

7 M-T1M1-07



The dot plot shows the number of crates of plums filled by each of 12 workers during one morning at a hypothetical orchard. What is the median number of crates filled?

A 4 **B** 6.5 **C** 7 **D** 8

8 M-T1M1-08

In the xy-plane, line ℓ has the equation $2y - 7x = 6$. Line m is perpendicular to line ℓ. What is the slope of line m?

A $-7/2$ **B** $-2/7$ **C** $2/7$ **D** $7/2$

9 M-T1M1-09

A conveyor belt at a shipping warehouse moves packages at a constant speed of 48 feet per minute. What is this speed in inches per second? (12 inches = 1 foot)

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

10 M-T1M1-10

What is the sum of the solutions to the equation $4x^2 + 5x - 6 = 0$?

A $-5/4$ **B** 1 **C** 5/4 **D** 11/4

11 M-T1M1-11

When a rental car with a full tank is driven on a highway, the function $G(m) = 14 - m/28$ estimates the number of gallons of fuel left in the tank after m miles. What is the best interpretation of 28 in this context?

A The fuel tank holds 28 gallons when it is full.

B The car uses 28 gallons of fuel per mile.

C The car is driven about 28 miles for each gallon of fuel used.

D The car can be driven 28 miles before the tank is empty.

12 M-T1M1-12

$y > x - 3$

$2x + 3y \leq 12$

Which point (x, y) is a solution to the given system of inequalities?

A (0, 5) **B** (2, −1) **C** (3, 2) **D** (5, 0)

13 M-T1M1-13

$6x + 5y = 15$

$x - 5y = 20$

The solution to the given system of equations is (x, y). What is the value of y?

Student-produced response. Write your answer on the answer sheet or below.

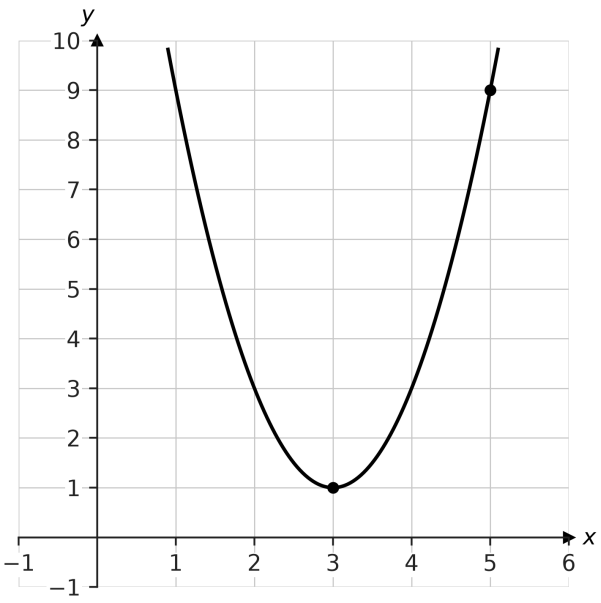
Answer: _____

14 M-T1M1-14

A grain silo on a hypothetical farm is a right circular cylinder with an interior diameter of 6 meters and an interior height of 15 meters. The silo is filled with grain to 80% of its volume. Which choice is closest to the volume of the grain, in cubic meters?

A 226 **B** 339 **C** 424 **D** 1,357

15 M-T1M1-15



The graph of the quadratic function f is shown in the xy-plane. Which equation could define f?

A $f(x) = 2(x + 3)^2 + 1$ **B** $f(x) = (x - 3)^2 + 1$ **C** $f(x) = 2(x - 3)^2 - 1$ **D** $f(x) = 2(x - 3)^2 + 1$

16 M-T1M1-16

Preferred New Park Feature (hypothetical data)

Age group	Splash pad	Dog run	Total
Under 18	48	32	80
18 or older	60	100	160
Total	108	132	240

In a hypothetical survey, 240 park visitors chose a preferred new feature; the table shows the results. A visitor is selected at random from those who are under 18 or prefer the splash pad, or both. What is the probability that this visitor is 18 or older and prefers the splash pad?

A 1/4 **B** 3/7 **C** 5/9 **D** 7/12

17 M-T1M1-17

For $x > 0$, the expression $(x^3 \cdot \sqrt[3]{(x^2)})/\sqrt{(x^5)}$ is equivalent to x^k, where k is a constant. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

18 M-T1M1-18

$3x^2 - 12x + k = 0$

In the given equation, k is a constant. The equation has exactly one real solution. What is the value of k?

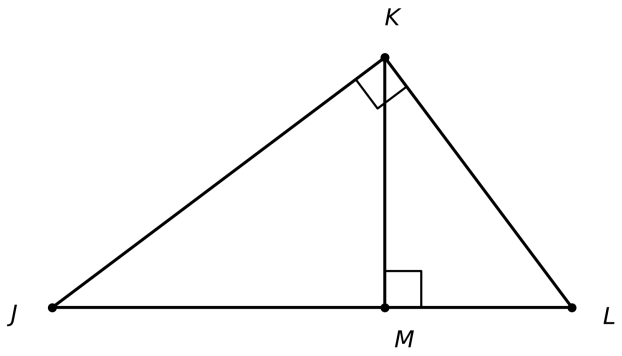
A 12 **B** 36 **C** 48 **D** 144

19 M-T1M1-19

A shipping clerk sent 96 packages in one day: small ones weighing 5 pounds each and large ones weighing 12 pounds each, for a total of 690 pounds. Shipping costs $7 per small package and $15 per large package. What was the total shipping charge for the 96 packages?

A $450 **B** $462 **C** $912 **D** $1,200

20 M-T1M1-20



In the figure, triangle JKL has a right angle at K, and segment KM is perpendicular to segment JL. If JL = 25 and sin J = 3/5, what is the length of KM?

A 12 **B** 15 **C** 16 **D** 20

---

21 M-T1M1-21

The function f is defined by $f(x) = a(b^x)$, where a and b are positive constants. If f(0) = 300 and f(2) = 192, what is the value of f(3)?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

---

22 M-T1M1-22

The graph of y = f(x) in the xy-plane is a parabola that opens downward and has its vertex at (−3, 7). The function g is defined by g(x) = −f(x − 5) + 2. Which statement about the graph of y = g(x) is true?

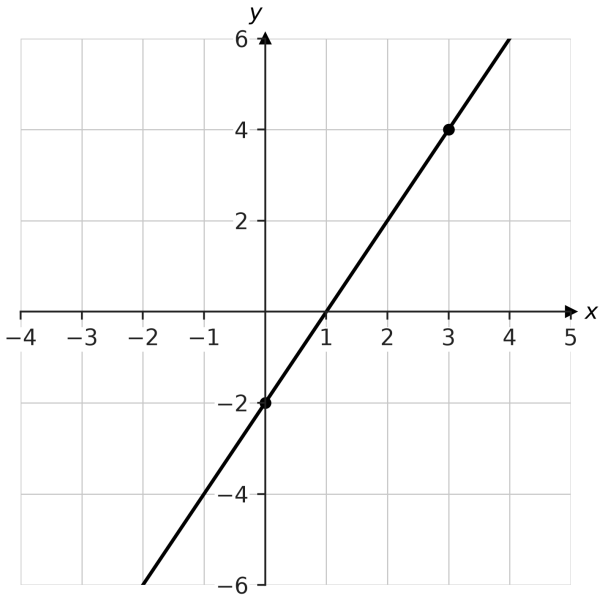
A It has a maximum value of 9 at x = 2.
B It has a minimum value of −5 at x = −8.
C It has a minimum value of −9 at x = 2.
D It has a minimum value of −5 at x = 2.

STOP. If you finish before time is called, check your work in this module only.

Practice Test 1: Math, Module 2

22 questions. 35 minutes. A calculator is allowed on every question. Use the reference sheet before Module 1.

1 M-T1M2-01



The graph of the linear function f is shown in the xy-plane. Which equation defines f?

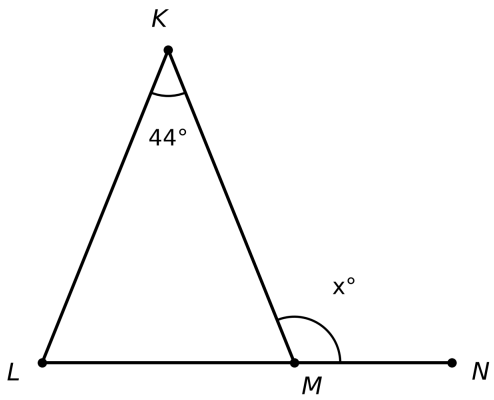
A f(x) = (1/2)x − 2 **B** f(x) = 2x + 1 **C** f(x) = −2x − 2 **D** f(x) = 2x − 2

2 M-T1M2-02

Which expression is equivalent to $x^2 + 2x - 35$?

A (x + 7)(x − 5) **B** (x − 7)(x + 5) **C** (x + 7)(x + 5) **D** (x + 35)(x − 1)

3 M-T1M2-03



In the figure, KL = KM, and point N lies on the extension of segment LM. What is the value of x?

A 68 **B** 88 **C** 112 **D** 136

4 M-T1M2-04

On a scale drawing of a proposed park, 1 inch represents 40 feet. A walking path is 6.5 inches long on the drawing. What is the actual length, in feet, of the walking path?

Student-produced response. Write your answer on the answer sheet or below.

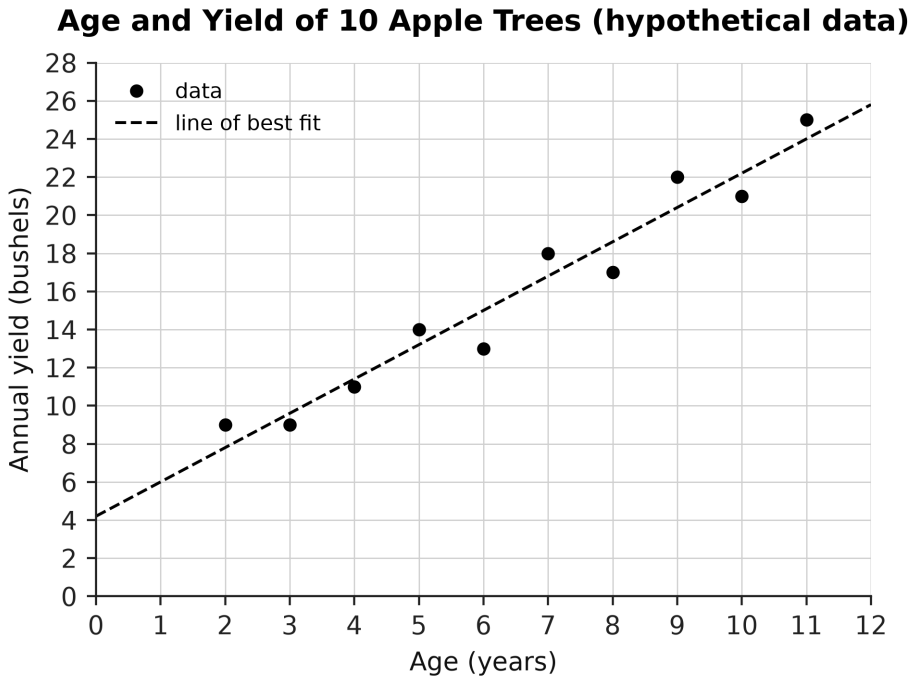
Answer: _____

5 M-T1M2-05

What value of x satisfies the equation $x/4 + (x - 2)/6 = 3$?

A 7/5 **B** 38/5 **C** 8 **D** 16

6 M-T1M2-06



The scatterplot shows the age and the annual yield of 10 apple trees at a hypothetical orchard, along with the line of best fit $y = 1.8x + 4.2$. How many more bushels did the 7-year-old tree yield than the line of best fit predicts?

A 1.2 **B** 1.8 **C** 16.8 **D** 18.0

7 M-T1M2-07

What is the solution to the equation $\sqrt{(x + 37)} = x - 5$?

A −1 **B** 1 **C** 4 **D** 12

8 M-T1M2-08

For the linear function f, $f(1) = 10$ and $f(5) = -2$. What is the value of $f(-4)$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

9 M-T1M2-09

A town is buying benches at $350 each and trees at $120 each for a new park. It will buy exactly 12 benches, and the total cost of the benches and trees must be at most $8,000. What is the greatest number of trees the town can buy?

A 31 **B** 32 **C** 63 **D** 66

10 M-T1M2-10

A playground in a park design is shaped like a rectangle that measures 30 meters by 18 meters, with a semicircle attached along one of its 18-meter sides so that this side is the semicircle's diameter. Which choice is closest to the total area, in square meters, of the playground?

A 568 **B** 667 **C** 794 **D** 1,049

11 M-T1M2-11

$7x + 4y = 37$
$5x + 8y = 38$
If (x, y) is the solution to the given system of equations, what is the value of $x + y$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

12 M-T1M2-12

Which expression is equivalent to $(27x^6)^{2/3}$, where $x > 0$?

A $3x^4$ **B** $18x^4$ **C** $9x^4$ **D** $9x^{20/3}$

13 M-T1M2-13

In a hypothetical survey, a shipping company selected a random sample of 600 of its customers. Of the customers sampled, 54% said they would pay extra for Saturday delivery. The margin of error for this estimate is 4 percentage points. Which conclusion is most appropriate?

A Exactly 54% of all the company's customers would pay extra for Saturday delivery.

B It is plausible that between 50% and 58% of all the company's customers would pay extra for Saturday delivery.

C The margin of error means the percentage among the 600 sampled customers could be any value from 50% to 58%.

D It is plausible that between 50% and 58% of the customers of every shipping company would pay extra for Saturday delivery.

14 M-T1M2-14

A hypothetical orchard applies 120 kilograms of a slow-release fertilizer to a field. The amount of fertilizer remaining in the soil decreases by 15% each week. Which function gives the amount A(d), in kilograms, of fertilizer remaining d days after it is applied?

A $A(d) = 120(0.85)^{d/7}$ **B** $A(d) = 120(0.85)^{7d}$

C $A(d) = 120(0.15)^{d/7}$ **D** $A(d) = 120(1 - 0.15/7)^{d}$

15 M-T1M2-15

$(2x - 1)^2 - 5(2x - 1) - 6 = 0$

What is the greatest solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

16 M-T1M2-16

A rental car costs $25 per day plus $0.40 per mile. The equation $25d + 0.4m = 400$ gives the combinations of d days and m miles that cost exactly $400. If d increases by 2 and the total cost stays $400, how does m change?

A It decreases by 20 miles.

B It decreases by 50 miles.

C It decreases by 125 miles.

D It increases by 125 miles.

17 M-T1M2-17

In triangle RST, angle T is a right angle and $\sin R = 21/29$. Triangle XYZ is similar to triangle RST, with vertices X, Y, and Z corresponding to vertices R, S, and T, respectively. If $XZ = 40$, what is the length of YZ?

A 21

B 840/29

C 800/21

D 42

18 M-T1M2-18

The function f is defined by $f(x) = a(x - 4)^2 + c$, where a and c are constants. If $f(2) = 11$ and $f(7) = 26$, what is the minimum value of f?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

19 M-T1M2-19

$(k + 1)x + 4y = 10$

$3x + ky = 6$

In the given system of equations, k is a constant. What is the sum of all values of k for which the system has no solution?

A −12

B −1

C 1

D 3

20 M-T1M2-20

$x^2 - 2kx + (k + 6) = 0$

In the given equation, k is a constant. For which of the following values of k does the equation have no real solutions?

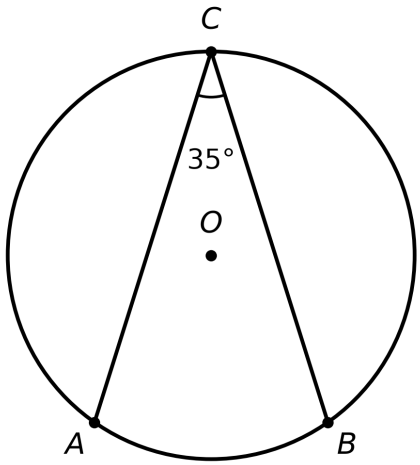
A −3

B 2

C 3

D 6

21 M-T1M2-21



In the figure, points A, B, and C lie on a circle with center O and a radius of 36. Angle ACB measures 35°. The length of minor arc AB is kπ, where k is a constant. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

22 M-T1M2-22

The function f is defined by $f(x) = 3(2^x)$. The function g is defined by $g(x) = f(x + 2)$. If g(x) is written in the form $g(x) = a(2^x)$, where a is a constant, what is the value of a?

A 3/4 **B** 6 **C** 7 **D** 12

STOP. If you finish before time is called, check your work in this module only.

CHAPTER 30

Practice Test 1: Answers, Scoring and Review

A practice test pays off in the review, not in the score. This chapter shows you how to check your answers, calculate percent correct, find your weakest domains and turn every miss into a plan for the weeks before Practice Test 2. Set aside two to three hours for the full review, split across two days if you like. The answer key, the score tally and the explanations for all 98 questions follow this chapter's text.

How to check your answers

The answer key that follows lists the correct answer for every question, module by module, along with the skill each question tests and the chapter to review.

1. Go through one module at a time. Mark each question you answered correctly, each one you missed and each one you left blank.
2. Circle every question you marked "g" as a guess, even if the guess was right. A correct guess counts toward your total, but you will review it as a miss.
3. A student-produced response is correct if it gives the keyed value in an allowed form. The key lists examples, not every equivalent entry: 3/4, .75 and 0.75, for example, name the same value. Apply Chapter 15's special rules for long or repeating decimals. If you wrote the right value in a form the entry rules do not allow, such as the mixed number 3 1/2, count it as wrong, because it would not be accepted on test day.
4. Fill in the Practice Test 1 Score Tally after the answer key. It groups the questions by domain, so write your number correct in each domain row and in each section's total row.

Score your test

Enter the number correct in each module in Table A. Add the two modules to get each section total. All 98 questions count in this book's review; it does not include hidden unscored pretest items. A correct guess counts in your total, but still belongs in your review list.

Table A: Correct-count worksheet

Section	Module 1 correct	Module 2 correct	Section total	Percent correct
Reading and Writing	___ of 27	___ of 27	___ of 54	___%
Math	___ of 22	___ of 22	___ of 44	___%
Whole test	—	—	___ of 98	___%

NOTE These fixed practice tests have not been statistically calibrated to the SAT. Their correct counts cannot support a defensible conversion to the 200–800 section scales or the 400–1600 total scale. Percent correct describes this set of questions; it is not a percentile rank, a college-readiness measure or a prediction of your SAT score. For practice scores on the SAT scale, use the College Board's full-length tests in Bluebook.

Calculate percent correct

Divide the number correct by the number of questions, then multiply by 100. Round to the nearest whole percent for your worksheet. Keep the original counts beside the percentage so that you can see the sample size.

Table B: Calculation examples

Result	Calculation	Percent correct
41 Reading and Writing answers correct	41 ÷ 54 × 100	about 76%
30 Math answers correct	30 ÷ 44 × 100	about 68%
71 answers correct across both sections	71 ÷ 98 × 100	about 72%

The whole-test percentage uses all 98 questions. Do not add the section percentages or average them: the sections contain different numbers of questions. Use section and domain results to choose what to study rather than relying only on the whole-test total.

Table C: Timing and confidence check

Measure	Reading and Writing	Math
Questions still blank when time ended	___	___
Answers guessed mainly because time was short	___	___
Correct answers you could not explain confidently	___	___

These rows describe different problems and can overlap, so do not add them to form a score. Review rushed and uncertain answers even when they were correct. If you finish extra questions after time expires, record that work separately from your timed result.

ONLINE (OPTIONAL) The website's scoring guide explains how section and total scores work. This book reports correct counts and percent correct; any online estimates are for study planning, not official scores.

skillbyskillprep.com/scoring

Read your domain results

Your section totals summarize this sitting. Your domain results help you choose what to study next. Copy your number correct for each domain from the score tally into Table D. Then divide by the number of questions and multiply by 100 to get a percent. For example, 11 correct out of 15 in Craft and Structure is 11 ÷ 15 × 100, or about 73%.

Table D: Domain review guide

Domain	Questions on this test	Your number correct	Percent correct	Chapters to review
Craft and Structure	15	___	___	5, 6, 7

Domain	Questions on this test	Your number correct	Percent correct	Chapters to review
Information and Ideas	14	___	___	8, 9, 10
Standard English Conventions	14	___	___	11, 12
Expression of Ideas	11	___	___	13, 14
Algebra	15	___	___	16, 17, 18, 19
Advanced Math	15	___	___	20, 21, 22
Problem-Solving and Data Analysis	7	___	___	23, 24, 25
Geometry and Trigonometry	7	___	___	26, 27, 28

Then use Table E to plan your next step.

Table E: Suggested review priorities

These bands are study-planning suggestions, not validated mastery levels. A percentage alone cannot tell you why you missed questions. Inspect the actual errors before choosing an action.

Percent correct in a domain	Starting review action
Below 50%	Begin with the lessons and worked examples for the skills you missed; then try the Practice Sets.
50–74%	Read each missed answer's explanation, identify whether the problem was knowledge, reasoning or pacing, and schedule redos.
75–89%	Target the specific missed skills and try harder or timed mixed questions where appropriate.
90% and above	Maintain with short review, check every uncertain answer and confirm the skill on unfamiliar questions.

Two cautions apply. First, the smaller domains move fast: Problem-Solving and Data Analysis and Geometry and Trigonometry have only 7 questions each, so one miss costs about 14 percentage points. Second, a domain percent can hide a single weak skill. Use the Skill column of the answer key to see which skills inside a domain caused your misses, and start with the chapter that teaches that skill.

Review every miss

Read the explanation for every question you missed, left blank or guessed, and for any question you got right but felt unsure about. Each explanation tells you why the correct answer works, why each wrong choice fails and which chapter to revisit. Then classify each miss once, using Table F.

Table F: Why did I miss it?

Miss type	How to recognize it	Fix
Content gap	I did not know the rule or method.	Relearn it in the chapter named in the explanation.

Miss type	How to recognize it	Fix
Trap	I chose a tempting wrong answer.	Name the trap type from the explanation and add it to your error log.
Misread	I answered a different question than the one asked.	Underline the task word before solving.
Careless	I knew it but slipped.	Add a 5-second check step to your method.
Time	I rushed or guessed at the end.	Practice the two-pass method from Chapter 3.

Follow this routine for each miss:

1. Read the explanation, including the reason your choice was wrong.
2. Decide which miss type from Table F fits best, and tally it.
3. Log the miss in the error log (Appendix E) within 24 hours. Write the rule or fix in your own words, not copied from the explanation.
4. Redo the question cold after 3 days and again after 10 days, and mark whether you got it right.
5. Look at your tallies. If one miss type dominates, fix the habit as well as the content. Several Time misses call for pacing practice, not more lessons.

Next steps: the weeks before Practice Test 2

Give yourself at least one week between finishing this review and taking Practice Test 2. Two to four weeks is better if your test date allows it. Use that time in this order:

1. **Fix your two weakest domains first.** Follow the Table E actions for the two domains with the lowest percent, starting with the chapters named in your most frequent misses.
2. **Redo your error log.** A redo you get right without help shows that a fix has stuck; one you miss again goes back on the list.
3. **Practice under time.** Once you can explain the methods for the skills you have been reviewing, add timed mixed sets at about 71 seconds per Reading and Writing question and 95 seconds per Math question.
4. **Take a Bluebook practice test.** The free full-length practice tests in the College Board's Bluebook app are adaptive and use the on-screen tools you will see on test day, and report practice scores on the SAT scale. Chapter 2 shows where they fit in your plan.
5. **Take Practice Test 2.** When the error log looks under control, turn to Chapter 31 and take it under the same conditions as this test.

Practice Test 1 Answer Key

Reading and Writing, Module 1

#	Answer	Skill	Review
1	B	Words in Context	Ch. 5, p. 69
2	A	Words in Context	Ch. 5, p. 69
3	D	Words in Context	Ch. 5, p. 69
4	D	Words in Context	Ch. 5, p. 69
5	C	Words in Context	Ch. 5, p. 69
6	A	Text Structure and Purpose	Ch. 6, p. 82
7	B	Text Structure and Purpose	Ch. 6, p. 82
8	C	Cross-Text Connections	Ch. 7, p. 98
9	A	Central Ideas and Details	Ch. 8, p. 112
10	C	Central Ideas and Details	Ch. 8, p. 112
11	D	Command of Evidence: Quantitative	Ch. 9, p. 128
12	B	Command of Evidence: Textual	Ch. 9, p. 128
13	B	Command of Evidence: Quantitative	Ch. 9, p. 128
14	A	Inferences	Ch. 10, p. 150
15	C	Inferences	Ch. 10, p. 150
16	C	Boundaries	Ch. 11, p. 164
17	A	Form, Structure, and Sense	Ch. 12, p. 180
18	D	Boundaries	Ch. 11, p. 164
19	C	Form, Structure, and Sense	Ch. 12, p. 180
20	B	Boundaries	Ch. 11, p. 164
21	D	Form, Structure, and Sense	Ch. 12, p. 180
22	A	Boundaries	Ch. 11, p. 164
23	C	Transitions	Ch. 13, p. 195
24	B	Transitions	Ch. 13, p. 195
25	A	Rhetorical Synthesis	Ch. 14, p. 208
26	D	Rhetorical Synthesis	Ch. 14, p. 208
27	B	Rhetorical Synthesis	Ch. 14, p. 208

Reading and Writing, Module 2

#	Answer	Skill	Review
1	C	Words in Context	Ch. 5, p. 69
2	D	Words in Context	Ch. 5, p. 69
3	A	Words in Context	Ch. 5, p. 69
4	B	Words in Context	Ch. 5, p. 69
5	B	Text Structure and Purpose	Ch. 6, p. 82
6	D	Text Structure and Purpose	Ch. 6, p. 82
7	A	Cross-Text Connections	Ch. 7, p. 98
8	C	Central Ideas and Details	Ch. 8, p. 112
9	B	Command of Evidence: Quantitative	Ch. 9, p. 128
10	D	Command of Evidence: Textual	Ch. 9, p. 128
11	C	Command of Evidence: Quantitative	Ch. 9, p. 128
12	A	Command of Evidence: Textual	Ch. 9, p. 128
13	A	Inferences	Ch. 10, p. 150
14	D	Inferences	Ch. 10, p. 150
15	B	Form, Structure, and Sense	Ch. 12, p. 180
16	C	Boundaries	Ch. 11, p. 164
17	D	Form, Structure, and Sense	Ch. 12, p. 180
18	A	Boundaries	Ch. 11, p. 164
19	C	Form, Structure, and Sense	Ch. 12, p. 180
20	B	Boundaries	Ch. 11, p. 164
21	D	Form, Structure, and Sense	Ch. 12, p. 180
22	C	Transitions	Ch. 13, p. 195
23	A	Transitions	Ch. 13, p. 195
24	B	Transitions	Ch. 13, p. 195
25	D	Rhetorical Synthesis	Ch. 14, p. 208
26	C	Rhetorical Synthesis	Ch. 14, p. 208
27	B	Rhetorical Synthesis	Ch. 14, p. 208

Math, Module 1

#	Answer	Skill	Review
1	B	Linear equations in one variable	Ch. 16, p. 239
2	D	Percentages	Ch. 23, p. 328
3	C	Lines, angles, and triangles	Ch. 26, p. 372
4	A	Linear functions	Ch. 17, p. 251
5	19	Linear equations in one variable	Ch. 16, p. 239
6	A	Equivalent expressions	Ch. 20, p. 289
7	C	One-variable data: distributions and measures of center and spread	Ch. 24, p. 340
8	B	Linear equations in two variables	Ch. 17, p. 251
9	9.6 or 48/5	Ratios, rates, proportional relationships, and units	Ch. 23, p. 328
10	A	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
11	C	Linear functions	Ch. 17, p. 251
12	C	Linear inequalities in one or two variables	Ch. 19, p. 278
13	-3	Systems of two linear equations in two variables	Ch. 18, p. 264
14	B	Area and volume	Ch. 26, p. 372
15	D	Nonlinear functions	Ch. 22, p. 313
16	B	Probability and conditional probability	Ch. 25, p. 357
17	7/6 or 1.166 or 1.167	Equivalent expressions	Ch. 20, p. 289
18	A	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
19	C	Systems of two linear equations in two variables	Ch. 18, p. 264
20	A	Right triangles and trigonometry	Ch. 27, p. 389
21	153.6 or 768/5	Nonlinear functions	Ch. 22, p. 313
22	D	Nonlinear functions	Ch. 22, p. 313

Math, Module 2

#	Answer	Skill	Review
1	D	Linear functions	Ch. 17, p. 251
2	A	Equivalent expressions	Ch. 20, p. 289
3	C	Lines, angles, and triangles	Ch. 26, p. 372
4	260	Ratios, rates, proportional relationships, and units	Ch. 23, p. 328
5	C	Linear equations in one variable	Ch. 16, p. 239
6	A	Two-variable data: models and scatterplots	Ch. 24, p. 340
7	D	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
8	25	Linear functions	Ch. 17, p. 251
9	A	Linear inequalities in one or two variables	Ch. 19, p. 278
10	B	Area and volume	Ch. 26, p. 372
11	25/4 or 6.25	Systems of two linear equations in two variables	Ch. 18, p. 264
12	C	Equivalent expressions	Ch. 20, p. 289
13	B	Inference from sample statistics and margin of error	Ch. 25, p. 357
14	A	Nonlinear functions	Ch. 22, p. 313
15	7/2 or 3.5	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
16	C	Linear equations in two variables	Ch. 17, p. 251
17	D	Right triangles and trigonometry	Ch. 27, p. 389
18	-1	Nonlinear functions	Ch. 22, p. 313
19	B	Systems of two linear equations in two variables	Ch. 18, p. 264
20	B	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
21	14	Circles	Ch. 28, p. 405
22	D	Nonlinear functions	Ch. 22, p. 313

Practice Test 1 Score Tally

Reading and Writing

Domain	Questions	Number correct
Craft and Structure	15	
Information and Ideas	14	
Standard English Conventions	14	
Expression of Ideas	11	
Total	54	

Math

Domain	Questions	Number correct
Algebra	15	
Problem-Solving and Data Analysis	7	
Geometry and Trigonometry	7	
Advanced Math	15	
Total	44	

Explanations: Reading and Writing, Module 1

1. Correct answer: B RW-T1M1-01

Words in Context · Craft and Structure · Easier

B *regular*

The text says the walk happens "every evening," that neither person "ever discussed" whether to go, and that the two "were always there." The walk is a dependable routine, and the tide comes and goes on a steady schedule. "Regular," meaning happening at fixed, predictable times, completes the comparison.

A "Unusual" is the opposite of what the text describes: a walk taken every evening without discussion is ordinary, not out of the ordinary.

C Nothing in the text suggests risk; a folding chair and a thermos of tea on a pier describe a calm scene, so "dangerous" has no support.

D The text never says the walk is hard or exhausting, and the comparison to the tide stresses routine rather than effort.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: A RW-T1M1-02

Words in Context · Craft and Structure · Easier

A *conserve*

The text explains that sea otters burn energy quickly and must eat a large share of their body mass each day to keep up. Rest is how an otter saves energy, and repeated disturbance interrupts that rest. "Conserve," meaning to save or avoid using up, completes the logic of the final sentence.

B Energy is not something an otter shows off; the sentence is about whether a resting otter can save energy, so "display" does not fit.

C The text says nothing about otters gauging or calculating their energy; the issue is whether they can hold on to it, which "measure" does not express.

D This reverses the logic. Disturbance makes otters use up energy, so saying a disturbed otter cannot "waste" energy contradicts the text.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: D RW-T1M1-03

Words in Context · Craft and Structure · Medium

D *respond to*

The text describes a two-way exchange: footwork "can signal the guitarist" to change, and the guitar "can, in turn, lead the dancer" into new steps. Each performer reacts to what the other does, so "respond to" is the most logical and precise choice. The last sentence, about performances unfolding differently, confirms that the exchange happens in the moment.

A The text says each performer's choices change what the other does, which is the opposite of the two performers ignoring each other.

B The performers are closely linked, which makes this tempting, but the text describes signals and changes, not copying: footwork prompts a new rhythm rather than repeating one.

C Nothing suggests a rivalry or contest; the exchanges are cooperative signals that move the piece forward, so "compete with" adds an unsupported idea.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: D RW-T1M1-04

Words in Context · Craft and Structure · Medium

D *reproduce*

The text says later scribes copied from "whatever copy was nearby" rather than from the original and that "a single error could appear in dozens of manuscripts." For an error to spread that way, each scribe must have copied it into a new manuscript. "Reproduce," meaning to make a copy of, precisely describes this.

A If scribes had usually corrected slips, errors would not have spread to dozens of manuscripts, so "correct" contradicts the end of the sentence.

B "Conceal" means to hide something. The passage explains how existing errors were repeated in later copies, not attempts to hide them.

C "Predict" means to forecast something before it happens. Predicting an error does not explain how an existing mistake spread through later copies.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: C RW-T1M1-05

Words in Context · Craft and Structure · Harder

C *distrust*

The blank names the attitude villagers kept toward a settler "from distant parts." The clause that follows says this attitude "would have prevented any surprise" if his good conduct "had ended in the commission of a crime." People who would not be surprised by a crime expect the worst, so "distrust" fits. The earlier picture of outsiders as unexplained mysteries supports it.

A Admiration would make a settler's crime surprising, not expected, so this choice clashes with the clause about villagers feeling no surprise.

B Villagers find outsiders mysterious, which makes "curiosity" tempting, but curiosity does not explain why a settler's crime would cause no surprise.

D The mention of "skill in handicraft" may suggest envy, but envy does not lead people to expect a crime; only distrust accounts for that lack of surprise.

Review: Chapter 5, Words in Context (page 69)

6. Correct answer: A RW-T1M1-06

Text Structure and Purpose · Craft and Structure · Easier

A *To suggest that something invisible can be recognized by its effects on the things around it*

Each stanza begins by admitting that no one "has seen the wind," then explains that we know it is there "when the leaves hang trembling" or "when the trees bow down their heads." The poem's purpose is to show that the wind, though unseen, can be known through what it does to the leaves and trees.

B The poem mentions leaves and trees only as signs of the wind; it says nothing about seasons or how plants change over time.

C The speaker makes no argument about how people should spend their time; the poem simply observes how the wind's presence is detected.

D The poem describes trembling leaves and bowing trees but never compares the wind's strength in two different settings.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: B RW-T1M1-07

Text Structure and Purpose · Craft and Structure · Medium

B *It describes the first step in the self-reinforcing process that the rest of the text explains.*

The first sentence contrasts how ice and water handle sunlight. The underlined sentence applies that contrast: melting exposes dark water, which absorbs more energy. The next sentence shows that this extra heat causes more melting, and the last sentence names the cycle a positive feedback. The underlined sentence therefore sets the cycle in motion.

A The underlined sentence supports the idea of positive feedback rather than challenging it; it is the first step in the cycle that the last sentence defines.

C The sentence describes what happens after ice melts, when the water is no longer covered; it is not an example of ice shielding water from warming.

D The first sentence states that ice reflects more sunlight than water does, but neither it nor the underlined sentence explains why that difference exists.

Review: Chapter 6, Text Structure and Purpose (page 82)

8. Correct answer: C RW-T1M1-08

Cross-Text Connections · Craft and Structure · Harder

C *By suggesting that the conclusion may hold for a simple task such as card sorting but is too broad to apply to all tasks*

Text 2 says an audience strengthens a person's most practiced responses, which helps on "simple or well-rehearsed" tasks but hurts on "complex or newly learned" ones. Card sorting is a simple task, so Text 2's author would accept the finding while objecting that the conclusion, which covers "people's performance" in general, reaches too far.

A Text 2 never discusses speed versus accuracy, and it predicts that an audience helps on simple tasks like card sorting, so it gives no reason to expect sloppier work.

B This is too strong. Text 2 says higher arousal improves simple or well-rehearsed tasks, so its author would not reject Text 1's finding outright.

D Text 2 says an audience makes errors more likely on complex or newly learned tasks, so its author would not accept a conclusion that covers every task.

Review: Chapter 7, Cross-Text Connections (page 98)

9. Correct answer: A RW-T1M1-09

Central Ideas and Details · Information and Ideas · Easier

A *Hearing an old recording helps Ama realize that she has improved more than she thought.*

At first Ama is "sure she had made no progress at all." When she hears the first-day recording, she winces at the "scratchy, halting notes" and wonders whether that is really how she sounded, then plays the whole piece straight through. The text centers on Ama recognizing her progress by comparing her past playing with her present playing.

B Ama hears her teacher's repeated "Again," but she never stops practicing; the text ends with her playing the piece straight through.

C Ama winces at a recording of her own early playing, which shows how far she has come, not a preference for listening over playing.

D Ama is looking for a photo when she finds the recording, but no photo reminds her of anything; the recording is what matters.

Review: Chapter 8, Central Ideas and Details (page 112)

10. Correct answer: C RW-T1M1-10

Central Ideas and Details · Information and Ideas · Medium

C *The squirrels' alarm calls appear to communicate what kind of predator is near, not merely that danger is present.*

The text shows that each predator produced a different call, that each call led to a different escape behavior, and

that recordings alone triggered the same responses. Together these details indicate that the calls carry specific information about the type of predator, which is the idea the whole study supports.

A The text describes different responses to the two predators, not different levels of fear, so this claim goes beyond the evidence.

B The squirrels responded to recorded calls as they had to calls made when real predators were present. Responding similarly does not establish that they cannot distinguish a recording from a live call; they might treat both as warnings.

D The text uses recordings as a method in one study; it never compares methods or calls playback the most reliable approach.

Review: Chapter 8, Central Ideas and Details (page 112)

11. Correct answer: D RW-T1M1-11

Command of Evidence: Quantitative · Information and Ideas · Easier

D *Long Shoal.*

The table shows that live coral cover at Long Shoal rose from 28 percent in 2016 to 30 percent in 2024, a slight increase. At each of the other three sites, the 2024 value is lower than the 2016 value, so Long Shoal is the only site that completes the statement.

A North Point's coral cover fell from 42 percent to 38 percent, so it is one of the three sites with a decline, not the site with an increase.

B Turtle Bay had the sharpest decline, from 35 percent to 21 percent, which is the opposite of the increase the statement describes.

C Coral Gardens had the highest cover in both years, but its cover fell from 51 percent to 44 percent rather than rising.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: B RW-T1M1-12

Command of Evidence: Textual · Information and Ideas · Medium

B *"with certain vials she gave cautions, standing in the doorway, and there were other doses which had to be accompanied on their healing way as far as the gate, while she muttered long chapters of directions, and kept up an air of secrecy and importance to the last."*

The claim is that Mrs. Todd treats handing over some remedies as a solemn, secret occasion. This quotation shows exactly that: she escorts certain doses "as far as the gate," mutters "long chapters of directions," and keeps up "an air of secrecy and importance to the last." These details turn a simple sale into a small ceremony.

A This quotation describes Mrs. Todd's size and how her skirts brush her plants; it says nothing about how she hands remedies to customers.

C This quotation shows a friendly relationship between Mrs. Todd and the village doctor, not any ceremony in how she delivers remedies.

D This quotation praises Mrs. Todd's skill at perfecting a drink through experiments, which concerns making a product, not a solemn handoff to customers.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: B RW-T1M1-13

Command of Evidence: Quantitative · Information and Ideas · Harder

B *The rate of wrong answers rose from 3 percent with one actor to 30 percent with three actors but stayed between 30 and 32 percent with three, four, or six actors.*

The conclusion has two parts: adding actors increased conformity at first, and then additional actors "made little difference." This choice captures both: the rate climbed from 3 percent to 30 percent as the group grew from one to three actors, then held between 30 and 32 percent for groups of three, four, and six.

A These values are accurate, but comparing only the smallest and largest groups shows an overall increase and hides the leveling off that the conclusion describes.

C This is accurate, but naming the single highest value says nothing about the early rise or the later plateau that the conclusion depends on.

D This misreads the graph: the rate peaked at 32 percent with four actors and was 31 percent with six, so it did not rise steadily.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

14. Correct answer: A RW-T1M1-14

Inferences · Information and Ideas · Medium

A *visit water holes at a wider range of times, including after dark.*

According to the hypothesis, zebras drink around midday because lions visit after dark. In a reserve without lions, that reason disappears, so zebras would have no need to avoid the water holes at night. Their visits would most likely spread across more of the day and night, including after dark.

B Keeping the same schedule when lions are absent would not support the idea that zebras choose midday to avoid lions at night; some other factor could be setting the timing.

C Nothing in the hypothesis suggests zebras would stop drinking; removing lions removes a danger, so visits would not disappear.

D The text gives no information about other grazing animals, so a comparison with them cannot follow from the hypothesis.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: C RW-T1M1-15

Inferences · Information and Ideas · Harder

C *the stationary features of streets rather than the amount of traffic on them.*

The text explains that long exposures recorded objects that stayed still, such as buildings and lampposts, while moving walkers and carriages left no lasting image. These photographs can therefore document fixed street features more reliably than the amount of moving traffic.

A The photographs cannot show population, and the text draws no link between a city's population and what appears in a street photograph.

B This reverses the text's point: moving crowds vanished during long exposures, so the photographs are unreliable evidence of how crowded streets were.

D The passage explains the apparently empty streets through exposure times. It does not connect what appears in the photographs to the activities residents valued most.

Review: Chapter 10, Inferences (page 150)

16. Correct answer: C RW-T1M1-16

Boundaries · Standard English Conventions · Easier

C *poles; warm*

The blank falls between two independent clauses: "Ocean currents act like a giant conveyor belt… toward the poles" and "warm water flowing north… at similar latitudes." Each could stand alone as a sentence. Two independent clauses cannot be joined by a comma alone or by no punctuation, so a semicolon is needed to mark the boundary.

A A comma alone between two independent clauses creates a comma splice, the error of joining two complete sentences with only a comma.

B With no punctuation, the two independent clauses run together as a fused sentence, so the reader cannot tell where the first idea ends.

D The conjunction "and" could join the clauses after a comma, but the comma after "and" wrongly separates the conjunction from the clause it introduces.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: A RW-T1M1-17

Form, Structure, and Sense · Standard English Conventions · Easier

A *reveals*

The subject of the verb is the singular noun "bundle," not "letters" or "partners," which sit in phrases between the subject and the verb. A singular subject needs a singular verb, so "reveals" is correct. Reading the sentence without the interrupting phrases, "A single bundle… reveals details," confirms the match.

B "Reveal" is a plural verb that agrees with "letters," a noun inside the phrase "of these letters," rather than with the singular subject "bundle."

C "Have revealed" is plural, so it does not agree with the singular subject "bundle"; the nearby plural noun "letters" makes it tempting.

D "Are revealing" uses the plural helping verb "are," which does not agree with the singular subject "bundle."

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: D RW-T1M1-18

Boundaries · Standard English Conventions · Medium

D *scene; however,*

"A long exposure brightens a dim scene" and "it also records any movement…" are independent clauses. Here, "however" introduces the second clause: a semicolon separates the complete clauses, and a comma after "however" sets off the transition.

A Commas on both sides of "however" treat it as an interrupter, leaving two independent clauses joined by commas alone, which is a comma splice.

B With no punctuation, the two independent clauses run together, and "however" is not set off from the clause it introduces.

C The second semicolon incorrectly treats "however" as an independent clause between two semicolons. The transition cannot stand alone as a sentence.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: C RW-T1M1-19

Form, Structure, and Sense · Standard English Conventions · Medium

C *raises*

The subject "a hibernating bat" needs a main verb, and none appears elsewhere in the clause; "Every few weeks" is only an introductory phrase. "Raises" is the only finite verb among the choices, meaning a verb that can serve as a sentence's main verb, and its present tense matches "is broken up" in the first sentence.

A "Raising" is a participle, which cannot serve as a main verb by itself, so the sentence would be a fragment with no verb for "bat."

B "To raise" is an infinitive, which cannot act as a sentence's main verb; the subject "a hibernating bat" would be left without one.

D "Having raised" is a nonfinite participle phrase; it cannot be a main verb, so the sentence would still lack one.

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: B RW-T1M1-20

Boundaries · Standard English Conventions · Medium

B *shoe—*

The phrase "the motion of tying a shoe" is an appositive, a noun phrase that renames the noun beside it, here "a single gesture." It interrupts the main clause, so it must be set off by a matching pair of marks. Because it opens with a dash, it must close with a dash, letting the sentence continue: "a single gesture… becomes the source."

A A comma cannot close an interruption that opens with a dash; the two marks that set off a supplementary phrase must match.

C A semicolon separates independent clauses, and here it would cut the subject "a single gesture" off from its verb "becomes."

D Without a closing dash, the appositive never ends, and "shoe becomes" wrongly reads as if the shoe, not the gesture, is the subject.

Review: Chapter 11, Boundaries (page 164)

21. Correct answer: D RW-T1M1-21

Form, Structure, and Sense · Standard English Conventions · Harder

D *had learned*

The sentence places the learning before a past reference point: European ships began arriving in the late fifteenth century. The past perfect, "had" plus a past participle, clearly marks that earlier learning. Among these choices, "had learned" fits the stated sequence.

A The present tense "learn" describes current or habitual action, which conflicts with the sentence's clear reference to events centuries in the past.

B The future tense "will learn" presents the action as still to come, contradicting the reference to sailors before the late fifteenth century.

C The present perfect "have learned" connects prior learning to the present, but this sentence locates the learning before a specific reference point in the distant past. "Had learned" supplies that past reference.

Review: Chapter 12, Form, Structure, and Sense (page 180)

22. Correct answer: A RW-T1M1-22

Boundaries · Standard English Conventions · Harder

A *striking: although*

The sentence contains two independent clauses: "The results were striking" and "most of them left out or misplaced basic features." The "although" clause is an introductory part of the second one. A colon can join two independent clauses when the second explains the first, and here it explains why the results were striking.

B The "although" clause belongs to the second independent clause, so a comma after "striking" joins two independent clauses with only a comma, creating a comma splice.

C With no punctuation after "striking," the two independent clauses run together, so the reader cannot tell where the first idea ends.

D A comma after "although" separates the word from the clause it introduces, and the comma before it still joins two independent clauses with a comma alone.

Review: Chapter 11, Boundaries (page 164)

23. Correct answer: C RW-T1M1-23

Transitions · Expression of Ideas · Easier

C *By contrast,*

The first sentence says ballet dancers make movement look effortless and seem free of gravity. The second says many modern choreographers "emphasized weight and effort," with dancers falling and pressing into the floor. These are opposing approaches, so a contrast transition is needed, and "By contrast" signals that difference.

A "Similarly" signals that the two sentences describe alike ideas, but resisting gravity and pressing into the floor are opposite approaches.

B "For example" would present modern dance as an instance of ballet's weightless style, but the second sentence describes a different approach.

D "As a result" suggests that ballet's style caused the modern choreographers' choices; the text presents a difference, not a cause.

Review: Chapter 13, Transitions (page 195)

24. Correct answer: B RW-T1M1-24

Transitions · Expression of Ideas · Harder

B *Consequently,*

The first two sentences build a chain of causes: warming lowers how much oxygen water can hold and weakens the mixing that carries oxygen downward. The final sentence reports the outcome, a decline in oxygen below the surface. "Consequently" signals that this decline follows from the processes just described.

A "In contrast" signals an opposing idea, but declining oxygen is exactly what the earlier sentences would lead a reader to expect.

C "Nevertheless" suggests the decline happens despite the earlier information, when those sentences actually explain why it happens.

D "Instead" introduces an alternative that replaces the preceding action or idea. Declining oxygen is a consequence of warming and reduced mixing, not a replacement for those processes.

Review: Chapter 13, Transitions (page 195)

25. Correct answer: A RW-T1M1-25

Rhetorical Synthesis · Expression of Ideas · Easier

A *When Mansa Musa traveled to Mecca in 1324, his caravan reportedly included some 60,000 people.*

The goal is to emphasize how large the caravan was. This is the only choice that states its size, using the historical report that it included "some 60,000 people," and it ties that number to the 1324 pilgrimage. The word "reportedly" keeps the claim as cautious as the notes.

B This sentence identifies Mansa Musa and the date of his pilgrimage, but it says nothing about how many people traveled with him.

C This sentence emphasizes Mansa Musa's spending in Cairo, which suggests wealth but does not describe the size of the caravan.

D This sentence describes Mansa Musa's influence on Timbuktu and never mentions his caravan at all.

Review: Chapter 14, Rhetorical Synthesis (page 208)

26. Correct answer: D RW-T1M1-26

Rhetorical Synthesis · Expression of Ideas · Medium

D *English botanist and photographer Anna Atkins created Photographs of British Algae: Cyanotype Impressions, which first appeared in 1843, by laying algae specimens on chemically treated paper and exposing them to sunlight.*

An audience unfamiliar with Atkins needs to learn who she was and what the work is. This choice identifies Atkins as an "English botanist and photographer," names the work and the year it first appeared, and briefly explains how its images were made, giving newcomers the context they need.

A This gives the work's date and links it to the cyanotype, but it never says who made the work or what it shows, so newcomers would be lost.

B This explains part of Atkins's method, but it never names *Photographs of British Algae: Cyanotype Impressions*, the work the student wants to introduce.

C This names the work and its subject, but it refers to "Atkins" without explaining who she was, which an unfamiliar audience would need.

Review: Chapter 14, Rhetorical Synthesis (page 208)

27. Correct answer: B RW-T1M1-27

Rhetorical Synthesis · Expression of Ideas · Harder

B *Because the enclosure held both a squirrel's own nuts and an equal number buried by the researchers, a squirrel relying on smell alone would be expected to find the researchers' nuts as often as its own.*

The goal is to explain how the design compared memory with smell alone. This choice identifies the equal numbers of nuts buried by a squirrel and by researchers in its enclosure, then states the smell-only prediction. Under that stated prediction, finding its own nuts more often would support memory rather than smell alone.

A This reports the study's main result but not how the design made that result meaningful, so it does not explain how the two explanations were tested.

C This describes part of the procedure but leaves out the researchers' nuts, the feature that let the biologists distinguish memory from smell.

D This states the biologists' conclusion without explaining how the study's design led to it, so it does not address the goal.

Review: Chapter 14, Rhetorical Synthesis (page 208)

Explanations: Reading and Writing, Module 2

1. Correct answer: C RW-T1M2-01

Words in Context · Craft and Structure · Easier

C *relocate*

When storms destroy kelp, the animals that rely on it lose their food and shelter, and the text says they go "to other parts of the coast until the kelp grows back." "Relocate," meaning to move to a new place, fits this context precisely.

A "Return" means to go back to a place one has been before. The text does not say these animals previously occupied the other coastal areas, so that added meaning is unsupported.

B "Contribute" means to give something toward a shared goal, which does not describe animals moving away from destroyed kelp.

D "Surrender" means to give up or yield; it does not make sense with "to other parts of the coast" and adds an idea of defeat.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: D RW-T1M2-02

Words in Context · Craft and Structure · Medium

D *concealed*

The narrator says some residents were poor but "were like the Spartans," who were known for enduring hardship without complaint, and adds that "We none of us spoke of money." Wearing "a smiling face" over their difficulties means hiding them, so "concealed" is the most logical choice. Here "smart" means a stinging pain.

A "Confessed" means admitted openly, which contradicts the next sentence: no one in Cranford spoke of money.

B "Multiplied" would mean the residents increased their own hardship, an idea the text never suggests.

C The residents still had "some difficulty in making both ends meet" and deliberately avoided the subject of money, so they hid their hardship rather than forgetting it.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: A RW-T1M2-03

Words in Context · Craft and Structure · Harder

A *absorbed in*

The text explains that people overestimate how much others notice them and "struggle to judge how little attention others actually pay them." That error arises when people are deeply focused on their own point of view. "Absorbed in," meaning fully occupied by, describes that focus and explains why the hat wearers guessed so high.

B Being skeptical of one's own perspective would lead people to question their assumptions, which would reduce the error rather than explain it.

C This is the opposite of the text's point: people misjudge others' attention because they are too focused on their own perspective, not distanced from it.

D The text never says people dislike their own perspective, and dissatisfaction would not explain why they overestimate how much others notice them.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: B RW-T1M2-04

Words in Context · Craft and Structure · Harder

B *innocent*

The narrator stresses that strangers would read Hepzibah's scowl as "bitter anger and ill-will," then insists "it was no such thing": she actually felt "reverence" for the portrait, and the scowl came from her near-sightedness. A scowl with no ill will behind it is harmless in intent, so "innocent" is the most precise choice.

A "Calculated" means deliberately planned, but the text traces the scowl to near-sightedness and an effort to see, not to a plan to look a certain way.

C "Spiteful" describes the ill will that strangers wrongly assume; the narrator rejects that reading by saying "it was no such thing."

D "Theatrical" suggests a performance for onlookers, but Hepzibah scowls simply to see the portrait clearly, with no audience in mind.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: B RW-T1M2-05

Text Structure and Purpose · Craft and Structure · Medium

B *It presents a setback that leads to Kofi's embarrassment and to his uncle's help.*

Before the underlined sentence, Kofi launches the boat and waits for the wind to carry him out. The underlined sentence shows the launch going wrong: the boat circles back while his sister laughs. The sentences that follow show the consequences, Kofi's burning face and his uncle's quiet repair, so the underlined sentence presents the setback that drives the rest of the scene.

A The underlined sentence describes what happens at the launch; the text never gives Kofi's reasons for building the boat.

C Nothing indicates the neighbors expected failure; they came "to watch," and only his sister laughs after the boat circles back.

D The underlined sentence shows a problem with the boat, not Kofi's skill; his uncle, not Kofi, finds and fixes the loose rudder pin.

Review: Chapter 6, Text Structure and Purpose (page 82)

6. Correct answer: D RW-T1M2-06

Text Structure and Purpose · Craft and Structure · Harder

D *It notes how a group of animals is known to hunt, reports findings that complicate that picture, and offers an explanation that accounts for those findings.*

The text first states that these bats are "best known for hunting by echolocation." It then reports that noise blocking the echoes barely hurt hunting on moonlit nights but did on dark ones, which complicates that picture. Finally, it presents the researchers' explanation: the bats shift between senses as the amount of light changes.

A The text explains echolocation briefly, but it never questions the researchers' method; the noise experiment is presented as revealing, not flawed.

B This overstates the findings: the bats still depended on echolocation on dark nights, so the text does not call the behavior rare or rank vision above it overall.

C The text does compare moonlit and dark nights, but it makes no recommendation about when future studies should be done.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: A RW-T1M2-07

Cross-Text Connections · Craft and Structure · Harder

A *By contending that Text 1 draws too sharp a line, since even photographs taken without staging reflect choices that shape what viewers understand*

Text 1 says a photograph stops being a document once the photographer intervenes. Text 2 counters that "every photograph involves intervention," since choices about position, framing, and timing shape understanding, and it argues that honesty is the real test. Text 2's author would therefore say that Text 1's line between documents and illustrations is too sharp.

B Text 2 does not endorse a ban on staging; it argues that intervention is unavoidable and that honesty, not the absence of intervention, is what matters.

C Text 2 never says staged images are more truthful; it says only that intervention by itself does not decide a photograph's value as a record.

D Text 2 says nothing about whether viewers notice staging; its argument concerns the choices behind every photograph, not what viewers can detect.

Review: Chapter 7, Cross-Text Connections (page 98)

8. Correct answer: C RW-T1M2-08

Central Ideas and Details · Information and Ideas · Medium

C *She supports Chidi outwardly while privately feeling the loss that his departure will bring.*

In front of Chidi, Mrs. Okafor "nodded and smiled" and asked practical questions, showing support. Once he leaves the room, she smooths the letter and looks "for a long time at the empty chair across from her," an image of the absence she will soon feel. The text shows outward support paired with private sadness.

A Nothing suggests she doubts Chidi's talent; she reads the acceptance letter twice and asks practical questions about his move.

B The text gives no sign that Chidi applied without her knowledge, and her calm, sensible questions show no anger.

D Her questions about train fares and coats concern Chidi's move, not hers, and nothing indicates she plans to live in the city.

Review: Chapter 8, Central Ideas and Details (page 112)

9. Correct answer: B RW-T1M2-09

Command of Evidence: Quantitative · Information and Ideas · Easier

B *Cedar.*

Cedar's average nightly distance was 7.8 kilometers in summer and 10.4 kilometers in winter, the highest value in each column of the table. No other fox exceeds Cedar in either season, so Cedar completes the statement.

A Juniper averaged 6.2 kilometers in summer and 8.9 in winter, both less than Cedar's averages, so Juniper did not travel farthest in either season.

C Willow had the lowest averages in both seasons, 5.1 and 7.3 kilometers, which is the opposite of what the statement requires.

D Aspen ranks second in both seasons, at 6.9 and 9.6 kilometers, but Cedar's averages of 7.8 and 10.4 kilometers are higher.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

10. Correct answer: D RW-T1M2-10

Command of Evidence: Textual · Information and Ideas · Medium

D *"You do not know half the wisdom and prudence I possess, because I have never been tried."*

The claim has two parts: Agnes believes her family underestimates her, and she blames the fact that her abilities have never been tested. In this quotation she tells her mother, "You do not know half the wisdom and prudence I possess," and gives the reason directly: "because I have never been tried."

A This shows Agnes's wish to help her family, but it says nothing about her family underestimating her or about her abilities going untested.

B This expresses Agnes's modest view of what she could teach and her fondness for children, not a complaint that her family misjudges her.

C This shows Agnes distracted while drawing, which hints at her plans but does not show her claiming that her family underestimates her.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

11. Correct answer: C RW-T1M2-11

Command of Evidence: Quantitative · Information and Ideas · Harder

C *The number of registered merchants rose at every point shown from 1200 to 1260 and fell at every point shown from 1270 to 1300.*

The historian argues that the fair grew until the road opened in 1265 and then declined. The graph shows registrations rising at every point from 40 in 1200 to 130 in 1260, then falling at every point after the road's completion, from 105 in 1270 to 60 in 1300. This choice captures both halves of that pattern.

A This is accurate (60 versus 40), but comparing only the first and last years hides both the growth before 1265 and the decline after it.

B This is accurate, but naming the peak alone does not show that growth stopped and a steady decline began after the road was completed.

D This reverses the graph: registrations rose from 1200 to 1260 and fell after 1265, the opposite of what this choice states.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: A RW-T1M2-12

Command of Evidence: Textual · Information and Ideas · Harder

A *When colonies hosting mainly type D algae and colonies hosting mainly type C algae were grown side by side in tanks at the same high temperature, the type D colonies bleached less.*

If type D colonies still bleach less when both groups experience the same high temperature, cooler reef locations cannot explain the difference observed in that tank comparison. The result supports a role for algae type and weakens the proposed location explanation, although it does not by itself eliminate every possible difference between the colonies.

B This supports the alternative explanation that cooler locations could account for the lower bleaching observed among type D colonies on the reef.

C A shared ability to supply food through photosynthesis does not show whether the two algae types differ in their effects on coral heat tolerance.

D Recovery after bleaching is a separate question; this finding says nothing about whether type D algae reduced bleaching in the first place.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: A RW-T1M2-13

Inferences · Information and Ideas · Medium

A *fewer algae near the surface, because fewer nutrients rose from the deep ocean.*

The text links a chain of events: winds drive upwelling, upwelling brings nutrients to the surface, and those nutrients "feed tiny algae." When the winds were unusually weak, less upwelling would occur, so fewer nutrients would reach the surface and fewer algae would grow there.

B This reverses the logic: weaker winds mean less upwelling, so less cold deep water would reach the surface, not more.

C The text ties fish to algae, which depend on upwelled nutrients; it never links calmer water to more fish, so this is unsupported.

D The text says upwelled nutrients "feed tiny algae," so the claim that algae depend only on sunlight contradicts it.

Review: Chapter 10, Inferences (page 150)

14. Correct answer: D RW-T1M2-14

Inferences · Information and Ideas · Harder

D *landowners' goals and expectations for their estates than about workers' own experiences.*

The text says the handbooks were written by "wealthy landowners" from "the perspective of owners seeking profit" and show how estates "were meant to run," while few writings survive from the workers themselves. The handbooks therefore reveal owners' goals and expectations better than they reveal workers' experiences.

A The passage does not discuss whether ordinary farmers developed techniques independently of landowners. Its stated limitation concerns the owners' perspective and the absence of workers' own accounts.

B This is the opposite of the text's point: because laborers left few writings, the handbooks are a weak source on how workers experienced rural life.

C The passage notes that few workers' writings survive but gives no explanation for that scarcity. Its conclusion concerns what the owners' handbooks reveal.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: B RW-T1M2-15

Form, Structure, and Sense · Standard English Conventions · Easier

B *its*

The blank needs a possessive pronoun showing that the aperture belongs to "a camera lens." "Its" is the possessive form of "it," so "its aperture" correctly means "the lens's aperture." Possessive pronouns such as its, their, and whose do not take apostrophes.

A "It's" is a contraction of "it is" or "it has," which makes no sense here: "through it is aperture."

C "Its'" is not a word in Standard English; the possessive form of "it" never takes an apostrophe.

D The antecedent is one inanimate camera lens, so "its" is the appropriate possessive pronoun here. "Their" does not agree with that antecedent in this context.

Review: Chapter 12, Form, Structure, and Sense (page 180)

16. Correct answer: C RW-T1M2-16

Boundaries · Standard English Conventions · Medium

C *source;*

The sentence lists three items, and each item contains its own internal comma ("water shrews, which hunted…"). In a series like this, semicolons separate the items so that readers can tell where one ends and the next begins. The semicolon before "and beavers" confirms the pattern, so a semicolon belongs after "source."

A A comma would blur the boundary between the first and second items, because each item already contains a comma; the series needs semicolons, as before "and beavers."

B With no punctuation, "source" runs directly into "river otters," merging the first two items of the series.

D The words since the first colon form a list item, not an independent clause introducing a new list or explanation. A second colon would not correctly separate the parallel items.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: D RW-T1M2-17

Form, Structure, and Sense · Standard English Conventions · Medium

D *they*

The pronoun refers back to "Caravanserais," the inns that "offered water, shelter, and a gate." "Caravanserais" is a plural noun, and a pronoun must agree in number with the noun it refers to, its antecedent. The plural pronoun "they" is therefore correct.

A "It" is singular, but its antecedent, "Caravanserais," is plural, so the pronoun does not agree in number.

B "This" is singular and seems to point to a single idea, but the inns themselves, a plural noun, are what offered water and shelter.

C "That" is a singular pronoun, so it cannot refer to the plural "Caravanserais," and it leaves unclear what offered shelter.

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: A RW-T1M2-18

Boundaries · Standard English Conventions · Medium

A *learn was*

The subject of the second sentence is the long noun phrase "The style that the largest number of students said they hoped to learn," and its verb is "was." No punctuation should separate a subject from its verb, even when the subject is long, so the words should run together without a mark.

B A single comma between the subject and its verb incorrectly separates them; nothing in the subject is an interrupting phrase that would call for commas.

C A colon must follow an independent clause, but "The style… hoped to learn" is only a subject, so a colon cannot come after it.

D A semicolon separates independent clauses, but the words before it are only a subject, and the words after it have no subject of their own.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: C RW-T1M2-19

Form, Structure, and Sense · Standard English Conventions · Harder

C *forms*

The subject "This water" is followed by a nonrestrictive clause, "which may not return to the surface for centuries," but the sentence still needs a main verb for "This water." "Forms" is the only finite verb among the choices, and its present tense matches "sinks" and "begins" in the first sentence.

A "Forming" is a participle and cannot supply the main verb by itself, without an auxiliary. Here, "This water" would still lack a main verb, leaving a fragment.

B "Having formed" is a nonfinite participle phrase, so it leaves the subject "This water" without a main verb and produces a fragment.

D "To form" is an infinitive, which cannot act as a sentence's main verb, so the subject "This water" would still lack a verb.

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: B RW-T1M2-20

Boundaries · Standard English Conventions · Harder

B *lost,*

The clause "many of which have survived even though the documents they once closed were lost" is a nonrestrictive element, extra information that interrupts the main clause "These seals... give historians a detailed record." It opens with a comma after "seals," so it must close with a matching comma before the verb "give."

A A dash cannot close an interruption that began with a comma; the marks that set off a supplementary element must match.

C A semicolon separates independent clauses, but here it would cut the subject "These seals" off from its verb "give."

D Without a closing comma, the interrupting clause never ends, so "lost give historians" runs together and hides the main verb "give."

Review: Chapter 11, Boundaries (page 164)

21. Correct answer: D RW-T1M2-21

Form, Structure, and Sense · Standard English Conventions · Harder

D *she often waited an hour or more for the light to change before taking a single picture.*

The introductory phrase "Setting up the camera again at each new site along the valley" must describe the subject of the clause that follows. Only "she," referring to the photographer, names someone who can set up a camera, so this choice correctly connects the modifier to its subject.

A This makes "an hour or more" the thing setting up the camera, a dangling modifier, because a span of time cannot set up a tripod.

B This places "the light" right after the introductory phrase, illogically suggesting that the light set up the camera at each site.

C This makes "a single picture" the subject the phrase describes, illogically implying that the picture set up the camera.

Review: Chapter 12, Form, Structure, and Sense (page 180)

22. Correct answer: C RW-T1M2-22

Transitions · Expression of Ideas · Easier

C *Next,*

The text lists the stages of care in order, marked by "First" and "Finally." The sentence with the blank describes the stage that comes between feeding and release, so a sequence transition is needed. "Next" correctly signals the following step.

A "Nevertheless" signals that something happens despite what came before, but moving to the pool simply follows the feeding stage.

B "For instance" introduces an example of the previous idea, but learning to catch fish is a new stage, not an example of being fed formula.

D "In contrast" signals a difference between two ideas, but the sentence continues a step-by-step sequence rather than opposing the feeding stage.

Review: Chapter 13, Transitions (page 195)

23. Correct answer: A RW-T1M2-23

Transitions · Expression of Ideas · Medium

A *For example,*

The first sentence states a general finding: people are less likely to help when others are present. The second sentence describes a specific experiment whose results show that pattern, comparing participants who were alone with those beside silent strangers. "For example" correctly introduces this illustration of the general claim.

B "However" signals a contrast, but the experiment's results agree with the finding in the first sentence rather than challenging it.

C "Similarly" introduces a second, comparable point, but the experiment is not a separate finding; it is an instance of the first sentence's claim.

D "As a result" suggests the experiment was caused by the finding, but the experiment illustrates the finding rather than following from it.

Review: Chapter 13, Transitions (page 195)

24. Correct answer: B RW-T1M2-24

Transitions · Expression of Ideas · Harder

B *Likewise,*

The first sentence explains that film choreographers lose parts of a dance to framing and cuts. The second describes photographers facing the same kind of problem: a frame and a single instant capture only part of a flowing movement. Because the second situation parallels the first, "Likewise" is the logical transition.

A "Instead" signals an alternative that replaces what came before, but the photographers' challenge parallels the choreographers' challenge rather than replacing it.

C "For this reason" implies that the choreographers' situation causes the photographers' challenge, but the two groups face similar problems independently.

D "Even so" signals a concession or contrast, but nothing in the photographers' situation goes against what is said about choreographers.

Review: Chapter 13, Transitions (page 195)

25. Correct answer: D RW-T1M2-25

Rhetorical Synthesis · Expression of Ideas · Easier

D *A sentry stood guard on about 90 percent of foraging days in the North group but on only about 60 percent in the South group.*

The goal is to emphasize a difference in sentry behavior. This choice compares how often each group posted a sentry, about 90 percent of foraging days for the North group versus about 60 percent for the South group, and the words "but on only" highlight the gap.

A This describes what the two groups have in common, their shared reserve, rather than any difference between them.

B This explains what sentries do in general but does not compare the two groups at all.

C This highlights a real difference between the groups, but it concerns group size, not the sentry behavior the student wants to emphasize.

Review: Chapter 14, Rhetorical Synthesis (page 208)

26. Correct answer: C RW-T1M2-26

Rhetorical Synthesis · Expression of Ideas · Medium

C *In both studies, participants preferred the items they had encountered more often, whether those items were shapes or melodies.*

The goal is a similarity between the two studies' findings. This choice states the shared result, that participants in both studies preferred the items they had encountered more often, and the phrase "in both studies" makes the similarity explicit while acknowledging that the items differed.

A This emphasizes a difference between the studies' materials, not a similarity, and it says nothing about what either study found.

B This defines the effect the studies examined but does not mention either study's findings or compare them.

D This contrasts details of the two procedures rather than pointing out what the studies' findings have in common.

Review: Chapter 14, Rhetorical Synthesis (page 208)

27. Correct answer: B RW-T1M2-27

Rhetorical Synthesis · Expression of Ideas · Medium

B *By allowing dances to be recorded in written symbols, labanotation gave choreographic works a lasting written record and became part of dance education at colleges and universities.*

Explaining significance means showing why something mattered. This choice does that by connecting what labanotation does, recording dances in written symbols, to its effects: choreographic works gained a lasting written record, and the system became part of dance education at colleges and universities.

A This tells when the system was presented and describes its staff, but it does not explain why labanotation mattered.

C This explains how the symbols work, which describes the system rather than its significance.

D This gives background about Laban and the system's publication but says nothing about its importance or influence.

Review: Chapter 14, Rhetorical Synthesis (page 208)

Explanations: Math, Module 1

1. Correct answer: B M-T1M1-01

Linear equations in one variable · Algebra · Easier

B *7*

Collect the x-terms on one side and the constants on the other. Subtracting 2x from both sides gives 3x + 14 = 35, and subtracting 14 gives 3x = 21. Dividing by 3 gives x = 7. Check: 5(7) + 14 = 49 and 2(7) + 35 = 49, so both sides match.

1. Subtract 2x from both sides: 3x + 14 = 35.
2. Subtract 14 from both sides: 3x = 21.
3. Divide both sides by 3: x = 7.

A This comes from adding 5x and 2x to get 7x = 21. Moving 2x to the other side requires subtracting it, which leaves 3x.

C This comes from adding 14 to 35 instead of subtracting it, giving 3x = 49. Subtract 14 from both sides to undo the + 14.

D This is the value of 3x, not x. The last step, dividing both sides by 3, was skipped.

Review: Chapter 16, Linear Equations in One Variable (page 239)

2. Correct answer: D M-T1M1-02

Percentages · Problem-Solving and Data Analysis · Easier

D *$64.40*

A 15% increase multiplies the original amount by 1 + 0.15 = 1.15. This year's rate is 56 × 1.15 = 64.40, or $64.40 per day. You can also find the increase, 0.15 × 56 = 8.40 dollars, and add it to the original $56.

1. Let r be this year's daily rate, in dollars.
2. A 15% increase means r = 56 × (1 + 0.15) = 56 × 1.15.
3. r = 64.40, so the rate is $64.40 per day.

A This is only the amount of the increase, 0.15 × 56. The question asks for the new rate, so the increase must be added to $56.

B This is 85% of $56, the result of a 15% decrease. The rate went up, so multiply by 1.15, not 0.85.

C This adds 15 cents to $56. A percent is a share of the original amount, and 15% of $56 is $8.40.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

3. Correct answer: C M-T1M1-03

Lines, angles, and triangles · Geometry and Trigonometry · Easier

C *15*

Corresponding sides of similar triangles are proportional. Side AB corresponds to DE, so the scale factor from triangle ABC to triangle DEF is 10/4 = 2.5. Side EF corresponds to BC, so x = 6 × 2.5 = 15. Equivalently, solve the proportion x/6 = 10/4.

1. Match corresponding sides: AB corresponds to DE, and BC corresponds to EF.
2. Scale factor from triangle ABC to triangle DEF: 10/4 = 2.5.
3. x = EF = 2.5 × BC = 2.5 × 6 = 15.

A This pairs sides that do not correspond, using x/10 = 4/6. Match AB with DE and BC with EF before setting up the proportion.

B This adds the difference DE − AB = 6 to BC. Similar triangles keep the same ratio between sides, not the same difference.

D This multiplies DE, not BC, by the scale factor 2.5. The side that corresponds to EF is BC, which has length 6.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: A M-T1M1-04

Linear functions · Algebra · Easier

A *f(x) = 3x + 5*

The slope is the change in f(x) divided by the change in x: (14 − 8)/(3 − 1) = 6/2 = 3. Substitute the point (1, 8) into f(x) = 3x + b: 8 = 3 + b, so b = 5. Check with (5, 20): 3(5) + 5 = 20.

1. Slope m = (14 − 8)/(3 − 1) = 6/2 = 3.
2. Write f(x) = 3x + b and substitute x = 1 and f(x) = 8: 8 = 3 + b, so b = 5.
3. Check x = 5: 3(5) + 5 = 20, which matches the table.

B This uses 8, the value of f(1), as the y-intercept. The y-intercept is f(0), which is 3 less than f(1), or 5.

C This uses 6, the change in f(x) from row to row, as the slope. The x-values also change by 2, so the slope is 6/2 = 3.

D This fits only the first row, f(1) = 8. It gives f(3) = 24, but the table shows f(3) = 14.

Calculator option: Enter the table into the calculator and fit a linear model with the regression feature; it reports a slope of 3 and an intercept of 5, an exact fit.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

5. Correct answer: 19 M-T1M1-05

Linear equations in one variable · Algebra · Easier

Multiply both sides by 3/2, the reciprocal of 2/3, to undo the coefficient: x − 4 = 15. Then add 4 to both sides: x = 19. A common wrong answer is 32/3, which comes from multiplying by 2/3 instead of its reciprocal. Another is 21, which comes from multiplying only the x by 2/3.

1. Multiply both sides by 3/2: x − 4 = 10 × (3/2) = 15.
2. Add 4 to both sides: x = 19.

3. Check: (2/3)(19 − 4) = (2/3)(15) = 10.

Calculator option: Graph y = (2/3)(x − 4) and y = 10; the two lines intersect where x = 19.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: A M-T1M1-06

Equivalent expressions · Advanced Math · Easier

A *(5x − 6)(5x + 6)*

The expression is a difference of two squares, because $25x^2 = (5x)^2$ and $36 = 6^2$. The pattern $a^2 - b^2 = (a - b)(a + b)$ gives (5x − 6)(5x + 6). Check by expanding: $25x^2 + 30x - 30x - 36 = 25x^2 - 36$.

1. Write each term as a square: $25x^2 = (5x)^2$ and $36 = 6^2$.
2. Apply $a^2 - b^2 = (a - b)(a + b)$ with a = 5x and b = 6.
3. Result: (5x − 6)(5x + 6).

B Expanding gives $25x^2 - 60x + 36$, which has a middle term and a positive constant. A difference of squares factors into a sum times a difference.

C Expanding gives $25x^2 + 144x - 36$. Splitting $25x^2$ as 25x times x keeps the middle terms from canceling.

D Expanding gives $25x^2 - 324$. The constant 36 is 6^2, so each factor needs 6, not 18, which is half of 36.

Calculator option: Graph $y = 25x^2 - 36$ and each choice to look for mismatches. Confirm the matching choice by expanding its factors; a graph alone does not prove that two expressions are equivalent.

Review: Chapter 20, Equivalent Expressions (page 289)

7. Correct answer: C M-T1M1-07

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

C *7*

With 12 values, the median is the mean of the 6th and 7th values in order. Counting dots from the left: three 4s are values 1 to 3, the 5 is value 4, the 6 is value 5, and the two 7s are values 6 and 7. Both middle values are 7, so the median is 7 crates.

1. There are n = 12 values, so the median is the mean of the 6th and 7th values in order.
2. Running counts from the left: 3 values through 4 crates, 4 through 5, 5 through 6, and 7 through 7 crates.
3. The 6th and 7th values are both 7, so the median is 7 crates.

A This is the number of workers who filled 8 crates, the tallest column. The median comes from the middle positions in the ordered data, not a column height.

B This is the mean: the 12 values total 78, and 78/12 = 6.5. The question asks for the median, the middle value.

D This is the mode, the value with the most dots. The median comes from counting to the middle of the ordered data.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

8. Correct answer: B M-T1M1-08

Linear equations in two variables · Algebra · Medium

B *−2/7*

Solve for y to find the slope of line ℓ: 2y = 7x + 6, so y = (7/2)x + 3, and the slope is 7/2. Perpendicular lines have slopes that are negative reciprocals, so the slope of line m is −2/7. Check: (7/2)(−2/7) = −1.

1. Solve 2y − 7x = 6 for y: y = (7/2)x + 3.
2. The slope of line ℓ is 7/2.
3. The slope of line m is the negative reciprocal of 7/2, which is −2/7.

A This changes only the sign of 7/2. A perpendicular slope is the negative reciprocal, so the fraction must also be flipped.

C This is the reciprocal of 7/2 without the sign change. Perpendicular slopes multiply to −1, but (7/2)(2/7) = 1.

D This is the slope of line ℓ itself. A line with the same slope is parallel to line ℓ, not perpendicular to it.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: 9.6 or 48/5 M-T1M1-09

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

Chain conversion factors so the unwanted units cancel: 48 feet per minute × 12 inches per foot × 1 minute per 60 seconds = 576/60 = 9.6 inches per second. A common wrong answer is 576, which converts feet to inches but not minutes to seconds. Another is 0.8, which converts minutes but not feet.

1. Convert feet to inches: 48 × 12 = 576 inches per minute.
2. Convert minutes to seconds by dividing by 60: 576/60 = 9.6.
3. The speed is 9.6 inches per second.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

10. Correct answer: A M-T1M1-10

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

A *−5/4*

Factor the quadratic: $4x^2 + 5x - 6 = (4x - 3)(x + 2)$. Setting each factor equal to 0 gives x = 3/4 and x = −2. Their sum is 3/4 + (−2) = −5/4. As a check, the sum of the solutions of $ax^2 + bx + c = 0$ is −b/a, which is −5/4 here.

1. Factor: $4x^2 + 5x - 6 = (4x - 3)(x + 2)$.
2. Solve each factor: 4x − 3 = 0 gives x = 3/4, and x + 2 = 0 gives x = −2.

3. Add the solutions: $3/4 + (-2) = -5/4$.

B This treats $4x - 3 = 0$ as if it gave $x = 3$, then adds 3 and −2. Dividing by 4 gives the actual first solution, 3/4.

C This uses −3/4 and 2, which have the wrong signs. The factor $4x - 3$ gives a positive solution, and $x + 2$ gives a negative one.

D This adds 3/4 and 2, changing the sign of only one solution. The factor $x + 2$ equals zero when $x = -2$.

Calculator option: Graph $y = 4x^2 + 5x - 6$ and read its x-intercepts, 0.75 and −2; their sum is −1.25.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

11. Correct answer: C M-T1M1-11

Linear functions · Algebra · Medium

C *The car is driven about 28 miles for each gallon of fuel used.*

The term $m/28$ is the fuel used after m miles. Every 28 miles uses $28/28 = 1$ gallon, so the car travels about 28 miles per gallon; the slope is $-1/28$ gallon per mile. The constant 14 is the full tank, and the tank is empty when $m = 14 \times 28 = 392$ miles.

1. $G(0) = 14$, so a full tank holds 14 gallons.
2. Increasing m by 28 decreases G(m) by $28/28 = 1$ gallon.
3. So 28 is the number of miles driven per gallon of fuel used.

A The full-tank amount is $G(0) = 14$ gallons. The number 28 divides the miles, so it describes fuel efficiency, not tank size.

B This reverses the rate. Dividing m by 28 means one gallon is used every 28 miles, so the car uses 1/28 gallon per mile.

D The tank is empty when $14 - m/28 = 0$, which happens at $m = 392$ miles, not at 28 miles.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

12. Correct answer: C M-T1M1-12

Linear inequalities in one or two variables · Algebra · Medium

C *(3, 2)*

Test each point in both inequalities. For (3, 2): $2 > 3 - 3 = 0$ is true, and $2(3) + 3(2) = 12 \leqslant 12$ is true, because $\leqslant$ allows equality. So (3, 2) is a solution to the system. Each other choice fails at least one of the inequalities.

1. Substitute each point into $y > x - 3$ and into $2x + 3y \leqslant 12$.
2. For (3, 2): $2 > 0$ is true, and $12 \leqslant 12$ is true.
3. Every other choice fails at least one inequality, so (3, 2) is the only listed point that satisfies the system.

A This point satisfies $y > x - 3$, but $2(0) + 3(5) = 15$, which is greater than 12, so the second inequality fails.

B Here $2 - 3 = -1$, so y equals the boundary value. The first inequality is strict, so a point on its boundary line is not a solution.

D This point satisfies $2x + 3y \leqslant 12$, since $10 + 0 = 10$, but $0 > 5 - 3 = 2$ is false.

Calculator option: Graph both inequalities; (3, 2) lies in the region shaded by both, and (2, −1) sits on the dashed line $y = x - 3$.

Review: Chapter 19, Linear Inequalities (page 278)

13. Correct answer: -3 M-T1M1-13

Systems of two linear equations in two variables · Algebra · Medium

The y-terms have opposite coefficients, so add the equations to eliminate y: $7x = 35$, which gives $x = 5$. Substitute into the second equation: $5 - 5y = 20$, so $-5y = 15$ and $y = -3$. Entering 5 gives the value of x instead of y, and entering 3 drops the negative sign.

1. Add the equations: $(6x + x) + (5y - 5y) = 15 + 20$, so $7x = 35$.
2. Solve for x: $x = 5$.
3. Substitute $x = 5$ into $x - 5y = 20$: $5 - 5y = 20$, so $-5y = 15$ and $y = -3$.

Calculator option: Graph both lines and select their intersection point, (5, −3); the y-coordinate is the answer.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

14. Correct answer: B M-T1M1-14

Area and volume · Geometry and Trigonometry · Medium

B *339*

Use the cylinder formula $V = \pi r^2 h$ with radius $r = 6/2 = 3$ meters: $V = \pi(3^2)(15) = 135\pi$, or about 424.1 cubic meters, for the full silo. The grain fills 80% of it: $0.8 \times 135\pi = 108\pi$, which is about 339 cubic meters.

1. Radius $r = 6/2 = 3$ meters; height $h = 15$ meters.
2. Full volume: $V = \pi(3^2)(15) = 135\pi \approx 424.1$ cubic meters.
3. Grain volume: $0.8 \times 135\pi = 108\pi \approx 339.3$, or about 339 cubic meters.

A This is 80% of $2\pi rh = 90\pi$, the area of the curved side. Volume uses $\pi r^2 h$, which squares the radius.

C This is the full volume of the silo, $135\pi \approx 424$. The grain fills only 80% of that volume.

D This uses the diameter, 6, as the radius, which makes the volume four times too large. The radius is half the diameter.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

15. Correct answer: D M-T1M1-15

Nonlinear functions · Advanced Math · Medium

D *$f(x) = 2(x - 3)^2 + 1$*

A parabola with vertex (h, k) can be written $f(x) = a(x - h)^2 + k$. The vertex is (3, 1), so $f(x) = a(x - 3)^2 + 1$. The graph passes through (5, 9), so $9 = a(5 - 3)^2 + 1 = 4a + 1$, which gives $a = 2$. Thus $f(x) = 2(x - 3)^2 + 1$.

1. Read the vertex (h, k) = (3, 1) and write $f(x) = a(x - 3)^2 + 1$.
2. Substitute the marked point (5, 9): $9 = a(5 - 3)^2 + 1 = 4a + 1$.
3. Solve: $a = 2$, so $f(x) = 2(x - 3)^2 + 1$.

A Vertex form uses $x - h$, so $(x + 3)^2$ places the vertex at $x = -3$. The vertex of the graph is at $x = 3$.

B This has the correct vertex but $a = 1$, so $f(5) = 5$. The graph passes through (5, 9), which requires $a = 2$.

C The $- 1$ places the vertex at $(3, -1)$, below the x-axis. The vertex of the graph is at (3, 1).

Calculator option: Graph each choice on the same screen; only $2(x - 3)^2 + 1$ passes through both (3, 1) and (5, 9).

Review: Chapter 22, Nonlinear Functions (page 313)

16. Correct answer: B M-T1M1-16

Probability and conditional probability · Problem-Solving and Data Analysis · Harder

B *3/7*

The selection group is everyone who is under 18 or prefers the splash pad: all 80 visitors under 18 plus the 60 older visitors who prefer the splash pad, for 80 + 60 = 140 visitors. Of these, 60 are 18 or older and prefer the splash pad, so the probability is 60/140 = 3/7.

1. Group: all 80 visitors under 18, plus the 60 visitors 18 or older who prefer the splash pad: 80 + 60 = 140.
2. Event: 18 or older and prefers the splash pad: 60 visitors.
3. Probability = 60/140 = 3/7.

A This is 60/240, which uses all 240 visitors as the denominator. The visitor is chosen only from the 140 people in the stated group.

C This is 60/108, which uses only the visitors who prefer the splash pad. The group also includes the 32 visitors under 18 who prefer the dog run.

D This is 140/240, the probability that a visitor belongs to the selection group. The question asks about a smaller event within that group.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

17. Correct answer: 7/6 or 1.166 or 1.167 M-T1M1-17

Equivalent expressions · Advanced Math · Harder

Rewrite each radical as a rational exponent: $\sqrt[3]{(x^2)} = x^{2/3}$ and $\sqrt{(x^5)} = x^{5/2}$. Multiplying adds exponents and dividing subtracts them, so $k = 3 + 2/3 - 5/2 = 18/6 + 4/6 - 15/6 = 7/6$. A common wrong answer is 2, from writing $\sqrt[3]{(x^2)}$ as $x^{3/2}$; $-1/2$ comes from multiplying 3 by 2/3 instead of adding.

1. Rewrite the radicals: $\sqrt[3]{(x^2)} = x^{2/3}$ and $\sqrt{(x^5)} = x^{5/2}$.
2. Numerator: $x^3 \cdot x^{2/3} = x^{3 + 2/3} = x^{11/3}$.
3. Divide: $x^{11/3 - 5/2} = x^{22/6 - 15/6} = x^{7/6}$, so $k = 7/6$.

Calculator option: Check by evaluating the expression at $x = 2$ and comparing it with $2^{7/6}$; both are about 2.245.

Review: Chapter 20, Equivalent Expressions (page 289)

18. Correct answer: A M-T1M1-18

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

A *12*

A quadratic equation $ax^2 + bx + c = 0$ has exactly one real solution when its discriminant, $b^2 - 4ac$, equals 0. Here $a = 3$, $b = -12$, and $c = k$, so $(-12)^2 - 4(3)(k) = 144 - 12k = 0$, which gives $k = 12$. Check: $3x^2 - 12x + 12 = 3(x - 2)^2$, which has the single solution $x = 2$.

1. Exactly one real solution means the discriminant $b^2 - 4ac$ equals 0.
2. Substitute $a = 3$, $b = -12$, and $c = k$: $144 - 12k = 0$.
3. Solve: $k = 12$.

B This is $(b/2)^2 = 36$, which completes the square only when the leading coefficient is 1. With $a = 3$, the constant must be $3 \times 4 = 12$.

C This comes from dropping the factor 4 and solving $144 - 3k = 0$. The discriminant is $b^2 - 4ac$, so the product $4ac$ is $12k$.

D This sets k equal to $b^2 = 144$ and leaves out 4a. The condition for one solution is $b^2 = 4ac$, not $b^2 = c$.

Calculator option: Graph $y = 3x^2 - 12x + k$ with a slider for k; the parabola just touches the x-axis when $k = 12$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

19. Correct answer: C M-T1M1-19

Systems of two linear equations in two variables · Algebra · Harder

C *$912*

Let s be the number of small packages and L the number of large ones. Then $s + L = 96$ and $5s + 12L = 690$. Multiply the first equation by 5 and subtract it from the second: $7L = 210$, so $L = 30$ and $s = 66$. The charge is $7(66) + 15(30) = 462 + 450$, or \$912.

1. Let s = number of small packages and L = number of large packages: $s + L = 96$ and $5s + 12L = 690$.

2. Multiply the first equation by 5 to get $5s + 5L = 480$; subtract it from the second: $7L = 210$, so $L = 30$.

3. Then $s = 96 - 30 = 66$.

4. Charge: $7(66) + 15(30) = 462 + 450 = 912$ dollars.

A This is the charge for the 30 large packages only, 15×30. The 66 small packages add another \$462.

B This is the charge for the 66 small packages only, 7×66. The 30 large packages add another \$450.

D This swaps the counts, charging \$7 for 30 packages and \$15 for 66. Solving the system gives 66 small and 30 large packages.

Calculator option: Graph $x + y = 96$ and $5x + 12y = 690$ and select the intersection, (66, 30); then compute the charge from those counts.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

20. Correct answer: A M-T1M1-20

Right triangles and trigonometry · Geometry and Trigonometry · Harder

A *12*

In right triangle JKL, JL is the hypotenuse. Because $\sin J = 3/5$, the sides are in the ratio 3 : 4 : 5, so $\cos J = 4/5$ and $JK = 25 \times 4/5 = 20$. In right triangle JMK, JK is the hypotenuse and KM is opposite angle J, so $KM = 20 \times 3/5 = 12$. Check with area: $(1/2)(20)(15) = (1/2)(25)(KM)$ gives $KM = 12$.

1. In triangle JKL, $\sin J = 3/5$ gives $\cos J = 4/5$ (a 3-4-5 ratio), so $JK = JL \times \cos J = 25 \times 4/5 = 20$.

2. In right triangle JMK, JK is the hypotenuse and KM is the side opposite angle J.

3. $KM = JK \times \sin J = 20 \times 3/5 = 12$.

B This is $KL = 25 \times 3/5$, the leg opposite angle J in the large triangle. KM is a different segment: the height from K to JL.

C This is $JM = 20 \times 4/5$, which uses cosine in triangle JMK. KM is opposite angle J in that triangle, so it uses sine.

D This is JK, the hypotenuse of the small triangle JMK. KM is the leg opposite angle J, so $KM = 20 \times \sin J$.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

21. Correct answer: 153.6 or 768/5 M-T1M1-21

Nonlinear functions · Advanced Math · Harder

Because $b^0 = 1$, $f(0) = a = 300$. Then $f(2) = 300b^2 = 192$, so $b^2 = 0.64$, and because b is positive, $b = 0.8$. So $f(3) = 300(0.8)^3 = 300(0.512) = 153.6$. Entering 138 assumes equal drops, a linear model; entering 61.44 multiplies f(2) by 0.32, half of 0.64, not its square root.

1. $f(0) = a \times b^0 = a$, so $a = 300$.

2. $f(2) = 300b^2 = 192$, so $b^2 = 0.64$ and $b = 0.8$ (b is positive).

3. $f(3) = 300(0.8)^3 = 300(0.512) = 153.6$.

Review: Chapter 22, Nonlinear Functions (page 313)

22. Correct answer: D M-T1M1-22

Nonlinear functions · Advanced Math · Harder

D *It has a minimum value of −5 at x = 2.*

Apply the changes in order. Replacing x with $x - 5$ shifts the graph 5 units right, moving the vertex to (2, 7). The negative sign reflects the graph across the x-axis, so it opens upward with vertex (2, −7). Adding 2 shifts it up to (2, −5). An upward-opening parabola has a minimum, so g has a minimum value of −5 at $x = 2$.

1. $f(x - 5)$: shift right 5 units; the vertex moves to $(-3 + 5, 7) = (2, 7)$, and the graph still opens downward.

2. $-f(x - 5)$: reflect across the x-axis; the vertex becomes (2, −7), and the graph opens upward.

3. $-f(x - 5) + 2$: shift up 2 units; the vertex is (2, −5), so the minimum value is −5 at $x = 2$.

A This skips the reflection. The negative sign flips the parabola to open upward, so g has a minimum, and its vertex y-value is $-7 + 2$.

B This shifts the vertex 5 units left. In $f(x - 5)$, the graph moves right, because x must be 5 greater to give the same output.

C This reflects after adding 2, computing $-(7 + 2)$. In $-f(x - 5) + 2$, the reflection applies only to f, giving $-7 + 2 = -5$.

Review: Chapter 22, Nonlinear Functions (page 313)

Explanations: Math, Module 2

1. Correct answer: D M-T1M2-01

Linear functions · Algebra · Easier

D *f(x) = 2x − 2*

The line passes through (0, −2) and (3, 4). Its slope is (4 − (−2))/(3 − 0) = 6/3 = 2, and its y-intercept is −2 because the line crosses the y-axis at (0, −2). In slope-intercept form, f(x) = 2x − 2.

1. Slope m = (4 − (−2))/(3 − 0) = 6/3 = 2.
2. The y-intercept is b = −2, from the point (0, −2).
3. f(x) = mx + b = 2x − 2.

A This uses run over rise, 3/6, as the slope. Slope is the change in y divided by the change in x, which is 6/3 = 2.

B This uses the x-intercept, 1, as the constant term. In f(x) = mx + b, b is where the line crosses the y-axis, which is −2.

C A negative slope would make the line fall from left to right. The graph rises, so the slope is positive.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

2. Correct answer: A M-T1M2-02

Equivalent expressions · Advanced Math · Easier

A *(x + 7)(x − 5)*

Look for two numbers whose product is −35 and whose sum is 2. The pair 7 and −5 works, since 7 × (−5) = −35 and 7 + (−5) = 2. So $x^2 + 2x - 35 = (x + 7)(x - 5)$. Check by expanding: $x^2 - 5x + 7x - 35 = x^2 + 2x - 35$.

1. Look for numbers p and q with pq = −35 and p + q = 2.
2. The numbers p = 7 and q = −5 work.
3. So $x^2 + 2x - 35 = (x + 7)(x - 5)$.

B This expands to $x^2 - 2x - 35$. The numbers −7 and 5 multiply to −35 but add to −2, so the middle term has the wrong sign.

C This expands to $x^2 + 12x + 35$. With both signs positive, the constant term is +35 instead of −35.

D This expands to $x^2 + 34x - 35$. The numbers 35 and −1 multiply to −35, but they add to 34, not 2.

Calculator option: Graph $y = x^2 + 2x - 35$; its x-intercepts are −7 and 5, which match the factors (x + 7) and (x − 5).

Review: Chapter 20, Equivalent Expressions (page 289)

3. Correct answer: C M-T1M2-03

Lines, angles, and triangles · Geometry and Trigonometry · Easier

C *112*

Because KL = KM, the base angles at L and M are equal. The angles of triangle KLM sum to 180°, so each base angle measures (180 − 44)/2 = 68°. Angles KML and KMN form a straight line, so x = 180 − 68 = 112. Equivalently, an exterior angle equals the sum of the two remote interior angles: 44 + 68 = 112.

1. KL = KM, so angle L = angle M; each measures (180 − 44)/2 = 68°.
2. Angles KML and KMN are supplementary: x + 68 = 180.
3. Solve: x = 112.

A This is the interior angle KML, one of the base angles. The angle marked x is the exterior angle, which is supplementary to it.

B This assumes angle L also measures 44°. The equal sides KL and KM make the angles at L and M equal, not the angles at K and L.

D This is 180 − 44, the supplement of the angle at K. The exterior angle at M is supplementary to the interior angle at M.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: 260 M-T1M2-04

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Easier

Each inch on the drawing stands for 40 feet in the park, so multiply the drawing length by the scale: 6.5 inches × 40 feet per inch = 260 feet. A common wrong answer is 0.1625, which divides 6.5 by 40 instead of multiplying. Another is 46.5, which adds the two numbers.

1. Scale: 1 inch on the drawing represents 40 feet in the park.
2. Actual length = 6.5 × 40.
3. Actual length = 260 feet.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

5. Correct answer: C M-T1M2-05

Linear equations in one variable · Algebra · Medium

C *8*

Clear the fractions by multiplying every term by 12, the least common denominator: 3x + 2(x − 2) = 36. Distribute: 3x + 2x − 4 = 36, so 5x = 40 and x = 8. Check: 8/4 + 6/6 = 2 + 1 = 3.

1. Multiply every term by the least common denominator, 12: 3x + 2(x − 2) = 36.
2. Distribute and combine like terms: 5x − 4 = 36.
3. Solve: 5x = 40, so x = 8.

A This multiplies the left side by 12 but leaves the right side as 3. Every term, including the 3, must be multiplied by 12.

B This distributes the 2 only to x, writing −2 instead of −4. Multiplying (x − 2)/6 by 12 gives 2(x − 2) = 2x − 4.

D This adds the fractions by adding numerators and denominators, writing $(2x - 2)/10$. Fractions need a common denominator before they are added.

Calculator option: Graph $y = x/4 + (x - 2)/6$ and $y = 3$; they intersect where $x = 8$.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: A M-T1M2-06

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Medium

A *1.2*

From the graph, the 7-year-old tree yielded 18 bushels. The line predicts $y = 1.8(7) + 4.2 = 16.8$ bushels for a tree of that age. The actual yield exceeds the prediction by $18 - 16.8 = 1.2$ bushels. This difference between the actual and predicted values is the residual for that tree.

1. Read the point for age 7: actual yield = 18 bushels.
2. Predicted yield: $y = 1.8(7) + 4.2 = 16.8$ bushels.
3. Difference: $18 - 16.8 = 1.2$ bushels.

B This is the slope of the line, the predicted increase in yield for each additional year of age. It does not compare this tree with its prediction.

C This is the predicted yield for a 7-year-old tree. The question asks how far the actual yield is above that prediction.

D This is the tree's actual yield read from the graph. Subtract the predicted yield, 16.8 bushels, to find the difference.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

7. Correct answer: D M-T1M2-07

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

D *12*

Square both sides: $x + 37 = (x - 5)^2 = x^2 - 10x + 25$, so $x^2 - 11x - 12 = 0$ and $(x - 12)(x + 1) = 0$. The candidates are 12 and −1. Squaring can create extraneous solutions, so check both: $\sqrt{49} = 7 = 12 - 5$ works, but $\sqrt{36} = 6$, while $-1 - 5 = -6$. The only solution is 12.

1. Square both sides: $x + 37 = x^2 - 10x + 25$.
2. Rearrange and factor: $x^2 - 11x - 12 = (x - 12)(x + 1) = 0$, so $x = 12$ or $x = -1$.
3. Check in the original equation: $x = 12$ gives $7 = 7$ (true), and $x = -1$ gives $6 = -6$ (false). The solution is 12.

A This solves the squared equation but is extraneous: $\sqrt{36} = 6$, while $-1 - 5 = -6$. A principal square root is never negative.

B This comes from factoring $x^2 - 11x - 12$ as $(x + 12)(x - 1)$. In the original equation, $\sqrt{38}$ is not equal to −4.

C This comes from squaring $x - 5$ as $x^2 + 25$, which drops the middle term $-10x$. In the original equation, $\sqrt{41}$ is not equal to −1.

Calculator option: Graph $y = \sqrt{(x + 37)}$ and $y = x - 5$; they intersect only once, at $x = 12$, which shows that −1 is extraneous.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

8. Correct answer: 25 M-T1M2-08

Linear functions · Algebra · Medium

The slope is $(-2 - 10)/(5 - 1) = -12/4 = -3$. Using $f(1) = 10$, write $f(x) = -3(x - 1) + 10$, which simplifies to $f(x) = -3x + 13$. Then $f(-4) = -3(-4) + 13 = 25$. Entering 13 gives $f(0)$, the y-intercept; entering −5 comes from using a slope of 3 instead of −3.

1. Slope $m = (-2 - 10)/(5 - 1) = -3$.
2. Use the point (1, 10): $f(x) = -3(x - 1) + 10 = -3x + 13$.
3. $f(-4) = -3(-4) + 13 = 12 + 13 = 25$.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: A M-T1M2-09

Linear inequalities in one or two variables · Algebra · Medium

A *31*

Let t be the number of trees. The cost condition is $350(12) + 120t \leq 8{,}000$, or $4{,}200 + 120t \leq 8{,}000$. Subtracting 4,200 gives $120t \leq 3{,}800$, so $t \leq 95/3$, or about 31.67. The number of trees must be a whole number, so the greatest possible value is 31. Buying 32 would cost $8,040.

1. Let t = number of trees: $350(12) + 120t \leq 8{,}000$.
2. Simplify: $120t \leq 3{,}800$, so $t \leq 95/3 \approx 31.67$.
3. t must be a whole number, so the greatest value is 31.

B This rounds 31.67 up. Buying 32 trees costs $4,200 + 3,840 = $8,040, which is over the $8,000 limit.

C This subtracts the cost of only one bench, $350, from $8,000. The town buys 12 benches, which cost $4,200 in total.

D This divides $8,000 by $120 and ignores the benches. The 12 benches use $4,200 of the budget before any trees are bought.

Review: Chapter 19, Linear Inequalities (page 278)

10. Correct answer: B M-T1M2-10

Area and volume · Geometry and Trigonometry · Medium

B *667*

Add the areas of the two parts. The rectangle has area $30 \times 18 = 540$ square meters. The semicircle has diameter 18, so its radius is 9 and its area is $(1/2)\pi(9^2) = 40.5\pi$, or about 127.2 square meters. The total is about $540 + 127.2 = 667.2$, or 667 square meters.

1. Rectangle area: $30 \times 18 = 540$ square meters.

2. Semicircle: radius r = 18/2 = 9, so its area is $(1/2)\pi r^2 = 40.5\pi \approx 127.2$ square meters.
3. Total area ≈ 540 + 127.2 = 667.2, or about 667 square meters.

A This adds half the circumference, $(1/2)(2\pi)(9) \approx 28.3$, instead of half the area. Circumference measures length, not area.

C This adds the area of a full circle, $\pi(9^2) \approx 254.5$. Only a semicircle is attached, so use half of that area.

D This uses 18, the diameter, as the radius: $(1/2)\pi(18^2) \approx 508.9$. The radius is half the diameter, or 9 meters.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

11. Correct answer: 25/4 or 6.25 M-T1M2-11

Systems of two linear equations in two variables · Algebra · Medium

Adding the equations gives 12x + 12y = 75, because the x-coefficients 7 and 5 and the y-coefficients 4 and 8 each sum to 12. Dividing by 12 gives x + y = 75/12 = 25/4. Solving fully gives x = 4 and y = 9/4, which confirms it. Entering 75 skips the division; entering 4 gives x alone.

1. Add the equations: (7x + 5x) + (4y + 8y) = 37 + 38, so 12x + 12y = 75.
2. Divide both sides by 12: x + y = 75/12 = 25/4.
3. Check: x = 4 and y = 9/4 satisfy both equations, and 4 + 9/4 = 25/4.

Calculator option: Graph both lines and select their intersection, (4, 2.25); then add the coordinates.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

12. Correct answer: C M-T1M2-12

Equivalent expressions · Advanced Math · Medium

C $9x^4$

Raise each factor to the 2/3 power. For the coefficient, $27^{2/3} = (27^{1/3})^2 = 3^2 = 9$. For the variable, a power raised to a power multiplies the exponents: $(x^6)^{2/3} = x^{6 \times 2/3} = x^4$. The result is $9x^4$.

1. Apply the exponent to each factor: $(27x^6)^{2/3} = 27^{2/3} \cdot (x^6)^{2/3}$.
2. Coefficient: $27^{2/3} = (27^{1/3})^2 = 3^2 = 9$.
3. Variable: $(x^6)^{2/3} = x^4$, so the expression equals $9x^4$.

A This takes the cube root of 27, which is 3, but does not square it. The exponent 2/3 means a cube root and then a square: $3^2 = 9$.

B This multiplies 27 by 2/3. A fractional exponent on a number means a root and a power, not multiplication by the fraction.

D This adds the exponents 6 and 2/3. A power raised to a power multiplies the exponents: 6 × 2/3 = 4.

Calculator option: Evaluate the original expression and each choice at x = 2: the original gives 144, and only $9x^4$ also gives 144.

Review: Chapter 20, Equivalent Expressions (page 289)

13. Correct answer: B M-T1M2-13

Inference from sample statistics and margin of error · Problem-Solving and Data Analysis · Medium

B *It is plausible that between 50% and 58% of all the company's customers would pay extra for Saturday delivery.*

A margin of error describes how far a sample percent may plausibly be from the population percent. Adding and subtracting 4 gives a plausible range of 50% to 58%. Because the sample was chosen at random from this company's customers, the range applies to all of this company's customers, and only as a plausible range, not an exact value.

1. Sample percent: 54%; margin of error: 4 percentage points.
2. Plausible range: 54 − 4 = 50 to 54 + 4 = 58 percent.
3. The range describes the population that was sampled: all of this company's customers.

A A sample estimate is not exact for the whole population. The margin of error shows that the population percent could plausibly differ from 54%.

C The sample percent is known exactly: 54% of the 600. The margin of error describes uncertainty about all customers, not about the people surveyed.

D The sample came only from this company's customers, so it cannot support a conclusion about the customers of every shipping company.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

14. Correct answer: A M-T1M2-14

Nonlinear functions · Advanced Math · Harder

A $A(d) = 120(0.85)^{d/7}$

A 15% weekly decrease keeps 85% each week, so after w weeks the amount is $120(0.85)^w$. Time here is measured in days, and d days equal d/7 weeks, so $A(d) = 120(0.85)^{d/7}$. Check: after 7 days, A(7) = 120(0.85) = 102 kilograms, which is 15% less than 120.

1. Each week 85% remains, so the amount after w weeks is $120(0.85)^w$.
2. Convert days to weeks: w = d/7.
3. Substitute: $A(d) = 120(0.85)^{d/7}$. Check: A(7) = 102.

B The exponent 7d applies 7 weekly decreases for every day. After one day this gives $120(0.85)^7$, about 38 kilograms, which is far too much loss.

C The factor 0.15 is the share lost each week, not the share kept. A 15% decrease leaves 85%, so the decay factor is 0.85.

D Splitting the weekly rate evenly across 7 days does not compound correctly. After 7 days this gives about 103.1 kilograms, not 102.

Calculator option: Graph choice A and confirm that the curve passes through (7, 102) and (14, 86.7), the amounts after one and two weeks.

Review: Chapter 22, Nonlinear Functions (page 313)

15. Correct answer: 7/2 or 3.5 M-T1M2-15

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

Treat $2x - 1$ as a single quantity u. Then $u^2 - 5u - 6 = 0$ factors as $(u - 6)(u + 1) = 0$, so $u = 6$ or $u = -1$. Solving $2x - 1 = 6$ gives $x = 7/2$, and solving $2x - 1 = -1$ gives $x = 0$. The greater solution is 7/2. Entering 6 gives the value of u, not x.

1. Let $u = 2x - 1$, so the equation becomes $u^2 - 5u - 6 = 0$.
2. Factor: $(u - 6)(u + 1) = 0$, so $u = 6$ or $u = -1$.
3. Solve $2x - 1 = 6$ to get $x = 7/2$, and $2x - 1 = -1$ to get $x = 0$.
4. The greatest solution is 7/2.

Calculator option: Graph $y = (2x - 1)^2 - 5(2x - 1) - 6$; its x-intercepts are 0 and 3.5.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

16. Correct answer: C M-T1M2-16

Linear equations in two variables · Algebra · Harder

C *It decreases by 125 miles.*

Two more days cost $2 \times \$25 = \$50$ more, which leaves \$50 less to spend on miles. At \$0.40 per mile, \$50 pays for $50/0.4 = 125$ miles, so the number of miles decreases by 125. In slope terms, $m = 1{,}000 - 62.5d$, so each added day removes 62.5 miles.

1. Let d = number of days and m = number of miles: $25d + 0.4m = 400$.
2. Solve for m: $m = 1{,}000 - 62.5d$.
3. Increasing d by 2 changes m by $-62.5 \times 2 = -125$ miles.

A This multiplies the extra \$50 by 0.4 instead of dividing. Each mile costs \$0.40, so \$50 pays for $50/0.4 = 125$ miles.

B This is the extra cost in dollars, 2×25. The question asks for the change in miles, so divide \$50 by \$0.40 per mile.

D More rental days use more of the fixed \$400, which leaves less money for miles. The number of miles goes down, not up.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

17. Correct answer: D M-T1M2-17

Right triangles and trigonometry · Geometry and Trigonometry · Harder

D *42*

In triangle RST, $\sin R = ST/RS = 21/29$, so the sides are in the ratio RT : ST : RS = 20 : 21 : 29, since $20^2 + 21^2 = 29^2$. Similar triangles share these ratios. XZ corresponds to RT and YZ corresponds to ST, so $YZ/XZ = 21/20$ and $YZ = 40 \times 21/20 = 42$.

1. $\sin R = 21/29$ means $ST = 21k$ and $RS = 29k$ for some $k > 0$; then $RT = 20k$ by the Pythagorean theorem.
2. In the similar triangle XYZ, XZ corresponds to RT and YZ corresponds to ST.
3. In triangle XYZ, write $XZ = 20s$ and $YZ = 21s$. From $20s = 40$, $s = 2$, so $YZ = 42$.

A This copies the 21 from sin R as a length. The sine is a ratio, and here the scale factor is $40/20 = 2$, so $YZ = 21 \times 2$.

B This treats XZ as the hypotenuse and computes $40 \times 21/29$. XZ is a leg; the hypotenuse is XY, opposite the right angle at Z.

C This uses 20/21, the reciprocal of the ratio needed. YZ is opposite angle X and XZ is adjacent to it, so $YZ = XZ \times 21/20$.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

18. Correct answer: -1 M-T1M2-18

Nonlinear functions · Advanced Math · Harder

Substitute the two points: $f(2) = 4a + c = 11$ and $f(7) = 9a + c = 26$. Subtracting the first equation from the second gives $5a = 15$, so $a = 3$ and $c = 11 - 12 = -1$. Because $a > 0$, the parabola opens upward, and its minimum is at the vertex $(4, c)$. The minimum value is -1. Entering 4 gives the x-coordinate of the vertex; entering 3 gives a.

1. $f(2) = a(2 - 4)^2 + c = 4a + c = 11$.
2. $f(7) = a(7 - 4)^2 + c = 9a + c = 26$.
3. Subtract: $5a = 15$, so $a = 3$; then $c = 11 - 12 = -1$.
4. Because $a > 0$, the minimum value is $f(4) = c = -1$.

Review: Chapter 22, Nonlinear Functions (page 313)

19. Correct answer: B M-T1M2-19

Systems of two linear equations in two variables · Algebra · Harder

B *−1*

No solution requires parallel, distinct lines. Their coefficient pairs must be proportional, so the cross-products match: $k(k + 1) = 3 \times 4$. Thus $k^2 + k - 12 = (k + 4)(k - 3) = 0$, giving $k = -4$ or 3. At $k = 3$, the equations require $x + y = 2.5$ and $x + y = 2$. At $k = -4$, adding the equations gives $0 = 16$. Both are inconsistent, so the required sum is -1.

1. No solution requires proportional coefficients: $(k + 1)(k) = (3)(4)$.
2. So $k^2 + k - 12 = (k + 4)(k - 3) = 0$, which gives $k = -4$ or $k = 3$.
3. Check: $k = 3$ requires $x + y = 2.5$ and $x + y = 2$; $k = -4$ gives $-3x + 4y = 10$ and $3x - 4y = 6$, which add to $0 = 16$.
4. Sum: $-4 + 3 = -1$.

A This is the product of the two values, $(-4)(3)$. The question asks for their sum, which is $-4 + 3 = -1$.

C This adds 4 and −3, the solutions with their signs reversed. Setting $k + 4 = 0$ and $k - 3 = 0$ gives $k = -4$ and $k = 3$.

D This is only one of the two values. The value $k = -4$ also makes the lines parallel and distinct, so it belongs in the sum.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

20. Correct answer: B M-T1M2-20

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

B *2*

A quadratic equation has no real solutions when its discriminant is negative. Here $b^2 - 4ac = (-2k)^2 - 4(1)(k + 6) = 4k^2 - 4k - 24 = 4(k - 3)(k + 2)$. This product is negative only when $-2 < k < 3$. Of the choices, only $k = 2$ lies in that interval; its discriminant is $16 - 32 = -16$.

1. Discriminant: $(-2k)^2 - 4(1)(k + 6) = 4k^2 - 4k - 24 = 4(k - 3)(k + 2)$.
2. No real solutions requires $4(k - 3)(k + 2) < 0$, which means $-2 < k < 3$.
3. Only $k = 2$ is in that interval (its discriminant is −16).

A The discriminant is $4(9) - 4(3) = 24$, which is positive, so there are two real solutions. This choice comes from requiring a positive discriminant.

C The discriminant is $36 - 36 = 0$, so the equation $x^2 - 6x + 9 = 0$ has exactly one real solution, $x = 3$.

D The discriminant is $144 - 48 = 96$, which is positive, so there are two real solutions. Using $b = -k$ instead of $-2k$ can make this look valid.

Calculator option: Graph $y = x^2 - 2kx + (k + 6)$ with a slider for k; at $k = 2$ the whole parabola stays above the x-axis, while the other choices touch or cross it.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

21. Correct answer: 14 M-T1M2-21

Circles · Geometry and Trigonometry · Harder

An inscribed angle is half of the central angle that intercepts the same arc, so central angle AOB measures $2 \times 35° = 70°$. Arc length is that fraction of the circumference: $(70/360)(2\pi \times 36) = (7/36)(72\pi) = 14\pi$, so $k = 14$. Entering 7 uses 35° as the central angle; entering 252 finds the sector area instead of the arc length.

1. Central angle AOB = 2 × angle ACB = $2 \times 35° = 70°$.
2. Circumference = $2\pi(36) = 72\pi$.
3. Arc AB = $(70/360)(72\pi) = 14\pi$, so $k = 14$.

Review: Chapter 28, Circles (page 405)

22. Correct answer: D M-T1M2-22

Nonlinear functions · Advanced Math · Harder

D *12*

Substitute $x + 2$ into f: $g(x) = 3(2^{x+2})$. By the product rule for exponents, $2^{x+2} = 2^x \cdot 2^2 = 4(2^x)$. So $g(x) = 3 \cdot 4(2^x) = 12(2^x)$, and $a = 12$. For this function, shifting left 2 units multiplies every output by 4.

1. $g(x) = f(x + 2) = 3(2^{x+2})$.
2. $2^{x+2} = 2^x \cdot 2^2 = 4(2^x)$.
3. $g(x) = 3 \cdot 4(2^x) = 12(2^x)$, so $a = 12$.

A This uses 2^{-2}, as if the graph moved right. Replacing x with $x + 2$ multiplies the output by $2^2 = 4$, so $a = 3 \times 4$.

B This multiplies 3 by 2 instead of by 2^2. Increasing the exponent by 2 multiplies the output by 4, not by 2.

C This adds 3 and 2^2. Since $2^{x+2} = 2^x \cdot 2^2$, the factor 4 multiplies the coefficient 3; it is not added to it.

Calculator option: Graph $y = 3(2^{x+2})$ and $y = 12(2^x)$ as a visual check of the exponent calculation.

Review: Chapter 22, Nonlinear Functions (page 313)

CHAPTER 31

Practice Test 2

Practice Test 2 has the same layout as Practice Test 1: 98 new questions in four modules, with the timing and usual question order of the digital SAT. Its passages, data and problems are all different from those in Test 1, so it shows whether your review has carried over to questions you have never seen. After these instructions you will find an answer sheet, the two Reading and Writing modules, the Math reference sheet and the two Math modules.

When to take it

Take Test 2 at least one week after reviewing Test 1. The gap gives you time for spaced redos and chapter work before you test those methods on new questions. If your schedule is shorter, use the available time for focused review. If you have not yet redone most of the questions in your error log, finish that first.

What the test contains

Part	Questions	Time	Average pace
Reading and Writing, Module 1	27	32 minutes	about 71 seconds per question
Reading and Writing, Module 2	27	32 minutes	about 71 seconds per question
Break	—	10 minutes	—
Math, Module 1	22	35 minutes	about 95 seconds per question
Math, Module 2	22	35 minutes	about 95 seconds per question
Total	98	134 minutes of testing, plus the break	—

Recreate test conditions

Use the same setup as for Test 1 so that the two results can be compared fairly. Block out about two and a half hours in one sitting, ideally at the time of day your test is scheduled.

1. Choose a quiet room, leave your phone in another room and keep time with a separate timer or a watch without an alarm.
2. Have pencils, scratch paper and the calculator you plan to use on test day. The Desmos graphing calculator is built into the Bluebook app, and approved handheld calculators are also allowed; confirm your model on the College Board website.
3. Keep the Math reference sheet, printed before Math Module 1, open beside you during both Math modules.

4. Take one module at a time. Do not look ahead, and do not go back to a module after its time ends, just as on test day.
5. Stop at each STOP line when time runs out.
6. Take the full 10-minute break before Math. Move around, drink water and eat something; leave the answer key closed.

An honest note about Module 2

On test day, your second module in each section is chosen by your performance on the first: on average, it is either harder or easier than Module 1. This printed test cannot adapt. Each Module 2 here is a fixed set, somewhat harder than Module 1 and similar in spirit to the higher-difficulty path. Take it whatever your Module 1 result, and expect it to feel more demanding.

Using the answer sheet

Fill in one bubble for each multiple-choice question, and record answers module by module as you work. For each student-produced response, write your number, fraction or decimal on its labeled answer line. Apply the digital entry rules from Chapter 15 even though the printed line has more room:

- Use no more than 5 characters for a positive answer, or 6 for a negative answer, counting the minus sign.
- Enter a fraction or a decimal; if a fraction will not fit, switch to its decimal form.
- Turn a mixed number into an improper fraction or a decimal (7/2 or 3.5, not 3 1/2).
- Leave out percent signs, dollar signs, commas, units and symbols such as π.
- Reduce fractions. If a decimal is too long to fit, as a repeating decimal is, cut it off or round it so that it fills every available space.

Put your Test 1 lessons to work

Before you start, reread the miss-type tallies from your Chapter 30 review and choose one habit to practice during this test.

- If **Time** misses were common, write your Chapter 3 checkpoints on your scratch paper (Reading and Writing 22 / 12 / 3, Math 28 / 17 / 3 minutes left) and check them.
- If **Misread** misses were common, underline the task word in each question before you answer.
- If **Careless** misses were common, build in a 5-second check before you move on.
- If **Trap** misses were common, predict an answer before you read the choices.

Whatever your plan, answer every question. There is no penalty for wrong answers, so in the last minute of each module, fill in an answer for anything left blank, marking each guess with a small "g" for review.

When you finish

Write down how many questions you left unfinished in each module. Then turn to Chapter 32 to check your answers, calculate percent correct and compare your results with Practice Test 1.

Practice Test 2 Answer Sheet

Choose one bubble for multiple-choice questions. On rows labeled Answer, write a number, fraction or decimal. Follow the entry rules in Chapter 15.

Reading and Writing, Module 1

1 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ

Reading and Writing, Module 2

1 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ	19 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	11 Ⓐ Ⓑ Ⓒ Ⓓ	20 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	12 Ⓐ Ⓑ Ⓒ Ⓓ	21 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	13 Ⓐ Ⓑ Ⓒ Ⓓ	22 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	14 Ⓐ Ⓑ Ⓒ Ⓓ	23 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	15 Ⓐ Ⓑ Ⓒ Ⓓ	24 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	16 Ⓐ Ⓑ Ⓒ Ⓓ	25 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ	17 Ⓐ Ⓑ Ⓒ Ⓓ	26 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ	18 Ⓐ Ⓑ Ⓒ Ⓓ	27 Ⓐ Ⓑ Ⓒ Ⓓ

Practice Test 2 Answer Sheet (continued)

Choose one bubble for multiple-choice questions. On rows labeled Answer, write a number, fraction or decimal. Follow the entry rules in Chapter 15.

Math, Module 1

1 Ⓐ Ⓑ Ⓒ Ⓓ	**9** Answer: _____	**17** Answer: _____
2 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**18** Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Ⓐ Ⓑ Ⓒ Ⓓ	**19** Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
5 Answer: _____	**13** Answer: _____	**21** Answer: _____
6 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Ⓐ Ⓑ Ⓒ Ⓓ	
8 Ⓐ Ⓑ Ⓒ Ⓓ	**16** Ⓐ Ⓑ Ⓒ Ⓓ	

Math, Module 2

1 Ⓐ Ⓑ Ⓒ Ⓓ	**9** Ⓐ Ⓑ Ⓒ Ⓓ	**17** Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ	**10** Ⓐ Ⓑ Ⓒ Ⓓ	**18** Answer: _____
3 Ⓐ Ⓑ Ⓒ Ⓓ	**11** Answer: _____	**19** Ⓐ Ⓑ Ⓒ Ⓓ
4 Answer: _____	**12** Ⓐ Ⓑ Ⓒ Ⓓ	**20** Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ	**13** Ⓐ Ⓑ Ⓒ Ⓓ	**21** Answer: _____
6 Ⓐ Ⓑ Ⓒ Ⓓ	**14** Ⓐ Ⓑ Ⓒ Ⓓ	**22** Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ	**15** Answer: _____	
8 Answer: _____	**16** Ⓐ Ⓑ Ⓒ Ⓓ	

Practice Test 2: Reading and Writing, Module 1

27 questions. 32 minutes.

1 RW-T2M1-01

> The following text is from Anton Chekhov's 1887 short story "Kashtanka," translated by Constance Garnett.
>
> A young dog, a reddish mongrel, between a dachshund and a "yard-dog," very like a fox in face, was running up and down the pavement looking _____ from side to side. From time to time she stopped and, whining and lifting first one chilled paw and then another, tried to make up her mind how it could have happened that she was lost.

Which choice completes the text with the most logical and precise word or phrase?

A proudly **B** uneasily **C** lazily **D** playfully

---

2 RW-T2M1-02

> Many bacteria do not live alone. On a wet surface, such as a rock in a stream, single cells can attach and begin producing a sticky coating that holds them together. Over time, thousands of cells _____ into a slimy layer called a biofilm. Living in this crowded layer helps protect the bacteria from being washed away, and it can also make them harder to remove with cleaning chemicals.

Which choice completes the text with the most logical and precise word or phrase?

A scatter **B** shrink **C** dissolve **D** gather

---

3 RW-T2M1-03

> Sculptors who work with salvaged barn wood face a choice: sand away the nail holes, saw marks and weathered paint, or leave them in place. Those who keep these traces argue that the marks let each piece _____ its earlier life as part of a building, so that viewers can read a board's history in its surface before they notice the new form the artist has given it.

Which choice completes the text with the most logical and precise word or phrase?

A disclose **B** conceal **C** exaggerate **D** postpone

---

4 RW-T2M1-04

> Long before public unemployment insurance existed, groups of workers sometimes formed mutual aid societies. In a typical arrangement, members paid small weekly dues into a shared fund, and when one of them was injured or fell ill, the society drew on that fund to _____ part of the lost wages for a few weeks. Because members ran these societies themselves, the groups also gave workers practice in keeping accounts, holding elections and settling disputes.

Which choice completes the text with the most logical and precise word or phrase?

A forfeit **B** calculate **C** replace **D** withhold

---

5 RW-T2M1-05

> The following text is from Edith Wharton's 1917 novel *Summer*. Charity Royall once made a day trip from her small village, North Dormer, to the larger town of Nettleton.
>
> This initiation had shown her that North Dormer was a small place, and developed in her a thirst for information that her position as custodian of the village library had previously failed to _____. For a month or two she dipped feverishly and disconnectedly into the dusty volumes of the Hatchard Memorial Library; then the impression of Nettleton began to fade, and she found it easier to take North Dormer as the norm of the universe than to go on reading.

Which choice completes the text with the most logical and precise word or phrase?

A excite **B** document **C** conceal **D** interrupt

6 RW-T2M1-06

> The following text is Robert Louis Stevenson's 1885 poem "Where Go the Boats?"
>
> Dark brown is the river,
> Golden is the sand.
> It flows along for ever,
> With trees on either hand.
>
> Green leaves a-floating,
> Castles of the foam,
> Boats of mine a-boating—
> Where will all come home?
>
> On goes the river
> And out past the mill,
> Away down the valley,
> Away down the hill.
>
> Away down the river,
> A hundred miles or more,
> Other little children
> Shall bring my boats ashore.

Which choice best states the main purpose of the text?

A To describe the dangers that small boats face on a long journey down a river

B To convey a child's imagining of where the toy boats will travel

C To explain how the mill and the hills change the course of the river

D To compare the colors of the river with the colors of the leaves and foam

7 RW-T2M1-07

> In many turtle species, the temperature of the sand during part of incubation determines a hatchling's sex: in several species, warmer nests produce mostly females and cooler nests produce mostly males. Because of this link, a nesting beach that warms over many years could gradually produce far fewer males than it once did. A conservation team on a hypothetical nesting beach has therefore begun shading selected nests, hoping to keep the balance of hatchlings closer to even.

Which choice best describes the function of the underlined portion in the text as a whole?

A It introduces a new explanation for how hatchling sex is determined that replaces the one given earlier.

B It presents evidence that shading nests has increased the number of male hatchlings.

C It describes the method the team used to measure the temperature of sand on the beach.

D It identifies a possible consequence of the temperature effect that helps explain why the team shades nests.

---

8 RW-T2M1-08

> **Text 1**
> In a hypothetical study, two groups of students read the same science article. One group reread it twice; the other closed the article, wrote down everything they could recall and then checked their notes against it. A week later, the recall group remembered far more. The researchers concluded that rereading wastes study time and that students should replace it entirely with practice in recalling material from memory.
>
> **Text 2**
> Recall practice clearly strengthens memory, but it depends on having something accurate to recall. A student who has read a dense article only once may retrieve a confused or partial version of it, and practicing that version can make the errors stick. For difficult material, a second careful reading that builds a correct first understanding is often what makes later recall practice worthwhile.

Based on the texts, how would the author of Text 2 most likely respond to the researchers' conclusion in Text 1?

A By rejecting the claim that recall practice improves students' memory of what they read

B By arguing that the recall group remembered more only because its members felt more confident

C By suggesting that rereading can still play a useful role before recall practice, especially with difficult material

D By agreeing that students should stop rereading and rely only on recall practice

---

9 RW-T2M1-09

> The following text is from Rabindranath Tagore's 1913 poetry collection *The Crescent Moon*, which Tagore translated from Bengali into English himself.
>
> Day by day I float my paper boats one by one down the running stream.
>
> In big black letters I write my name on them and the name of the village where I live.
>
> I hope that someone in some strange land will find them and know who I am.
>
> I load my little boats with shiuli flowers from our garden, and hope that these blooms of the dawn will be carried safely to land in the night.

Which choice best states the main idea of the text?

A The speaker is learning to build boats that can survive a long trip downstream.

B The speaker plans to leave the village and travel to a strange land.

C The speaker gathers flowers each morning to decorate the family garden.

D The speaker sends paper boats down the stream in the hope of being known by someone far away.

---

10 RW-T2M1-10

> When a large volcano erupts, the towering cloud of ash is the most visible sign of its power, yet ash has little lasting effect on global temperatures. Most ash particles are heavy enough to fall back to the ground within days or weeks. The longer-lasting influence comes from sulfur dioxide gas. If an eruption is powerful enough to inject this gas into the stratosphere, the gas reacts to form tiny sulfate aerosol droplets that reflect some incoming sunlight, and these droplets can remain aloft for a year or more, slightly cooling Earth's surface.

Which choice best states the main idea of the text?

A An eruption's longer-term cooling effect comes mainly from sulfur gas that reaches high in the atmosphere rather than from the ash cloud.

B Volcanic ash clouds are dangerous mainly because the particles are heavy and fall to the ground quickly.

C Every volcanic eruption sends sulfur dioxide into the stratosphere and cools Earth's surface.

D Sulfur dioxide gas is the most visible product of a large volcanic eruption.

---

11 RW-T2M1-11

> For a class project, a student recorded male spring peepers, a small frog species, calling at a pond on four spring nights with different water temperatures. The student found that the frogs called more often in warmer water: at a water temperature of 17°C, males produced an average of _____

Average Calling Rate of Male Spring Peepers at Four Water Temperatures (hypothetical data)

Water temperature (°C)	Average calls per minute
8	38
11	52
14	67
17	81

Which choice most effectively uses data from the table to complete the statement?

A 38 calls per minute, compared with 81 calls per minute at 8°C.

B 52 calls per minute, compared with 67 calls per minute at 14°C.

C 81 calls per minute, compared with 38 calls per minute at 8°C.

D 67 calls per minute, compared with 52 calls per minute at 11°C.

12 RW-T2M1-12

> *Anne of Green Gables* is a 1908 novel by L. M. Montgomery. In one episode, Josie Pye dares Anne to walk along the top of a roof, and Anne's friend Diana warns her not to try it. A reader claims that Anne's concern for her own honor makes her determined to attempt the dangerous feat: _____

Which quotation from the novel most effectively illustrates the claim?

A "Anne tossed her red braids."

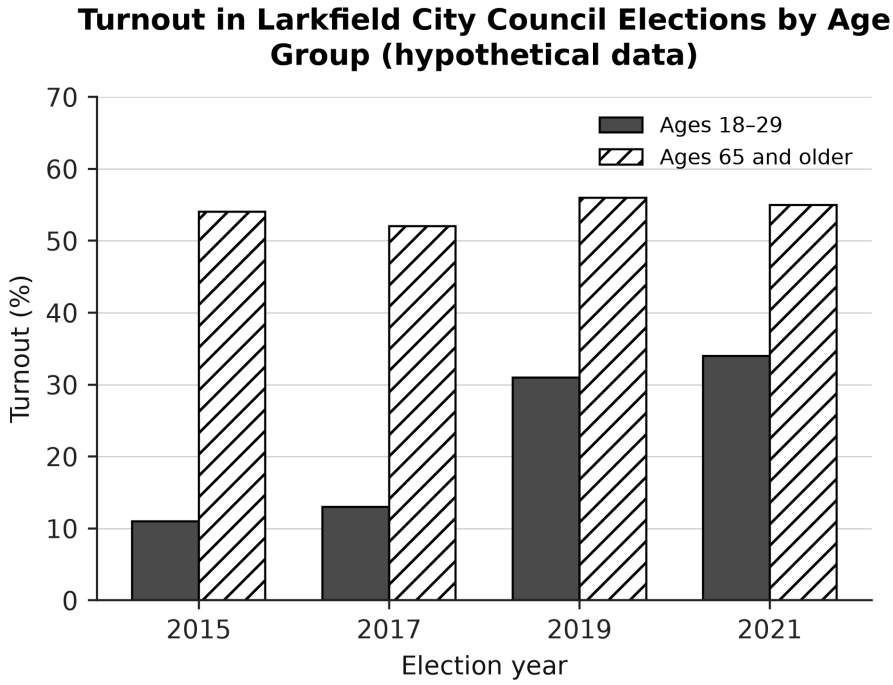
B "'Don't you do it, Anne,' entreated Diana. 'You'll fall off and be killed.'"

C "Josie descended from her perch, flushed with victory, and darted a defiant glance at Anne."

D "'I must do it. My honor is at stake,' said Anne solemnly. 'I shall walk that ridgepole, Diana, or perish in the attempt.'"

13 RW-T2M1-13

> The fictional city of Larkfield once held its city council elections on a different day from statewide elections, a schedule that tended to draw small crowds to the polls. Beginning in 2019, the city moved its elections to the same day as statewide voting. After examining turnout by age group, a political science student concluded that turnout rose much more among younger voters than among older voters after the schedule changed.



Which choice best describes data from the graph that support the student's conclusion?

A Turnout among voters aged 65 and older was higher than turnout among voters aged 18–29 in every election shown.

B Turnout among voters aged 18–29 was higher in 2017 than in 2015.

C Turnout among voters aged 65 and older was highest in 2019, the first year the elections were combined.

D Turnout among voters aged 18–29 rose from about 13% in 2017 to about 31% in 2019, while turnout among voters aged 65 and older stayed close to 55%.

14 RW-T2M1-14

> Bacteria usually pass genes to the next generation when a cell divides. Many bacteria, however, can also pass small rings of DNA called plasmids directly to neighboring cells, even cells of a different species, and some plasmids carry genes that let bacteria survive antibiotics. In a hypothetical experiment, a team grew an antibiotic-resistant strain beside a strain of a different species that could not survive a test dose of the antibiotic. After a few days, some cells of the second strain survived the same dose of the antibiotic. This result suggests that _____

Which choice most logically completes the text?

A some cells of the second strain had likely received resistance genes from the first strain.

B the antibiotic dose used in the experiment was too weak to harm any bacteria.

C bacteria can pass plasmids only to cells of their own species.

D the second strain had already been resistant before the two strains were grown together.

---

15 RW-T2M1-15

> Conservators can sometimes date a quilt by testing its dyes. Many synthetic dyes were first manufactured in known years, so a fabric containing such a dye cannot be older than the dye itself. A conservator examining a hypothetical quilt, which the owners' family says was sewn in 1850, finds that one patch contains a synthetic dye first produced decades later. However, that patch is attached with stitching unlike the stitching anywhere else on the quilt, and every other patch contains only plant-based dyes that were in use long before 1850. Taken together, these findings suggest that _____

Which choice most logically completes the text?

A the whole quilt was probably made decades later than the family believes.

B the quilt may date from around 1850, with the patch containing the later dye added during a repair.

C plant-based dyes had stopped being used by the time the quilt was sewn.

D testing a fabric's dyes cannot reveal anything useful about when it was made.

---

16 RW-T2M1-16

> Weather enthusiasts can now buy compact stations for their backyards. A basic home weather station uses separate sensors to measure air temperature, _____ and rainfall, and then it sends the readings to a display inside the house. Some models also upload the data to online maps, where neighbors can compare conditions on their own streets.

Which choice completes the text so that it conforms to the conventions of Standard English?

A humidity wind speed **B** humidity; wind speed **C** humidity, wind speed **D** humidity, wind speed;

---

17 RW-T2M1-17

> In a hypothetical textile town in 1912, workers at three mills walked off the job together after their wages were cut. The _____ demands, printed in the local newspaper the next morning, included restoring the old pay rate and limiting the workday to ten hours. Within a week, two of the three mills had agreed to negotiate.

Which choice completes the text so that it conforms to the conventions of Standard English?

A workers' **B** worker's **C** workers **D** workers's

18 RW-T2M1-18

> Many bronze sculptures begin as wax. The lost-wax method—in which an artist's wax model is coated in clay and heated until the wax melts and drains away, leaving a hollow mold that is then filled with molten _____ been practiced for thousands of years. Because the clay mold captures every detail of the wax, the method lets sculptors reproduce fine textures such as fingerprints and strands of hair.

Which choice completes the text so that it conforms to the conventions of Standard English?

A bronze, has **B** bronze—has **C** bronze; has **D** bronze has

19 RW-T2M1-19

> Crocodiles spend long periods basking on riverbanks, and their skin is built for it. The tissues beneath the ridged scales along the back of a crocodile _____ bony plates called osteoderms. These plates strengthen the skin like a flexible suit of armor, and blood vessels running through them may help the animal absorb heat from the sun.

Which choice completes the text so that it conforms to the conventions of Standard English?

A contains **B** is containing **C** has contained **D** contain

20 RW-T2M1-20

> Dyers who work with indigo often describe the first minutes after a dip as the most surprising part of the process. Cloth lifted out of an indigo vat looks yellowish green at _____ within minutes it turns blue as the dye reacts with oxygen in the air. Repeating the dip-and-air cycle many times produces deeper and deeper shades.

Which choice completes the text so that it conforms to the conventions of Standard English?

A first but **B** first, but, **C** first, but **D** first; but,

21 RW-T2M1-21

> Psychologists have long used simple memory tasks to estimate how much information a person can hold in mind at once. In one hypothetical version of such a task, adult participants saw a list of twenty unrelated words flashed on a screen and then wrote down as many as they could. Shown each list for only a few seconds, _____ The results fit a familiar pattern: people tend to remember the first and last items on a list better than the items in the middle.

Which choice completes the text so that it conforms to the conventions of Standard English?

A most participants recalled between five and nine words, often including the first and last items.

B between five and nine words were recalled by most participants, often including the first and last items.

C the recall of most participants was between five and nine words, often including the first and last items.

D between five and nine words, often including the first and last items, made up most participants' recall.

22 RW-T2M1-22

> In many small New England towns, local budgets are still approved at a town meeting, where residents vote directly instead of electing representatives to decide for them. The rules governing who may speak at such a meeting and for how _____ often set out in a printed guide that is distributed before the session begins, so that newcomers know how to take part.

Which choice completes the text so that it conforms to the conventions of Standard English?

A long, are **B** long are **C** long; are **D** long—are

23 RW-T2M1-23

> Many visitors picture ancient Greek statues as gleaming white marble, and most surviving examples do look that way today. _____ experts who study the surfaces of these statues have found traces of paint, showing that many were once colored in reds, blues and golds. Over the centuries, weather and burial wore most of that color away.

Which choice completes the text with the most logical transition?

A Similarly, **B** For example, **C** As a result, **D** However,

24 RW-T2M1-24

> Many soil bacteria release antibiotics, chemicals that kill or slow the growth of nearby competitors. Making these chemicals is expensive, though: a cell must divert energy and raw materials that it could otherwise use to grow and divide. _____ some bacteria increase antibiotic production when they detect chemical cues from competitors, concentrating the costly response where competition is present.

Which choice completes the text with the most logical transition?

A Nevertheless, **B** Likewise, **C** For this reason, **D** In other words,

25 RW-T2M1-25

> While researching a topic, a student has taken the following notes:
>
> • Frances Perkins was born in Boston in 1880 and graduated from Mount Holyoke College in 1902.
>
> • From 1910 to 1912, she was executive secretary of the Consumers' League of New York and lobbied for better wages and working conditions, especially for women and children.
>
> • In 1933, President Franklin D. Roosevelt named her secretary of labor.
>
> • She was the first woman appointed to a U.S. cabinet post.
>
> • She helped develop the Social Security Act and worked to secure passage of the Fair Labor Standards Act (1938).

The student wants to introduce Frances Perkins to an audience unfamiliar with her. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A In 1933, President Franklin D. Roosevelt chose a new secretary of labor, the first woman to hold a cabinet post.

B Frances Perkins, a labor reformer who became the first woman appointed to a U.S. cabinet post, served as secretary of labor under President Franklin D. Roosevelt.

C Perkins helped develop the Social Security Act and later worked to secure passage of the Fair Labor Standards Act (1938).

D The Consumers' League of New York lobbied for better wages and working conditions, especially for women and children.

---

26 RW-T2M1-26

> While researching a topic, a student has taken the following notes:
>
> • Woven fabric is made by interlacing two sets of threads at right angles.
>
> • In woven fabric, the lengthwise threads are called the warp and the crosswise threads are called the weft.
>
> • Knitted fabric is made by pulling loops of yarn through other loops.
>
> • A knitted fabric can be made from a single continuous strand of yarn.
>
> • Because its loops can open and close, knitted fabric usually stretches more than woven fabric does.

The student wants to emphasize a difference in how the two kinds of fabric are constructed. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Woven fabric has lengthwise threads, called the warp, and crosswise threads, called the weft.

B Knitted fabric usually stretches more than woven fabric does, because its loops can open and close.

C Woven fabric is made by interlacing two sets of threads at right angles, whereas knitted fabric is made by pulling loops of yarn through other loops.

D A knitted fabric can be made from a single continuous strand of yarn that is pulled through loops.

---

27 RW-T2M1-27

> While researching a topic, a student has taken the following notes:
>
> - A team of biologists studied a hypothetical lizard species that lives on two nearby islands.
> - Island A has many predatory birds; Island B has almost none.
> - On each island, the team slowly approached 40 lizards and recorded the "flight distance," or how close a person could get before the lizard ran.
> - Average flight distance was 3.1 meters on Island A and 0.9 meters on Island B.
> - The team concluded that lizards on the island with few predators were bolder.

The student wants to present the study's method and its main finding. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A By measuring how close a person could get before each lizard ran, the team found that lizards on Island B, which has almost no predatory birds, let people come much closer (0.9 meters on average) than lizards on Island A did (3.1 meters).

B The team slowly approached 40 lizards on each of two islands and recorded each lizard's flight distance.

C Lizards on Island B, which has almost no predatory birds, were judged to be bolder than lizards on Island A.

D Island A has many predatory birds, while Island B has almost none, so lizards on the two islands face very different risks.

STOP. If you finish before time is called, check your work in this module only.

Practice Test 2: Reading and Writing, Module 2

27 questions. 32 minutes.

1 RW-T2M2-01

> In the northern part of their range, eastern box turtles live in forests and meadows where winters can be very cold. Because a turtle cannot produce enough body heat to stay warm, it must find another way to _____ the season. In autumn, turtles burrow into soil or shelter under leaf litter and become inactive, usually remaining sheltered until warmer spring weather returns.

Which choice completes the text with the most logical and precise word or phrase?

A predict **B** celebrate **C** endure **D** shorten

2 RW-T2M2-02

> The following text is from Selma Lagerlöf's 1906 novel *The Wonderful Adventures of Nils*, translated from the Swedish by Velma Swanston Howard. A boy has just noticed a tiny elf sitting on the edge of a chest.
>
> He had an old, wrinkled and beardless face, and was dressed in a black frock coat, knee-breeches and a broad-brimmed black hat. He was very trim and smart, with his white laces about the throat and wrist-bands, his buckled shoes, and the bows on his garters.

As used in the text, what does the word "smart" most nearly mean?

A Neatly dressed **B** Intelligent **C** Painful **D** Quick

3 RW-T2M2-03

> Labor strikes are often remembered as sudden walkouts, but many were anything but _____. Organizers typically spent weeks gathering pledges from workers, collecting money for a relief fund to feed strikers' families and choosing a moment, such as a busy season, when a shutdown would cost employers the most. Such preparation could decide whether a strike lasted a few days or held together for several months.

Which choice completes the text with the most logical and precise word or phrase?

A costly **B** effective **C** organized **D** spontaneous

4 RW-T2M2-04

> The following text is from Katherine Mansfield's short story "Miss Brill," published in her 1922 collection *The Garden Party and Other Stories*. Miss Brill is sitting on a bench in a public garden on a Sunday afternoon.
>
> Only two people shared her "special" seat: a fine old man in a velvet coat, his hands clasped over a huge carved walking-stick, and a big old woman, sitting upright, with a roll of knitting on her embroidered apron. They did not speak. This was disappointing, for Miss Brill always looked forward to the conversation. She had become really quite _____, she thought, at listening as though she didn't listen, at sitting in other people's lives just for a minute while they talked round her.

Which choice completes the text with the most logical and precise word or phrase?

A curious **B** expert **C** hesitant **D** weary

5 RW-T2M2-05

> The following text is Sara Teasdale's 1920 poem "Stars."
>
> Alone in the night
> On a dark hill
> With pines around me
> Spicy and still,
>
> And a heaven full of stars
> Over my head,
> White and topaz
> And misty red;
>
> Myriads with beating
> Hearts of fire
> That aeons
> Cannot vex or tire;
>
> Up the dome of heaven
> Like a great hill,
> I watch them marching
> Stately and still,
>
> And I know that I
> Am honored to be
> Witness
> Of so much majesty.

Which choice best describes the overall structure of the text?

A The speaker describes a frightening night and then explains how she found her way home.

B The speaker compares the stars to fires and then argues that they will eventually burn out.

C The speaker lists the colors of the stars and then questions whether she is worthy of seeing them.

D The speaker describes her nighttime surroundings and the stars above her and then expresses how she feels about witnessing them.

6 RW-T2M2-06

> In many clouds, ice crystals grow into precipitation. Tiny particles such as dust can help cloud droplets freeze. Some bacteria also carry proteins on their surfaces that help water freeze at temperatures only a few degrees below 0°C, warmer than dust typically allows. In a hypothetical survey of fresh snow from several mountain sites, a team of microbiologists found that many of the most active ice-forming particles were biological. Even so, the team noted, it remains unclear how often these bacteria reach clouds in numbers large enough to affect how much precipitation falls.

Which choice best describes the overall structure of the text?

A It provides background on how ice forms in clouds, presents evidence about one possible source of ice-forming particles and then identifies a question that remains unresolved.

B It explains how bacteria help ice form and then argues that dust plays no role in producing rain or snow.

C It details the methods of a survey of mountain snow and then compares the survey's results with those of earlier surveys.

D It presents a debate between two groups of scientists and then explains why one group's view is correct.

---

7 RW-T2M2-07

> **Text 1**
>
> A hypothetical outdoor bronze monument has uneven green streaks caused by active corrosion rather than a stable patina. Rainwater and pollutants can promote corrosion that gradually erases fine details. Careful treatment and a maintained protective wax coating can slow further deterioration. A conservator argues that the damaging corrosion should be stopped and this monument's original dark finish restored.
>
> **Text 2**
>
> Restoring a bronze monument to its original finish can unsettle a community that has known it only as green. For decades the color has appeared in photographs, postcards and local memory, and some residents see it as part of the work itself. Conservators should of course stop any corrosion that is wearing away the metal, but they should also consult the public before changing a familiar landmark's appearance.

Based on the texts, both authors would most likely agree with which statement?

A Outdoor bronze statues should always be returned to the finish their sculptors intended.

B A green surface on a bronze statue causes no harm to the metal underneath.

C Corrosion that is eating away a bronze statue's surface should be halted.

D Residents should decide how every public monument is cleaned and maintained.

---

8 RW-T2M2-08

> The following text is from Johanna Spyri's 1880–1881 novel *Heidi*, translated into English by Marian Edwardes. Heidi, a young girl, has just come to live in her grandfather's mountain hut.
>
> Heidi was delighted, and began at once to examine all the nooks and corners to find out where it would be pleasantest to sleep. In the corner near her grandfather's bed she saw a short ladder against the wall; up she climbed and found herself in the hayloft. There lay a large heap of fresh sweet-smelling hay, while through a round window in the wall she could see right down the valley.
>
> "I shall sleep up here, grandfather," she called down to him, "It's lovely, up here. Come up and see how lovely it is!"
>
> "Oh, I know all about it," he called up in answer.

Based on the text, what is true about the grandfather's reaction to Heidi's choice of the hayloft?

A He worries that the hayloft is unsafe for a young child.

B He shows little excitement because the hayloft is already familiar to him.

C He is annoyed that Heidi did not choose to sleep near his bed.

D He is surprised to learn that the hayloft has a view of the valley.

---

9 RW-T2M2-09

> A student analyzing records from a hypothetical weather station in the central United States counted the average number of days with thunderstorms in each month from April through September. The student concluded that thunderstorms at the station were most common in midsummer, noting that July averaged _____.

Average Number of Thunderstorm Days per Month at a Weather Station (hypothetical data)

Month	Average thunderstorm days
April	3
May	6
June	9
July	12
August	10
September	5

Which choice most effectively uses data from the table to complete the statement?

A 12 thunderstorm days, more than any other month in the table.

B 9 thunderstorm days, more than any other month in the table.

C 10 thunderstorm days, fewer than June averaged.

D 3 thunderstorm days, fewer than any other month in the table.

---

10 RW-T2M2-10

> *American Indian Stories* is a 1921 book by Zitkala-Sa, a Yankton Sioux writer and reformer. In one of its autobiographical sections, she recalls that as a young girl living near the Missouri River, she enjoyed a sense of unrestrained freedom: _____

Which quotation from the book most effectively illustrates the claim?

A "A wigwam of weather-stained canvas stood at the base of some irregularly ascending hills."

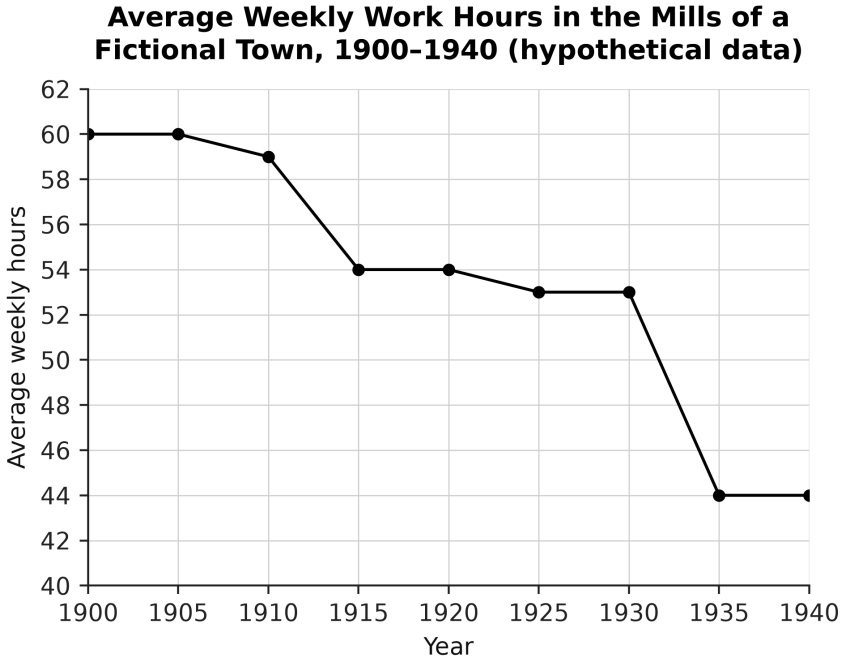
B "Often she was sad and silent, at which times her full arched lips were compressed into hard and bitter lines, and shadows fell under her black eyes."

C "My grown-up cousin, Warca-Ziwin (Sunflower), who was then seventeen, always went to the river alone for water for her mother."

D "Loosely clad in a slip of brown buckskin, and light-footed with a pair of soft moccasins on my feet, I was as free as the wind that blew my hair, and no less spirited than a bounding deer."

11 RW-T2M2-11

> The shortening of the factory workweek in the early twentieth century is often pictured as a slow, steady process. After studying payroll records from the textile mills of a fictional New England town, a student argues that the change there followed a different pattern: it came mainly in two sudden drops, each followed by a long period of little change.



Which choice best describes data from the graph that support the student's argument?

A Average weekly hours were lower in 1940 than in 1900.

B Average weekly hours fell between 1910 and 1915.

C Average weekly hours changed by one hour or less across most five-year intervals but fell sharply between 1910 and 1915 and again between 1930 and 1935.

D Average weekly hours fell by roughly the same amount in each five-year interval from 1900 to 1940.

12 RW-T2M2-12

> Certain soil bacteria colonize plant roots and protect the plants from disease-causing fungi. Studying a hypothetical root-dwelling bacterium, a team of microbiologists found that bean plants with the bacterium on their roots developed far less root rot, a fungal disease. The team hypothesizes that the protection comes from an antifungal chemical the bacterium releases, rather than from the bacterium simply crowding the fungus out by occupying space on the roots.

Which finding, if true, would most directly support the team's hypothesis?

A A mutant strain of the bacterium that covers roots as densely as the normal strain but cannot make the chemical gives bean plants no protection from root rot.

B Bean plants with a dense covering of the normal bacterium on their roots show less root rot than plants with a sparse covering.

C A different species of bacterium that occupies the same amount of root space as the normal strain protects bean plants equally well.

D The fungus that causes root rot grows more slowly in cool, dry soil than in warm, wet soil.

---

13 RW-T2M2-13

> How a volcano erupts depends partly on its magma. Magma rich in silica is thick and sticky, so gas bubbles cannot easily escape from it; pressure builds until the gas bursts out violently. Magma low in silica is runny, and its gas escapes more easily. A geologist studying a hypothetical island volcano finds that nearly all of the rock formed by its past eruptions is low in silica. This finding suggests that most of the volcano's past eruptions _____

Which choice most logically completes the text?

A were driven by gas that could not escape from thick magma.

B released their gas relatively gently, producing flowing lava rather than violent blasts.

C produced more silica-rich rock than any other volcano on the island.

D will be followed by eruptions of much thicker magma in the future.

---

14 RW-T2M2-14

> When people focus closely on one task, they often fail to notice unexpected events in plain view, a pattern known as inattentional blindness. In a hypothetical experiment, participants watched a video of people passing a ball and counted the passes. Midway through, a person in a bright costume walked across the screen. Participants given an easy counting task usually noticed the costumed walker, while those given a harder task, counting two kinds of passes separately, usually did not. These results suggest that _____

Which choice most logically completes the text?

A people who notice unexpected events are generally better at counting than people who do not.

B inattentional blindness occurs only when people watch videos rather than real scenes.

C a bright costume is less noticeable than a plain one when a task is demanding.

D how likely people are to miss an unexpected event may depend on how much of their attention the main task requires.

---

15 RW-T2M2-15

> Quilting guilds bring together people who sew quilts by hand or by machine. Every year since 1998, the fictional Maple Ridge Quilting Guild _____ an exhibition of its members' work in the town library, and the money raised by raffling one quilt goes to a local food bank.

Which choice completes the text so that it conforms to the conventions of Standard English?

A will hold **B** had held **C** has held **D** is holding

16 RW-T2M2-16

> A skin cell and a nerve cell seem to have little in common, yet they share a surprising feature. Nearly every cell in the human body carries the same set of _____ each type of cell, however, only some of those genes are switched on. Which genes are active determines which proteins a cell makes, and those proteins shape what the cell looks like and what it does.

Which choice completes the text so that it conforms to the conventions of Standard English?

A genes, in **B** genes in **C** genes? In **D** genes. In

17 RW-T2M2-17

> In the early twentieth century, some American cities tried a form of local government in which a small elected commission, rather than a single mayor, ran city departments. Each commissioner oversaw one area, such as public safety or finance. The commission as a whole also set the city's tax rate and held _____ meetings in public, where residents could watch decisions being made.

Which choice completes the text so that it conforms to the conventions of Standard English?

A its **B** their **C** it's **D** they're

18 RW-T2M2-18

> Some sculptures are designed to change shape on their own. The mobile, a hanging sculpture whose balanced parts drift and turn in moving _____ became a popular art form in the twentieth century. Because each part hangs from a rod that pivots freely, even a slight draft can rearrange the whole composition, so a viewer rarely sees the same arrangement twice.

Which choice completes the text so that it conforms to the conventions of Standard English?

A air **B** air; **C** air, **D** air—

19 RW-T2M2-19

> A hailstone begins as a tiny ice pellet high inside a thunderstorm. Its growth depends on the storm's strength: as powerful rising air currents _____ the pellet through regions of supercooled water—liquid droplets colder than 0°C—the droplets freeze onto its surface, and the pellet gains a new coat of ice. When the stone becomes too heavy for the updraft to support, it can fall toward the ground.

Which choice completes the text so that it conforms to the conventions of Standard English?

A carrying **B** carry **C** to carry **D** having carried

20 RW-T2M2-20

> When workers at the shipyards of a fictional port city walked off the job in 1919, the yard owners at first refused to meet with the workers' elected _____ by the third week of the strike, with orders for new ships piling up and penalties for late deliveries mounting, the owners agreed to talks. The settlement that followed shortened the workday by an hour but left wages unchanged, a compromise that satisfied neither side completely.

Which choice completes the text so that it conforms to the conventions of Standard English?

A representatives; however,
B representatives, however,
C representatives; however
D representatives however,

21 RW-T2M2-21

> A fictional weaving cooperative in the highlands shows one carpet in particular to every visitor. Woven entirely by hand over three years, _____ some of them made from plants that grow only at high elevations. Older members use the carpet to teach younger weavers how each of those dyes is prepared.

Which choice completes the text so that it conforms to the conventions of Standard English?

A the weaver colored the carpet with more than forty natural dyes,
B more than forty natural dyes color the carpet,
C the weaver's skill shows in the carpet's more than forty natural dyes,
D the carpet is colored with more than forty natural dyes,

22 RW-T2M2-22

> On February 20, 1943, after several weeks of earthquakes, a new volcano began erupting in a field in the Mexican state of Michoacán. By the end of its first year, the new cinder cone, named Paricutín, had risen about 336 meters (1,100 feet) above its base. Activity continued for several more years. _____ in 1952, the eruptions ended.

Which choice completes the text with the most logical transition?

A For instance,
B Similarly,
C Finally,
D Instead,

23 RW-T2M2-23

> Short-term memory can hold only a few separate items at once, but people can stretch that limit by grouping items into meaningful units, a strategy called chunking. _____ a ten-digit phone number is far easier to remember as three groups (an area code, a three-digit prefix and four final digits) than as ten unrelated digits. Expert chess players rely on the same strategy, recognizing whole arrangements of pieces at a glance.

Which choice completes the text with the most logical transition?

A In contrast,
B For instance,
C Nevertheless,
D Meanwhile,

24 RW-T2M2-24

> Bronze sculptors face practical problems when casting a large figure. They often cast it in sections, such as arms, torso and head, and weld the parts together afterward, hiding the seams under carefully finished surfaces. _____ makers of large ceramic sculptures often fire a figure in separate pieces, since one huge piece might crack in the kiln, and then join the pieces once they have cooled.

Which choice completes the text with the most logical transition?

A In contrast, **B** For example, **C** As a result, **D** Similarly,

25 RW-T2M2-25

> While researching a topic, a student has taken the following notes:
>
> - The wood frog is a small frog that lives in forests and woodlands.
> - Its range extends from the northeastern United States through most of Canada to central and southern Alaska.
> - It is tan to brown, with a dark mask across its face.
> - It often breeds in shallow temporary pools filled by snowmelt and rain.
> - Breeding dates vary with location and local weather.
> - The male's call sounds like a duck's quack.

The student wants to describe the kind of place where wood frogs breed. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Wood frogs breed at different times depending on location and local weather.

B Wood frogs often breed in shallow temporary pools filled by snowmelt and rain.

C The wood frog's range extends from the northeastern United States to central and southern Alaska.

D Male wood frogs make a call that sounds like a duck's quack.

26 RW-T2M2-26

> While researching a topic, a student has taken the following notes:
>
> - In 1910, textile workers in the fictional town of Elm Falls asked the school board to open a free night school.
> - Most of the workers had left school by age 14 to work in the mills.
> - The night school offered classes in English, arithmetic and civics after the mills closed at 6 p.m.
> - More than 300 workers enrolled in its first year.
> - Several of its graduates later became the first mill workers ever elected to the Elm Falls town council.

The student wants to emphasize the night school's significance for civic life in Elm Falls. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A The Elm Falls night school shaped the town's civic life: several of its graduates became the first mill workers ever elected to the town council.

B More than 300 workers enrolled in the Elm Falls night school during its first year.

C Classes in English, arithmetic and civics were offered at the night school after the mills closed at 6 p.m.

D Most Elm Falls textile workers had left school by age 14 to work in the mills.

---

27 RW-T2M2-27

> While researching a topic, a student has taken the following notes:
>
> - Lena Ortiz, a fictional textile artist, weaves wall hangings from strips of old denim.
> - Ortiz collects worn-out jeans from thrift stores near her home.
> - Tomas Reyes, a fictional sculptor, makes large animal figures from knotted fishing rope.
> - Reyes gathers rope that has washed up on beaches near his studio.
> - Ortiz's hangings are flat and meant for walls; Reyes's figures stand on the floor.

The student wants to emphasize a similarity between the materials the two artists use. Which choice most effectively uses relevant information from the notes to accomplish this goal?

A Ortiz's hangings are flat and meant for walls, while Reyes's animal figures stand on the floor.

B Reyes makes large animal figures from knotted fishing rope that he gathers on beaches near his studio.

C Ortiz and Reyes both work with discarded materials: Ortiz with worn-out jeans from thrift stores, and Reyes with rope that has washed up on beaches.

D Ortiz weaves strips of old denim into wall hangings that are meant to be displayed flat.

STOP. If you finish before time is called, check your work in this module only.

Math Reference Sheet

Use these facts during both Math modules. Your test-day reference sheet may present them differently.

Topic	Formula or fact
Circle	$A = \pi r^2$ $C = 2\pi r$
Rectangle	$A = \ell w$
Triangle	$A = (1/2)bh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Special right triangles	30°-60°-90°: sides x, $x\sqrt{3}$, 2x 45°-45°-90°: sides s, s, $s\sqrt{2}$
Rectangular prism	$V = \ell wh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = (4/3)\pi r^3$
Cone	$V = (1/3)\pi r^2 h$
Pyramid	$V = (1/3)\ell wh$
Angles and arcs	A circle has 360° of arc, or 2π radians. The angles of a triangle sum to 180°.

Practice Test 2: Math, Module 1

22 questions. 35 minutes. A calculator is allowed on every question. Use the reference sheet before Module 1.

1 M-T2M1-01

$6(x - 2) = 2x + 8$

What value of x is the solution to the given equation?

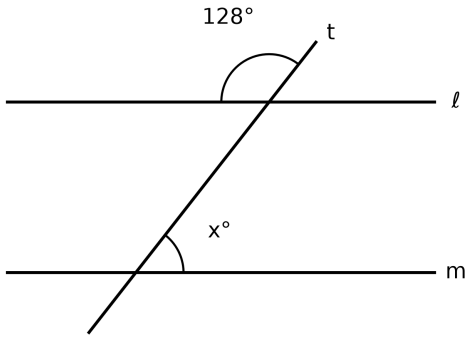
A −1 **B** 5/2 **C** 5 **D** 20

2 M-T2M1-02

A concert venue has 2,400 seats. For one show, 85% of the seats were filled. How many seats were empty?

A 15 **B** 360 **C** 2,040 **D** 2,760

3 M-T2M1-03



In the figure, line ℓ is parallel to line m, and both lines are cut by transversal t. What is the value of x?

A 38 **B** 52 **C** 128 **D** 232

4 M-T2M1-04

x	f(x)
0	−3
2	1
4	5
6	9

The table shows four values of x and their corresponding values of f(x), where f is a linear function. Which equation defines f?

A $f(x) = 2x - 3$ **B** $f(x) = 2x + 1$ **C** $f(x) = 4x - 3$ **D** $f(x) = (1/2)x - 3$

5 M-T2M1-05

$5x + 2 = 9 - 3x$

What is the solution to the given equation?

Student-produced response. Write your answer on the answer sheet or below.

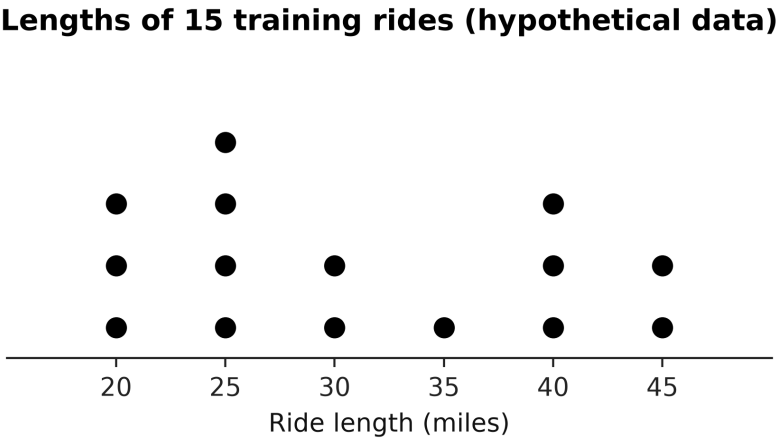
Answer: _____

6 M-T2M1-06

Which expression is equivalent to $(3x - 4)(x + 5)$?

A $3x^2 - 20$ **B** $3x^2 + 19x - 20$ **C** $3x^2 - 11x - 20$ **D** $3x^2 + 11x - 20$

7 M-T2M1-07



A cyclist recorded the length of each of 15 training rides. The dot plot shows the results. What is the median length, in miles, of these rides?

A 8 **B** 25 **C** 30 **D** 32.5

8 M-T2M1-08

In the xy-plane, line k passes through the points $(-2, 7)$ and $(4, -5)$. Which equation defines line k?

A $y = -2x + 3$ **B** $y = -2x + 11$ **C** $y = 2x + 11$ **D** $y = -(1/2)x + 6$

9 M-T2M1-09

A printing press produces 840 brochures every 6 minutes. At this rate, how many hours will it take the press to produce 21,000 brochures?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

10 M-T2M1-10

$4^{x-1} = 8^{x-3}$

What value of x satisfies the given equation?

A -11 **B** 2 **C** 5 **D** 7

11 M-T2M1-11

During a training ride on a fixed route, the distance d(t), in kilometers, that a cyclist still has to ride t minutes after starting is given by $d(t) = 54 - 0.4t$. Which statement is the best interpretation of the t-intercept of the graph of $y = d(t)$?

A The route is 54 kilometers long.

B The cyclist rides 0.4 kilometer each minute.

C The cyclist has 135 kilometers left to ride at the start.

D The cyclist finishes the route 135 minutes after starting.

---

12 M-T2M1-12

$y < 3x - 2$

$x + 2y \geqslant 7$

Which point (x, y) is a solution to the given system of inequalities?

A (4, 1) **B** (1, 5) **C** (3, 4) **D** (2, 4)

---

13 M-T2M1-13

$y = 3x - 7$

$5x - 2y = 9$

The solution to the given system of equations is (x, y). What is the value of $x + y$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

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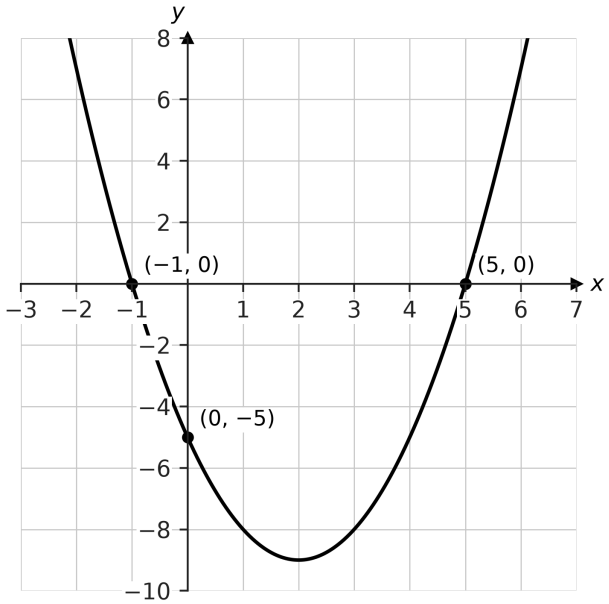
14 M-T2M1-14

An aquarium displays jellyfish in a tank shaped like a sphere with an interior diameter of 60 centimeters. What is the interior volume of the tank, in cubic centimeters?

A $1{,}200\pi$ **B** $3{,}600\pi$ **C** $27{,}000\pi$ **D** $36{,}000\pi$

---

15 M-T2M1-15



The quadratic function f is graphed in the xy-plane, as shown. Which equation could define f?

A $f(x) = (x + 1)(x - 5)$ **B** $f(x) = (x - 1)(x + 5)$ **C** $f(x) = -(x + 1)(x - 5)$ **D** $f(x) = 5(x + 1)(x - 5)$

16 M-T2M1-16

Survey of concert attendees (hypothetical data)

Seating section	Bought merchandise	Did not buy merchandise	Total
Floor	96	64	160
Lower balcony	54	81	135
Upper balcony	30	75	105
Total	180	220	400

In a hypothetical survey, a concert venue asked 400 attendees at one show where they sat and whether they bought merchandise. The table summarizes the results. If one of the surveyed attendees who did not sit on the floor is selected at random, what is the probability that this attendee bought merchandise?

A 21/100 **B** 7/20 **C** 2/5 **D** 7/15

17 M-T2M1-17

$(ax + 3)(2x - 5) = 8x^2 + bx - 15$

In the given equation, a and b are constants, and the equation is true for all values of x. What is the value of b?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

18 M-T2M1-18

$x/(x - 5) - 4/(x + 1) = 30/((x - 5)(x + 1))$

Which of the following gives all solutions to the given equation?

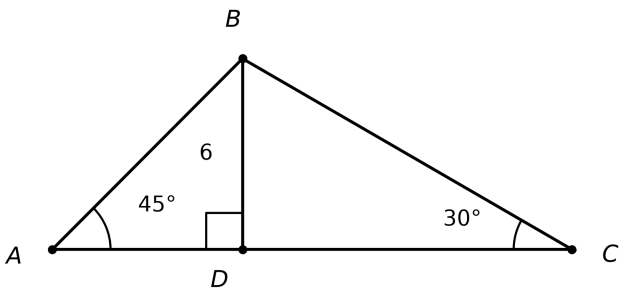
A −2 only **B** −2 and 5 **C** −5 and 2 **D** There is no solution.

19 M-T2M1-19

A concert venue sets up its floor in sections. Each standing section holds 120 people, and each seated section holds 75 people. For one show, the venue used 14 sections, all filled to capacity, for a total of 1,410 people. How many more people were in standing sections than in seated sections?

A 2 **B** 450 **C** 510 **D** 960

20 M-T2M1-20



In the figure, point D lies on side AC of triangle ABC, and segment BD is perpendicular to AC. The measure of angle A is 45°, the measure of angle C is 30°, and BD = 6. What is the length of AC?

A $6 + 2\sqrt{3}$ **B** $6 + 6\sqrt{3}$ **C** 18 **D** $6\sqrt{2} + 6\sqrt{3}$

21 M-T2M1-21

$f(x) = a(x - 2)(x + 6)$

The function f is defined by the given equation, where a is a constant. If $f(0) = 36$, what is the maximum value of f?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

22 M-T2M1-22

For the function f, $f(2x) = 4x^2 - 6x$ for all values of x. Which equation defines f?

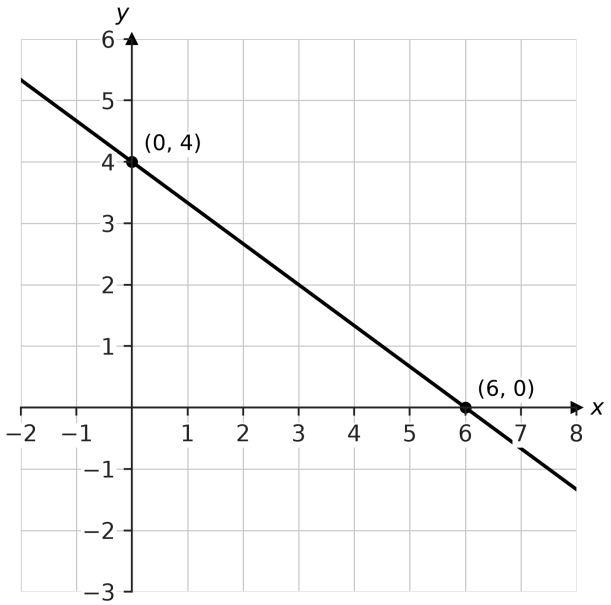
A $f(x) = 16x^2 - 12x$ **B** $f(x) = 4x^2 - 6x$ **C** $f(x) = 2x^2 - 3x$ **D** $f(x) = x^2 - 3x$

STOP. If you finish before time is called, check your work in this module only.

Practice Test 2: Math, Module 2

22 questions. 35 minutes. A calculator is allowed on every question. Use the reference sheet before Module 1.

1 M-T2M2-01



The graph of the linear function y = f(x) is shown. Which equation defines f?

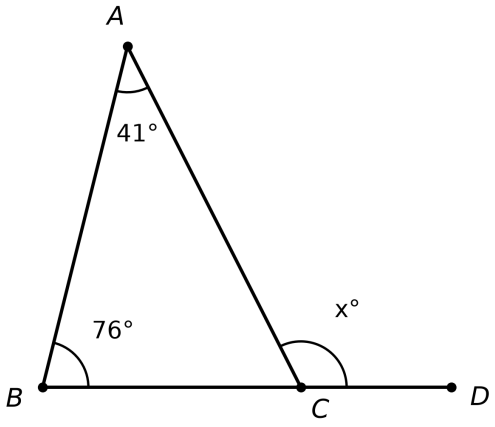
A f(x) = −(2/3)x + 4 **B** f(x) = −(3/2)x + 4 **C** f(x) = (2/3)x + 4 **D** f(x) = −(2/3)x + 6

2 M-T2M2-02

Which expression is equivalent to $(6x^2 + 9x)/(3x)$, for $x > 0$?

A $2x^2 + 3x$ **B** $6x + 9$ **C** $2x + 9$ **D** $2x + 3$

3 M-T2M2-03



In the figure, point D lies on the extension of side BC of triangle ABC. What is the value of x?

A 35 **B** 63 **C** 117 **D** 139

4 M-T2M2-04

A pump fills an aquarium tank at a constant rate of 18 gallons per minute. How many hours will it take the pump to fill an empty tank that holds 6,480 gallons?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

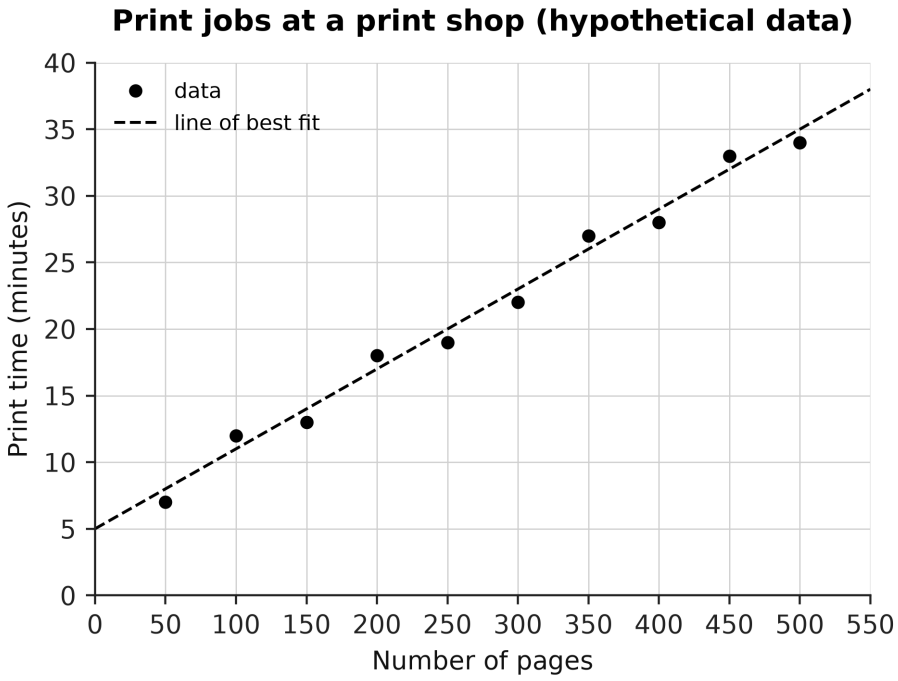
5 M-T2M2-05

$5(x - 2) + kx = 3x - 10$

In the given equation, k is a constant. The equation has infinitely many solutions. What is the value of k?

A −8 **B** −5 **C** −2 **D** 2

6 M-T2M2-06



The scatterplot shows the number of pages, x, and the print time, y, in minutes, for 10 jobs at a print shop. A line of best fit is also shown; it passes through the points (0, 5) and (500, 35). Which statement is the best interpretation of the slope of the line of best fit in this context?

A Each additional page increases the predicted print time by about 0.06 minute.

B Each additional minute of printing increases the predicted number of pages by about 0.06.

C A job with 0 pages has a predicted print time of about 0.06 minute.

D Each additional page increases the predicted print time by about 5 minutes.

7 M-T2M2-07

$y = x^2 - 3x - 4$

$y = x + 8$

Which ordered pair (x, y) is a solution to the given system of equations?

A (−2, −6) **B** (−2, 6) **C** (2, 10) **D** (6, −2)

8 M-T2M2-08

For the linear function f, f(1) = 7 and f(5) = 27. What is the value of f(−4)?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

9 M-T2M2-09

A print shop's large-format printer can run for at most 8 hours on a given day. Each poster takes 4 minutes to print, and each banner takes 15 minutes to print. Which inequality represents the possible numbers of posters, p, and banners, b, that the printer can produce in one day?

A $4p + 15b \geqslant 480$ **B** $4p + 15b \leqslant 8$ **C** $4p + 15b \leqslant 480$ **D** $15p + 4b \leqslant 480$

10 M-T2M2-10

An aquarium uses a feeding funnel shaped like a right circular cone. The base of the cone has a diameter of 18 centimeters, and the height of the cone is 20 centimeters. What is the volume of the funnel, in cubic centimeters?

A 60π **B** 540π **C** $1{,}620\pi$ **D** $2{,}160\pi$

11 M-T2M2-11

$6x - 9y = 15$

$kx - 3y = 5$

In the given system of equations, k is a constant. The system has infinitely many solutions. What is the value of k?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

12 M-T2M2-12

Which expression is equivalent to $3/(x - 2) - 2/(x + 1)$, for $x > 2$?

A $1/(2x - 1)$ **B** $(x - 1)/((x - 2)(x + 1))$ **C** $(x + 5)/((x - 2)(x + 1))$ **D** $(x + 7)/((x - 2)(x + 1))$

13 M-T2M2-13

In a hypothetical study, a concert venue selected 400 people at random from the 12,000 people on its email list. The venue randomly assigned 200 of these people to receive a text-message reminder about an upcoming show, and the other 200 received no reminder. A much greater proportion of the reminded group attended the show. A statistical analysis indicated that a difference this large was unlikely if the reminder had no effect. Which conclusion is best supported by this study?

A Sending a text-message reminder likely causes higher attendance among people on any concert venue's email list.

B No cause-and-effect conclusion is possible, because the study was observational.

C Sending a text-message reminder likely causes higher attendance among people on this venue's email list.

D The reminder affected attendance only for the 200 people who received it, so the results cannot be applied to anyone else on the list.

14 M-T2M2-14

The function $N(t) = 1{,}600(1.69)^{t/2}$ models the number of subscribers to a concert venue's newsletter t years after the venue opened. According to the model, which statement best describes how the number of subscribers changes?

A It increases by 30% each year.
B It increases by 34.5% each year.
C It increases by 69% each year.
D It increases by 169% every 2 years.

15 M-T2M2-15

$y = x^2 + 4x + c$
$y = -2x + 1$

In the given system of equations, c is a constant. The system has exactly one distinct real solution. What is the value of c?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

16 M-T2M2-16

A cyclist plans a 90-mile ride. She will ride x hours on flat roads at an average speed of 18 miles per hour and y hours on hilly roads at an average speed of 10 miles per hour, so $18x + 10y = 90$. If she spends 30 more minutes on flat roads and the total distance is still 90 miles, how does her time on hilly roads change?

A It decreases by about 17 minutes.
B It decreases by 54 minutes.
C It decreases by 108 minutes.
D It increases by 54 minutes.

17 M-T2M2-17

In right triangle ABC, angle C is the right angle. Also, $\sin A = (2k + 2)/37$ and $\cos B = (k + 7)/37$, where k is a constant. What is the value of $\tan B$?

A 12/37 **B** 12/35 **C** 35/37 **D** 35/12

18 M-T2M2-18

The function f is defined by $f(x) = ab^x$, where a and b are positive constants. If $f(2) = 50$ and $f(5) = 400$, what is the value of $f(0)$?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

19 M-T2M2-19

$5x - 2y = 6$
$ax - 8y = c$

In the given system of equations, a and c are constants. The system has infinitely many solutions. What is the value of $a + c$?

A −44 **B** 11 **C** 26 **D** 44

20 M-T2M2-20

$|3x - 6| = k$

In the given equation, k is a positive constant. The equation has two solutions, and the positive difference between the solutions is 10. What is the smaller solution?

A −3 **B** −4/3 **C** 7 **D** 15

21 M-T2M2-21

$x^2 + y^2 + 8x - 12y + c = 0$

In the xy-plane, the graph of the given equation is a circle with radius 9, where c is a constant. What is the value of c?

Student-produced response. Write your answer on the answer sheet or below.

Answer: _____

22 M-T2M2-22

The function f is defined by $f(x) = 2x^2 + bx + c$, where b and c are constants. If $f(0) = -6$ and $f(-1) = f(4)$, what is the value of $f(1)$?

A −16 **B** −10 **C** −6 **D** 2

STOP. If you finish before time is called, check your work in this module only.

CHAPTER 32

Practice Test 2: Answers, Scoring and Review

This chapter follows the same steps as Chapter 30: check your answers, calculate percent correct, read your domain results and review every miss. It then adds a side-by-side comparison of your two practice tests to help you decide how to spend your final weeks. The answer key, the score tally and the explanations for all 98 questions follow this chapter's text.

How to check your answers

The answer key that follows gives the correct answer for each question, grouped by module, with the skill tested and the chapter to review.

1. Score one module at a time. Mark every question you answered correctly, every miss and every blank.
2. Circle each question you marked "g." A correct guess still counts toward your score, but review it as a miss.
3. A student-produced response is correct if it gives the keyed value in an allowed form. The key lists examples, not every equivalent entry: 3/4, .75 and 0.75 name the same value. Apply Chapter 15's special rules for long or repeating decimals. Count it as wrong if the form breaks the entry rules (a mixed number such as 3 1/2, for example), because it would not be accepted on test day.
4. Fill in the Practice Test 2 Score Tally after the answer key, writing your number correct in each domain row and each section's total row.

Score your test

Enter the number correct in each module in Table A. Add the two modules to get each section total. All 98 questions count in this book's review; it does not include hidden unscored pretest items. A correct guess counts in your total, but still belongs in your review list.

Table A: Correct-count worksheet

Section	Module 1 correct	Module 2 correct	Section total	Percent correct
Reading and Writing	___ of 27	___ of 27	___ of 54	___%
Math	___ of 22	___ of 22	___ of 44	___%
Whole test	—	—	___ of 98	___%

NOTE These fixed practice tests have not been statistically calibrated to the SAT. Their correct counts cannot support a defensible conversion to the 200–800 section scales or the 400–1600 total scale. Percent correct describes this set of questions; it is not a percentile rank, a college-readiness measure or a prediction of your SAT score. For practice scores on the SAT scale, use the College Board's full-length tests in Bluebook.

Calculate percent correct

Divide the number correct by the number of questions, then multiply by 100. Round to the nearest whole percent for your worksheet. Keep the original counts beside the percentage so that you can see the sample size.

Table B: Calculation examples

Result	Calculation	Percent correct
41 Reading and Writing answers correct	41 ÷ 54 × 100	about 76%
30 Math answers correct	30 ÷ 44 × 100	about 68%
71 answers correct across both sections	71 ÷ 98 × 100	about 72%

The whole-test percentage uses all 98 questions. Do not add the section percentages or average them: the sections contain different numbers of questions. Use section and domain results to choose what to study rather than relying only on the whole-test total.

Table C: Timing and confidence check

Measure	Reading and Writing	Math
Questions still blank when time ended	___	___
Answers guessed mainly because time was short	___	___
Correct answers you could not explain confidently	___	___

These rows describe different problems and can overlap, so do not add them to form a score. Review rushed and uncertain answers even when they were correct. If you finish extra questions after time expires, record that work separately from your timed result.

Read your domain results

Copy your number correct for each domain from the score tally into Table D, then find the percent by dividing by the number of questions and multiplying by 100. For example, 10 correct out of 14 in Standard English Conventions is about 71%.

Table D: Domain review guide

Domain	Questions on this test	Your number correct	Percent correct	Chapters to review
Craft and Structure	15	___	___	5, 6, 7
Information and Ideas	14	___	___	8, 9, 10
Standard English Conventions	14	___	___	11, 12
Expression of Ideas	11	___	___	13, 14
Algebra	15	___	___	16, 17, 18, 19
Advanced Math	15	___	___	20, 21, 22

Domain	Questions on this test	Your number correct	Percent correct	Chapters to review
Problem-Solving and Data Analysis	7	___	___	23, 24, 25
Geometry and Trigonometry	7	___	___	26, 27, 28

Table E: Suggested review priorities

These bands are study-planning suggestions, not validated mastery levels. A percentage alone cannot tell you why you missed questions. Inspect the actual errors before choosing an action.

Percent correct in a domain	Starting review action
Below 50%	Begin with the lessons and worked examples for the skills you missed; then try the Practice Sets.
50–74%	Read each missed answer's explanation, identify whether the problem was knowledge, reasoning or pacing, and schedule redos.
75–89%	Target the specific missed skills and try harder or timed mixed questions where appropriate.
90% and above	Maintain with short review, check every uncertain answer and confirm the skill on unfamiliar questions.

Each of the two smallest domains has only 7 questions, so a single miss shifts its percent by about 14 points. Before you decide a whole domain is weak, check the Skill column of the answer key: repeated misses on one skill point to that chapter, while misses across skills need separate diagnosis. They may reflect content gaps, reasoning errors or pacing.

Review every miss

Read the explanation for every question you missed, left blank or guessed, and for any you answered correctly without being sure. Then classify each miss with Table F.

Table F: Why did I miss it?

Miss type	How to recognize it	Fix
Content gap	I did not know the rule or method.	Relearn it in the chapter named in the explanation.
Trap	I chose a tempting wrong answer.	Name the trap type from the explanation and add it to your error log.
Misread	I answered a different question than the one asked.	Underline the task word before solving.
Careless	I knew it but slipped.	Add a 5-second check step to your method.
Time	I rushed or guessed at the end.	Practice the two-pass method from Chapter 3.

Use the same routine as after Test 1:

1. Read the full explanation, including why your choice was wrong.

2. Assign one miss type and tally it.
3. Log the miss in the error log (Appendix E) within 24 hours, with the fix in your own words.
4. Redo the question without help after 3 days and again after 10 days.
5. Compare your tallies with Test 1. A miss type that shrank may show improvement; confirm it on new questions. One that grew deserves attention now.

Compare Test 1 and Test 2

Fill in Table G with your results from both tests. For "Weakest domain," write the domain with the lowest percent correct. Count blanks and time-driven guesses separately using Table C. A guessed answer is not an unanswered question.

Table G: Test 1 versus Test 2

Measure	Test 1	Test 2	Change
Reading and Writing correct (of 54)	___	___	___
Math correct (of 44)	___	___	___
Weakest domain	___	___	___
Questions still blank when time ended	___	___	___
Answers guessed mainly because time was short	___	___	___
Most common miss type	___	___	___

Read the table as a whole rather than one row at a time.

- **Your number correct rose and your weakest domain changed.** That is encouraging evidence of progress. Give your new weakest domain the most time, and keep the old one warm with short timed sets.
- **Your number correct rose, but the weakest domain stayed the same.** Inspect which skills in that domain still need work. Go back to its worked examples and Practice Sets rather than only rereading the lessons.
- **Your number correct stayed about the same.** These two tests have different questions and have not been statistically equated, so a small change does not by itself establish a change in ability. Look at the miss-type row: if Time or Careless misses dominate, practice pacing and checking rather than adding new content.
- **Fewer questions were left unanswered.** You completed more questions, which may reflect better pacing. Check whether accuracy held up and whether time-driven guesses also fell.

Keep Bluebook scores separate

Use a separate log for scores reported by the College Board's full-length Bluebook practice tests. Record whether it was your first attempt, since seeing questions before can change the result. These are practice scores, not guarantees of your test-day score.

Date and practice test	First attempt?	Reading and Writing score	Math score	Total score
—	—	—	—	—
—	—	—	—	—

Next steps

Your two practice tests and your error log now show where your remaining points are. Take at least one of the free full-length practice tests in the College Board's Bluebook app, which are adaptive and report practice scores on the SAT scale, and review it with the same routine you used here. Then turn to Chapter 33, "The Final Two Weeks and Beyond," for a day-by-day plan for the last stretch, test-day logistics and what to do after your scores arrive.

Practice Test 2 Answer Key

Reading and Writing, Module 1

#	Answer	Skill	Review
1	B	Words in Context	Ch. 5, p. 69
2	D	Words in Context	Ch. 5, p. 69
3	A	Words in Context	Ch. 5, p. 69
4	C	Words in Context	Ch. 5, p. 69
5	A	Words in Context	Ch. 5, p. 69
6	B	Text Structure and Purpose	Ch. 6, p. 82
7	D	Text Structure and Purpose	Ch. 6, p. 82
8	C	Cross-Text Connections	Ch. 7, p. 98
9	D	Central Ideas and Details	Ch. 8, p. 112
10	A	Central Ideas and Details	Ch. 8, p. 112
11	C	Command of Evidence: Quantitative	Ch. 9, p. 128
12	D	Command of Evidence: Textual	Ch. 9, p. 128
13	D	Command of Evidence: Quantitative	Ch. 9, p. 128
14	A	Inferences	Ch. 10, p. 150
15	B	Inferences	Ch. 10, p. 150
16	C	Boundaries	Ch. 11, p. 164
17	A	Form, Structure, and Sense	Ch. 12, p. 180
18	B	Boundaries	Ch. 11, p. 164
19	D	Form, Structure, and Sense	Ch. 12, p. 180
20	C	Boundaries	Ch. 11, p. 164
21	A	Form, Structure, and Sense	Ch. 12, p. 180
22	B	Boundaries	Ch. 11, p. 164
23	D	Transitions	Ch. 13, p. 195
24	C	Transitions	Ch. 13, p. 195
25	B	Rhetorical Synthesis	Ch. 14, p. 208
26	C	Rhetorical Synthesis	Ch. 14, p. 208
27	A	Rhetorical Synthesis	Ch. 14, p. 208

Reading and Writing, Module 2

#	Answer	Skill	Review
1	C	Words in Context	Ch. 5, p. 69
2	A	Words in Context	Ch. 5, p. 69
3	D	Words in Context	Ch. 5, p. 69
4	B	Words in Context	Ch. 5, p. 69
5	D	Text Structure and Purpose	Ch. 6, p. 82
6	A	Text Structure and Purpose	Ch. 6, p. 82
7	C	Cross-Text Connections	Ch. 7, p. 98
8	B	Central Ideas and Details	Ch. 8, p. 112
9	A	Command of Evidence: Quantitative	Ch. 9, p. 128
10	D	Command of Evidence: Textual	Ch. 9, p. 128
11	C	Command of Evidence: Quantitative	Ch. 9, p. 128
12	A	Command of Evidence: Textual	Ch. 9, p. 128
13	B	Inferences	Ch. 10, p. 150
14	D	Inferences	Ch. 10, p. 150
15	C	Form, Structure, and Sense	Ch. 12, p. 180
16	D	Boundaries	Ch. 11, p. 164
17	A	Form, Structure, and Sense	Ch. 12, p. 180
18	C	Boundaries	Ch. 11, p. 164
19	B	Form, Structure, and Sense	Ch. 12, p. 180
20	A	Boundaries	Ch. 11, p. 164
21	D	Form, Structure, and Sense	Ch. 12, p. 180
22	C	Transitions	Ch. 13, p. 195
23	B	Transitions	Ch. 13, p. 195
24	D	Transitions	Ch. 13, p. 195
25	B	Rhetorical Synthesis	Ch. 14, p. 208
26	A	Rhetorical Synthesis	Ch. 14, p. 208
27	C	Rhetorical Synthesis	Ch. 14, p. 208

Math, Module 1

#	Answer	Skill	Review
1	C	Linear equations in one variable	Ch. 16, p. 239
2	B	Percentages	Ch. 23, p. 328
3	B	Lines, angles, and triangles	Ch. 26, p. 372
4	A	Linear functions	Ch. 17, p. 251
5	7/8 or .875 or 0.875	Linear equations in one variable	Ch. 16, p. 239
6	D	Equivalent expressions	Ch. 20, p. 289
7	C	One-variable data: distributions and measures of center and spread	Ch. 24, p. 340
8	A	Linear equations in two variables	Ch. 17, p. 251
9	2.5 or 5/2	Ratios, rates, proportional relationships, and units	Ch. 23, p. 328
10	D	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
11	D	Linear functions	Ch. 17, p. 251
12	C	Linear inequalities in one or two variables	Ch. 19, p. 278
13	13	Systems of two linear equations in two variables	Ch. 18, p. 264
14	D	Area and volume	Ch. 26, p. 372
15	A	Nonlinear functions	Ch. 22, p. 313
16	B	Probability and conditional probability	Ch. 25, p. 357
17	-14	Equivalent expressions	Ch. 20, p. 289
18	A	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
19	C	Systems of two linear equations in two variables	Ch. 18, p. 264
20	B	Right triangles and trigonometry	Ch. 27, p. 389
21	48	Nonlinear functions	Ch. 22, p. 313
22	D	Nonlinear functions	Ch. 22, p. 313

Math, Module 2

#	Answer	Skill	Review
1	A	Linear functions	Ch. 17, p. 251
2	D	Equivalent expressions	Ch. 20, p. 289
3	C	Lines, angles, and triangles	Ch. 26, p. 372
4	6	Ratios, rates, proportional relationships, and units	Ch. 23, p. 328
5	C	Linear equations in one variable	Ch. 16, p. 239
6	A	Two-variable data: models and scatterplots	Ch. 24, p. 340
7	B	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
8	-18	Linear functions	Ch. 17, p. 251
9	C	Linear inequalities in one or two variables	Ch. 19, p. 278
10	B	Area and volume	Ch. 26, p. 372
11	2	Systems of two linear equations in two variables	Ch. 18, p. 264
12	D	Equivalent expressions	Ch. 20, p. 289
13	C	Evaluating statistical claims: observational studies and experiments	Ch. 25, p. 357
14	A	Nonlinear functions	Ch. 22, p. 313
15	10	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
16	B	Linear equations in two variables	Ch. 17, p. 251
17	D	Right triangles and trigonometry	Ch. 27, p. 389
18	12.5 or 25/2	Nonlinear functions	Ch. 22, p. 313
19	D	Systems of two linear equations in two variables	Ch. 18, p. 264
20	A	Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300
21	-29	Circles	Ch. 28, p. 405
22	B	Nonlinear functions	Ch. 22, p. 313

Practice Test 2 Score Tally

Reading and Writing

Domain	Questions	Number correct
Craft and Structure	15	
Information and Ideas	14	
Standard English Conventions	14	
Expression of Ideas	11	
Total	54	

Math

Domain	Questions	Number correct
Algebra	15	
Problem-Solving and Data Analysis	7	
Geometry and Trigonometry	7	
Advanced Math	15	
Total	44	

Explanations: Reading and Writing, Module 1

1. Correct answer: B RW-T2M1-01

Words in Context · Craft and Structure · Easier

B *uneasily*

The blank describes how the dog looks around. The text says she is "whining," lifting "chilled" paws and trying to work out how "she was lost." A lost, anxious dog would look from side to side uneasily, meaning in a worried, nervous way. Predicting a word like "nervously" before reading the choices leads straight to it.

A Nothing in the text suggests pride; a whining dog that cannot understand how she got lost is worried, not pleased with herself.

C The dog is "running up and down" and searching for a way home, which is active and anxious, not slow or idle.

D Playfulness clashes with the whining and confusion about being lost; the dog is distressed, not having fun.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: D RW-T2M1-02

Words in Context · Craft and Structure · Easier

D *gather*

The text says single cells "attach" and make a coating "that holds them together," and it later calls the result "this crowded layer." Cells that come together and stay together gather into a biofilm, so "gather" is the logical and precise choice.

A Opposite meaning: to scatter is to spread apart, but the cells are held together in a crowded layer.

B Nothing says the cells get smaller; the text describes many cells joining into a layer, not changing size.

C Dissolving would mean dispersing into a solution; the text instead describes intact cells gathering in a lasting, protective layer.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: A RW-T2M1-03

Words in Context · Craft and Structure · Medium

A *disclose*

Artists who keep nail holes and weathered paint want viewers to "read a board's history in its surface." For that to happen, the marks must reveal the wood's past. "Disclose," meaning to make known or reveal, fits: the visible traces let each piece disclose its earlier life as part of a building.

B Opposite meaning: concealing the wood's past matches sanding the marks away, the option these sculptors reject.

C The marks simply show the wood's past; nothing suggests they make that history seem larger or more dramatic than it was.

D Viewers read the history "before" noticing the new form, so the marks show the past immediately rather than delaying it.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: C RW-T2M1-04

Words in Context · Craft and Structure · Medium

C *replace*

The society "drew on that fund" when a member could not work because of injury or illness. Spending pooled money in that situation means paying the member, so the fund would replace, or make up for, part of the wages the worker lost. The purpose of the shared dues confirms this meaning.

A To forfeit is to give something up; the fund exists to help the injured member, not to cost that member more pay.

B Measuring the lost wages would not require drawing on the fund; spending the fund implies paying the member, not just counting the loss.

D Withholding wages would deny the injured member money, the opposite of what a shared benefit fund is for.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: A RW-T2M1-05

Words in Context · Craft and Structure · Harder

A *excite*

The trip "developed in her a thirst for information," and the contrast is with a library job that had previously failed to stir that interest. "Excite," meaning to arouse or awaken, precisely describes what the job had failed to do and the trip succeeded in doing.

B To document a thirst for information would mean to record it. The contrast concerns arousing curiosity, not making a record of it.

C The text contrasts the effects of the job and the trip on her curiosity; it does not suggest that her position concealed that curiosity.

D To interrupt an interest is to break it off. The passage instead contrasts the trip that stirred curiosity with the job that had failed to stir it.

Review: Chapter 5, Words in Context (page 69)

6. Correct answer: B RW-T2M1-06

Text Structure and Purpose · Craft and Structure · Easier

B *To convey a child's imagining of where the toy boats will travel*

The speaker watches "boats of mine" float away and asks, "Where will all come home?" The poem follows the river

"out past the mill" for "a hundred miles or more" and ends by imagining that "other little children / Shall bring my boats ashore." Its purpose is to convey a child's imagining of the boats' long journey.

A The poem never mentions danger; the boats simply float downstream, and the speaker pictures other children bringing them safely ashore.

C The mill, valley and hill show how far the river carries the boats; the poem does not explain any change in the river's course.

D The opening lines name colors to set the scene, but the poem moves on to the boats' journey instead of comparing colors.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: D RW-T2M1-07

Text Structure and Purpose · Craft and Structure · Medium

D *It identifies a possible consequence of the temperature effect that helps explain why the team shades nests.*

The sentence before the underlined portion explains that warmer nests produce mostly females. The underlined sentence draws out a consequence of that link: a warming beach could produce far fewer males. The final sentence then says the team "therefore" shades nests, so the underlined sentence supplies the reason for that action.

A The underlined sentence builds on the temperature explanation given earlier; it offers no competing account of how sex is determined.

B The text reports no results from shading; the underlined sentence predicts a possible future problem rather than presenting evidence of success.

C Nothing in the underlined sentence describes measuring sand; it predicts what long-term warming could do to hatchling ratios.

Review: Chapter 6, Text Structure and Purpose (page 82)

8. Correct answer: C RW-T2M1-08

Cross-Text Connections · Craft and Structure · Harder

C *By suggesting that rereading can still play a useful role before recall practice, especially with difficult material*

Text 1 concludes that rereading "wastes study time" and should be replaced "entirely." Text 2 accepts that recall practice "clearly strengthens memory" but argues that for difficult material a second careful reading builds the correct understanding that makes recall worthwhile. The author of Text 2 would therefore qualify the conclusion, not reject the finding.

A Too strong: Text 2 says recall practice "clearly strengthens memory," so its author accepts the finding and challenges only the conclusion about rereading.

B Neither text links the recall group's results to confidence; Text 2 discusses the accuracy of understanding, not students' feelings.

D This restates Text 1's conclusion, which Text 2 disputes by defending a second careful reading of difficult material.

Review: Chapter 7, Cross-Text Connections (page 98)

9. Correct answer: D RW-T2M1-09

Central Ideas and Details · Information and Ideas · Easier

D *The speaker sends paper boats down the stream in the hope of being known by someone far away.*

The speaker writes "my name" and "the name of the village where I live" on each boat and hopes "someone in some strange land will find them and know who I am." Every detail serves that hope, so the main idea is that the boats are the speaker's way of reaching a distant stranger.

A The poem never mentions building sturdier boats; the speaker simply floats paper boats and hopes someone finds them.

B The boats travel toward a strange land, but nothing suggests the speaker plans to go there.

C The flowers are loaded onto the boats; the poem does not describe decorating a garden.

Review: Chapter 8, Central Ideas and Details (page 112)

10. Correct answer: A RW-T2M1-10

Central Ideas and Details · Information and Ideas · Medium

A *An eruption's longer-term cooling effect comes mainly from sulfur gas that reaches high in the atmosphere rather than from the ash cloud.*

The text says ash has "little lasting effect" because it falls within days or weeks. It then explains that sulfur dioxide reaching the stratosphere reacts to form sunlight-reflecting droplets that stay aloft "for a year or more." These aerosols account for the longer-lasting cooling effect.

B The text mentions that ash falls quickly only to explain its short-lived climate effect; it says nothing about the danger of ash clouds.

C Too broad: the text says cooling happens only "if an eruption is powerful enough" to put the gas into the stratosphere.

D The text calls the ash cloud, not sulfur dioxide, "the most visible sign" of an eruption's power.

Review: Chapter 8, Central Ideas and Details (page 112)

11. Correct answer: C RW-T2M1-11

Command of Evidence: Quantitative · Information and Ideas · Easier

C *81 calls per minute, compared with 38 calls per minute at 8°C.*

The statement needs the value for 17°C plus a comparison that shows more calling in warmer water. The table lists 81 calls per minute at 17°C and 38 at 8°C, the highest and lowest rates. Choice C reports both values accurately and supports the student's finding.

A Reverses the data: the table shows 38 calls per minute at 8°C and 81 at 17°C.

B The table shows 52 calls per minute at 11°C, not 17°C, and this comparison would suggest fewer calls in warmer water.

D The table shows 67 calls per minute at 14°C, not 17°C, so this choice misreports the value the statement needs.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: D RW-T2M1-12

Command of Evidence: Textual · Information and Ideas · Medium

D *"'I must do it. My honor is at stake,' said Anne solemnly. 'I shall walk that ridgepole, Diana, or perish in the attempt.'"*

Choice D connects Anne's determination directly to her concern for her honor: she says "I must do it" because her "honor is at stake." Her declaration that she will walk the ridgepole "or perish in the attempt" also shows that she recognizes the danger but intends to proceed.

A Tossing her braids describes a gesture but does not explain why Anne decides to attempt the dangerous feat.

B This quotation expresses Diana's concern about the danger, not Anne's reason for deciding to proceed.

C This describes Josie's triumphant attitude toward Anne, not Anne's concern for her own honor or determination to walk the roof.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: D RW-T2M1-13

Command of Evidence: Quantitative · Information and Ideas · Harder

D *Turnout among voters aged 18–29 rose from about 13% in 2017 to about 31% in 2019, while turnout among voters aged 65 and older stayed close to 55%.*

The conclusion compares changes in turnout after the elections were combined in 2019. Choice D provides both age groups: turnout for ages 18–29 rose from about 13% to about 31% between 2017 and 2019, while turnout for ages 65 and older remained close to 55%. The younger group's increase was much larger.

A True, but comparing the two age groups within each year does not show how much either group's turnout changed after the election schedule changed.

B Both 2015 and 2017 precede the schedule change, so this small rise does not describe the pattern after elections were combined.

C True, but the older group's peak alone gives neither the size of its change nor the younger group's increase. A small peak is compatible with little change overall.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

14. Correct answer: A RW-T2M1-14

Inferences · Information and Ideas · Medium

A *some cells of the second strain had likely received resistance genes from the first strain.*

The text explains that plasmids can transfer resistance genes between neighboring cells, even across species. The second strain could not survive the test dose before growing beside the resistant strain, but some cells survived that same dose afterward. Receiving resistance genes from the neighboring strain is therefore a likely explanation for the change.

B The dose initially killed cells of the second strain, so it was not too weak to harm any bacteria.

C This contradicts the text, which says plasmids can pass to neighboring cells "even cells of a different species."

D Before the strains grew together, the second strain could not survive the test dose. That baseline contradicts the claim that it was already resistant.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: B RW-T2M1-15

Inferences · Information and Ideas · Harder

B *the quilt may date from around 1850, with the patch containing the later dye added during a repair.*

The later dye shows that one patch cannot date from 1850. But that patch is attached with unusual stitching, and every other patch uses only dyes available well before 1850. Together, these clues suggest the quilt itself could be from about 1850, while the odd patch was added later, for example during a repair.

A Too strong: one patch with a later dye does not date the whole quilt, especially since its stitching differs and the other patches fit 1850.

C The text says the plant-based dyes were "in use long before 1850" and appear in most patches; nothing says they had stopped being used.

D The text treats dye testing as informative: it shows the patch cannot be older than its dye, the key clue in the passage.

Review: Chapter 10, Inferences (page 150)

16. Correct answer: C RW-T2M1-16

Boundaries · Standard English Conventions · Easier

C *humidity, wind speed*

The sensors measure four items in a series: air temperature, humidity, wind speed and rainfall. Simple series items are separated by commas, matching the comma after "air temperature," so a comma belongs between "humidity" and "wind speed." A comma before the final "and" is optional in Standard English, so none is needed.

A Leaving out the comma between "humidity" and "wind speed" runs two separate list items together.

B A semicolon cannot separate simple list items when the rest of the list uses commas.

D A semicolon before "and rainfall" wrongly sets off the last item; simple items in a series are separated by commas, not semicolons.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: A RW-T2M1-17

Form, Structure, and Sense · Standard English Conventions · Easier

A *workers'*

The demands belong to the many workers from three mills, so the noun must be both plural and possessive. A plural noun that already ends in s forms its possessive by adding only an apostrophe after the s, which gives "workers'."

B "Worker's" is singular possessive, but the demands came from workers at three mills, not from one worker.

C "Workers" is plural but not possessive, so it cannot show that the demands belonged to the workers.

D "Workers's" is not a standard form; a plural noun ending in s takes only an apostrophe to show possession.

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: B RW-T2M1-18

Boundaries · Standard English Conventions · Medium

B *bronze—has*

The supplement describing the method opens with a dash after "method" and contains a comma of its own. A supplement that opens with a dash must close with a matching dash, so a dash belongs after "bronze," just before the main verb "has been practiced."

A A comma cannot close a supplement that opened with a dash; the marks around a supplement must match.

C A semicolon would cut the subject "method" off from its verb "has been practiced," and it cannot close a dash-opened supplement.

D With no closing mark, the sentence never signals where the interruption ends and the main clause resumes.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: D RW-T2M1-19

Form, Structure, and Sense · Standard English Conventions · Medium

D *contain*

The subject of the verb is the plural noun "tissues." The words "scales," "back" and "crocodile" appear in phrases that modify that subject. A plural subject needs a plural verb, and the present tense matches the rest of the text, so "contain" is correct.

A "Contains" is singular; it wrongly agrees with "crocodile," the nearest noun, instead of the plural subject "tissues."

B "Is containing" is singular, and the progressive form also does not fit a permanent feature such as bony plates.

C "Has contained" is singular, so it does not agree with the plural subject "tissues."

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: C RW-T2M1-20

Boundaries · Standard English Conventions · Medium

C *first, but*

Two independent clauses meet at the blank: "Cloth lifted out of an indigo vat looks yellowish green at first" and "within minutes it turns blue." Here, a comma before the coordinating conjunction "but" joins the clauses. No comma is needed immediately after "but" because the following words continue directly into the second clause.

A Leaving out the comma before "but" fails to mark the boundary between two independent clauses joined by a conjunction.

B The comma after "but" wrongly separates the conjunction from the clause it introduces.

D Like B, this puts a comma after "but," cutting the conjunction off from its clause; a comma before "but" is all these two clauses need.

Review: Chapter 11, Boundaries (page 164)

21. Correct answer: A RW-T2M1-21

Form, Structure, and Sense · Standard English Conventions · Harder

A *most participants recalled between five and nine words, often including the first and last items.*

The introductory phrase "Shown each list for only a few seconds" describes the people who saw the lists. A modifying phrase like this must be followed immediately by the noun it describes, so the clause must begin with "most participants." Only choice A places the participants right after the phrase.

B This places "words" right after the modifier, illogically saying that the words were shown each list.

C This places "the recall" after the modifier, suggesting that recall, an ability, was shown the lists rather than the participants.

D Again "words" follows the modifier, so the sentence says the words, not the participants, were shown each list.

Review: Chapter 12, Form, Structure, and Sense (page 180)

22. Correct answer: B RW-T2M1-22

Boundaries · Standard English Conventions · Harder

B *long are*

The subject of the sentence is the long phrase "The rules governing who may speak at such a meeting and for how long," and its verb is "are." No punctuation should separate a subject from its verb, however long the subject is, so "long are" is correct.

A A single comma between the subject and its verb "are" wrongly splits the core of the sentence.

C A semicolon joins independent clauses; here it would cut the subject off from its verb and leave two fragments.

D A single dash would interrupt the subject and verb with no matching dash and no supplement to set off.

Review: Chapter 11, Boundaries (page 164)

23. Correct answer: D RW-T2M1-23

Transitions · Expression of Ideas · Easier

D *However,*

The first sentence describes how ancient Greek statues look today: white. The next sentence reveals that many were once brightly painted. Because the second idea contrasts with the impression set up by the first, "However" is the logical transition.

A "Similarly" signals a likeness, but bright paint contrasts with the white appearance described before the blank.

B "For example" would introduce an instance of the previous claim, but the evidence of paint challenges that impression instead of illustrating it.

C "As a result" would mean the statues' whiteness caused the discovery of paint, which is not the relationship described.

Review: Chapter 13, Transitions (page 195)

24. Correct answer: C RW-T2M1-24

Transitions · Expression of Ideas · Harder

C *For this reason,*

The sentence before the blank stresses that making antibiotics is costly. The sentence after it describes how responding to competitors' chemical cues concentrates that costly activity where competition is present. This response follows logically from the cost, so "For this reason" is the best transition.

A "Nevertheless" would signal a contrast with the cost. Here, directing costly production toward competitors follows logically from that cost.

B "Likewise" signals a similar example, but the final sentence describes a result of the cost, not another costly activity.

D "In other words" introduces a restatement, but the final sentence adds a new behavior instead of rewording the point about cost.

Review: Chapter 13, Transitions (page 195)

25. Correct answer: B RW-T2M1-25

Rhetorical Synthesis · Expression of Ideas · Easier

B *Frances Perkins, a labor reformer who became the first woman appointed to a U.S. cabinet post, served as secretary of labor under President Franklin D. Roosevelt.*

To introduce someone to an unfamiliar audience, a sentence should name the person and explain who she was. Choice B gives Perkins's full name, identifies her as a labor reformer and the first woman appointed to a cabinet post, and states the office she held under Roosevelt.

A This never names Frances Perkins, so readers would not learn who the new secretary of labor was.

C This uses only her last name and lists achievements without explaining who she was, assuming the audience already knows her.

D This describes the Consumers' League's goals without ever mentioning Perkins, so it introduces an organization rather than the person.

Review: Chapter 14, Rhetorical Synthesis (page 208)

26. Correct answer: C RW-T2M1-26

Rhetorical Synthesis · Expression of Ideas · Medium

C *Woven fabric is made by interlacing two sets of threads at right angles, whereas knitted fabric is made by pulling loops of yarn through other loops.*

The goal has two parts: a difference, and a difference in construction. Choice C meets both. It uses "whereas" to set the two methods side by side: woven fabric interlaces two sets of threads at right angles, while knitted fabric pulls loops of yarn through other loops.

A This describes only the threads in woven fabric, so it presents no difference between the two kinds of fabric.

B This presents a difference in stretch, a property of the finished fabric, rather than a difference in how each fabric is made.

D This describes only how knitted fabric is made, without comparing it to woven fabric.

Review: Chapter 14, Rhetorical Synthesis (page 208)

27. Correct answer: A RW-T2M1-27

Rhetorical Synthesis · Expression of Ideas · Harder

A *By measuring how close a person could get before each lizard ran, the team found that lizards on Island B, which has almost no predatory birds, let people come much closer (0.9 meters on average) than lizards on Island A did (3.1 meters).*

The goal has two parts: the method and the main finding. Choice A gives both. It explains the method (measuring how close a person could get before each lizard ran) and the finding (lizards on predator-poor Island B allowed much closer approach, 0.9 meters versus 3.1 meters).

B This describes the method only; it never reports what the team found.

C This states the conclusion but leaves out the method, so readers cannot tell how boldness was measured.

D This gives background about the islands but neither the method nor the result of the study.

Review: Chapter 14, Rhetorical Synthesis (page 208)

Explanations: Reading and Writing, Module 2

1. Correct answer: C RW-T2M2-01

Words in Context · Craft and Structure · Easier

C *endure*

The passage describes how turtles in cold regions survive winter by finding shelter and becoming inactive until warmer weather returns. To endure the season means to live through its difficult conditions, which matches the purpose of that behavior.

A Burrowing and staying inactive is a way to live through winter, not a way to forecast when winter will arrive.

B Nothing suggests turtles enjoy winter; the text describes a strategy for surviving dangerous cold.

D Sheltering does not make winter shorter; it helps the turtle survive the cold season.

Review: Chapter 5, Words in Context (page 69)

2. Correct answer: A RW-T2M2-02

Words in Context · Craft and Structure · Medium

A *Neatly dressed*

"Smart" appears in a description of the elf's clothing: his frock coat, white laces, buckled shoes and garter bows. Paired with "trim," it describes a tidy, stylish appearance, so "neatly dressed" is the best meaning here. The common meaning of "smart" as "intelligent" does not fit a list of clothes.

B "Intelligent" is a common meaning of "smart," but this sentence describes the elf's clothes, not his mind.

C "Smart" can mean to sting, but nothing in the description involves pain; it lists details of the elf's outfit.

D Nothing in the sentence concerns speed; "trim and smart" describes how the elf is dressed, not how fast he moves.

Review: Chapter 5, Words in Context (page 69)

3. Correct answer: D RW-T2M2-03

Words in Context · Craft and Structure · Harder

D *spontaneous*

"Anything but" signals that many strikes were the opposite of the missing word. The next sentence lists weeks of preparation: gathering pledges, building a relief fund and choosing the timing. Something carefully prepared is the opposite of spontaneous, or unplanned, which also fits the contrast with "sudden walkouts."

A The contrast concerns advance planning, not whether a strike was expensive. Timing a shutdown to hurt employers does not support describing the strike as anything but costly.

B The text describes preparation, not results, so nothing supports saying that strikes were the opposite of effective.

C "Anything but organized" would mean disorganized, which contradicts the detailed preparation the text describes.

Review: Chapter 5, Words in Context (page 69)

4. Correct answer: B RW-T2M2-04

Words in Context · Craft and Structure · Harder

B *expert*

The sentence describes a skill Miss Brill believes she has developed: "listening as though she didn't listen" and "sitting in other people's lives" unnoticed. The structure "had become really quite ___ ... at" calls for a word naming skill, and her eagerness for "the conversation" shows she practices often. "Expert" fits both clues.

A Miss Brill is interested in others, but "quite curious at listening" does not work; the structure calls for a word describing skill, not interest.

C "Hesitant" contradicts the text: Miss Brill "always looked forward to the conversation" and is disappointed when the couple stays silent.

D "Weary" clashes with her eager anticipation; she is disappointed by the silence, not tired of listening.

Review: Chapter 5, Words in Context (page 69)

5. Correct answer: D RW-T2M2-05

Text Structure and Purpose · Craft and Structure · Medium

D *The speaker describes her nighttime surroundings and the stars above her and then expresses how she feels about witnessing them.*

The poem first sets the scene: the speaker is "alone in the night / On a dark hill" among pines. It then describes the stars, their colors, their "hearts of fire" and their stately motion. Only in the final stanza does the speaker give her response: she is "honored to be / Witness / Of so much majesty."

A The speaker is alone on a dark hill, but nothing suggests fear, and she never travels home; she stays and watches.

B The poem says the stars' fires are something "aeons / Cannot vex or tire," which suggests they endure rather than burn out.

C The speaker does mention colors, but she ends by stating that she is "honored," not by doubting whether she deserves the sight.

Review: Chapter 6, Text Structure and Purpose (page 82)

6. Correct answer: A RW-T2M2-06

Text Structure and Purpose · Craft and Structure · Harder

A *It provides background on how ice forms in clouds, presents evidence about one possible source of ice-forming particles and then identifies a question that remains unresolved.*

The text first explains that particles can help cloud droplets freeze. It then introduces ice-forming proteins on bacteria and a survey finding that many active particles were biological. Finally, "Even so" introduces an unresolved question about whether enough bacteria reach clouds to influence precipitation.

B Too strong: the text never says dust plays no role; it says only that many of the most active particles in the snow were biological.

C The survey's methods are barely described, and the text mentions no earlier surveys to compare it with.

D The text does not present two groups debating or choose a winner. It moves from background to evidence about biological particles and then to an open question.

Review: Chapter 6, Text Structure and Purpose (page 82)

7. Correct answer: C RW-T2M2-07

Cross-Text Connections · Craft and Structure · Harder

C *Corrosion that is eating away a bronze statue's surface should be halted.*

Text 1 presents a conservator who wants to stop damaging corrosion. Text 2 raises a concern about changing a familiar landmark's appearance but agrees that conservators "should of course stop any corrosion that is wearing away the metal." The shared position is that destructive corrosion should be halted.

A Text 1 supports restoring one monument's original finish, while Text 2 calls for public consultation before changing an appearance. Neither text establishes that all statues should always be restored.

B Both texts acknowledge that corrosion can damage the metal. Text 1 specifically distinguishes the monument's active corrosion from a stable patina.

D Too strong: Text 2 says conservators should consult the public, not that residents should make every decision, and Text 1 never mentions residents.

Review: Chapter 7, Cross-Text Connections (page 98)

8. Correct answer: B RW-T2M2-08

Central Ideas and Details · Information and Ideas · Medium

B *He shows little excitement because the hayloft is already familiar to him.*

Heidi is thrilled and urges her grandfather to "come up and see how lovely it is." He simply answers, "Oh, I know all about it." His calm reply shows that the hayloft holds no surprise for him because he already knows it well, so he does not share her excitement.

A The grandfather expresses no concern about safety; his only reply is that he already knows the hayloft.

C Nothing suggests annoyance or a wish for Heidi to sleep near him; his reply is calm and matter-of-fact.

D His reply, "Oh, I know all about it," shows familiarity with the hayloft rather than surprise about its view.

Review: Chapter 8, Central Ideas and Details (page 112)

9. Correct answer: A RW-T2M2-09

Command of Evidence: Quantitative · Information and Ideas · Easier

A *12 thunderstorm days, more than any other month in the table.*

The table lists 12 thunderstorm days for July, the highest value from April through September. Choice A reports this value accurately and explains why it supports the conclusion that thunderstorms at the station peaked in midsummer.

B Nine days is the June value, and it is not the highest in the table, since July averaged 12.

C Ten days is August's value, not July's, and August had more thunderstorm days than June did.

D Three days is April's value, the lowest in the table, which would not support a midsummer peak.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

10. Correct answer: D RW-T2M2-10

Command of Evidence: Textual · Information and Ideas · Medium

D *"Loosely clad in a slip of brown buckskin, and light-footed with a pair of soft moccasins on my feet, I was as free as the wind that blew my hair, and no less spirited than a bounding deer."*

The claim is that the narrator recalls enjoying unrestrained freedom as a girl. Choice D states this directly: light-footed and loosely clad, she was "as free as the wind that blew my hair" and as spirited as "a bounding deer." Both images suggest movement without limits.

A This describes the family's wigwam and the hills around it, not the girl's feelings or her sense of freedom.

B This describes her mother's sadness, not the narrator's own sense of freedom.

C This describes her cousin's chore of fetching water, which says nothing about the narrator's freedom.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

11. Correct answer: C RW-T2M2-11

Command of Evidence: Quantitative · Information and Ideas · Harder

C *Average weekly hours changed by one hour or less across most five-year intervals but fell sharply between 1910 and 1915 and again between 1930 and 1935.*

The argument has two parts: two sudden drops and long stretches of little change between them. Choice C captures both. Across most five-year intervals, hours stay nearly flat, but they fall from 59 to 54 between 1910 and 1915 and from 53 to 44 between 1930 and 1935.

A True, but an overall decline could also come from slow, steady change, so it does not show two sudden drops.

B True, but it identifies only one drop and ignores both the second drop and the flat periods the argument depends on.

D This misreads the graph and describes steady change, the very pattern the student argues against.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

12. Correct answer: A RW-T2M2-12

Command of Evidence: Textual · Information and Ideas · Harder

A *A mutant strain of the bacterium that covers roots as densely as the normal strain but cannot make the chemical gives bean plants no protection from root rot.*

The hypothesis credits the chemical rather than crowding. Choice A separates the two explanations: a mutant that crowds the roots just as densely but cannot make the chemical gives no protection. If crowding alone fails to protect the plants, the chemical is the likely cause, which directly supports the hypothesis.

B More bacteria could mean both more chemical and more crowding, so this result fits either explanation and cannot favor one.

C This finding does not say whether the other bacterium produces an antifungal chemical. Equal protection by another species therefore does not distinguish chemical action from crowding.

D How soil conditions affect the fungus says nothing about how the bacterium protects bean plants.

Review: Chapter 9, Command of Evidence: Textual and Quantitative (page 128)

13. Correct answer: B RW-T2M2-13

Inferences · Information and Ideas · Medium

B *released their gas relatively gently, producing flowing lava rather than violent blasts.*

The text links low-silica magma to runny lava from which gas "escapes more easily," in contrast to silica-rich magma, which builds pressure until gas bursts out violently. Since nearly all of the volcano's past rock is low in silica, its eruptions most likely released gas gently and produced flowing lava.

A This describes silica-rich magma, but the rock shows that this volcano's magma was low in silica.

C The rock is described as low in silica, and the text compares this volcano with no others on the island.

D The text gives no basis for predicting future magma; it concerns what past rock reveals about past eruptions.

Review: Chapter 10, Inferences (page 150)

14. Correct answer: D RW-T2M2-14

Inferences · Information and Ideas · Harder

D *how likely people are to miss an unexpected event may depend on how much of their attention the main task requires.*

Participants given the easier task usually noticed the walker, while those given the harder task usually missed it. The pattern supports the qualified inference that missing an unexpected event may depend on how much attention the main task requires. It does not establish any of the unrelated comparisons in the other choices.

A The experiment compared task difficulty, not counting skill, and nothing shows that people who noticed the walker counted more accurately.

B The experiment used only videos, so it cannot show that the effect happens only with videos; "only" goes beyond the evidence.

C Every participant saw the same bright costume, so the experiment offers no comparison with a plain one.

Review: Chapter 10, Inferences (page 150)

15. Correct answer: C RW-T2M2-15

Form, Structure, and Sense · Standard English Conventions · Easier

C *has held*

The phrase "every year since 1998" describes a repeated action that began in the past and continues into the present. The present perfect tense expresses this kind of ongoing action, so "has held" is correct.

A The future tense ignores "since 1998," which shows the exhibitions have already been taking place for years.

B The past perfect "had held" needs a later past event as a reference point, and the sentence gives none.

D "Is holding" describes something happening right now, not a yearly event repeated since 1998.

Review: Chapter 12, Form, Structure, and Sense (page 180)

16. Correct answer: D RW-T2M2-16

Boundaries · Standard English Conventions · Medium

D *genes. In*

"Nearly every cell in the human body carries the same set of genes" is a complete statement, and "In each type of cell, however, only some of those genes are switched on" is another. With no conjunction joining them, the first sentence should end with a period.

A A comma alone cannot join two independent clauses, so this choice creates a comma splice.

B With no punctuation, the two clauses run together, and "genes in each type of cell" misleadingly reads as one phrase.

C A question mark ends a direct question, but the first sentence is a statement.

Review: Chapter 11, Boundaries (page 164)

17. Correct answer: A RW-T2M2-17

Form, Structure, and Sense · Standard English Conventions · Medium

A *its*

The pronoun refers to "the commission as a whole," the subject of the sentence. "Commission" is a singular collective noun, and "as a whole" stresses that it acts as one body. A singular antecedent needs a singular possessive pronoun, so "its meetings" is correct.

B "Their" is plural, but its antecedent, "the commission as a whole," is singular.

C "It's" is a contraction of "it is" or "it has," not a possessive pronoun, so "it's meetings" is incorrect.

D "They're" means "they are"; it is neither possessive nor singular, so it cannot refer to the commission's meetings.

Review: Chapter 12, Form, Structure, and Sense (page 180)

18. Correct answer: C RW-T2M2-18

Boundaries · Standard English Conventions · Medium

C *air,*

The appositive "a hanging sculpture whose balanced parts drift and turn in moving air" renames "the mobile" and opens with a comma. A supplement that opens with a comma must close with a matching comma, which goes before the main verb "became."

A Without a closing comma, the appositive never ends, blurring where the supplement stops and the main clause resumes.

B A semicolon cannot close a supplement, and here it would cut the subject "the mobile" off from its verb "became."

D A dash cannot close a supplement that opened with a comma; the paired marks must match.

Review: Chapter 11, Boundaries (page 164)

19. Correct answer: B RW-T2M2-19

Form, Structure, and Sense · Standard English Conventions · Harder

B *carry*

The clause beginning with "as" has the subject "powerful rising air currents" and needs a finite verb to be complete. "Carry" is a finite present-tense verb that agrees with the plural subject and matches the present tense used throughout the text.

A "Carrying" is a participle, not a finite verb, so the clause beginning with "as" would have no main verb.

C "To carry" is an infinitive, which cannot serve as the main verb of the clause beginning with "as."

D "Having carried" is a nonfinite participle form, so it also leaves the "as" clause without a finite verb.

Review: Chapter 12, Form, Structure, and Sense (page 180)

20. Correct answer: A RW-T2M2-20

Boundaries · Standard English Conventions · Harder

A *representatives; however,*

"The yard owners at first refused to meet with the workers' elected representatives" and "by the third week of the strike… the owners agreed to talks" are independent clauses. When a conjunctive adverb such as "however" joins two independent clauses, a semicolon goes before it and a comma follows it.

B A comma before "however" creates a comma splice, because "however" cannot join two independent clauses the way "but" can.

C The semicolon is correct, but "however" at the start of the second clause needs a comma after it.

D With no punctuation before "however," the two independent clauses run together, and readers cannot tell which clause "however" belongs to.

Review: Chapter 11, Boundaries (page 164)

21. Correct answer: D RW-T2M2-21

Form, Structure, and Sense · Standard English Conventions · Harder

D *the carpet is colored with more than forty natural dyes,*

The introductory phrase "Woven entirely by hand over three years" describes the carpet, so "the carpet" must come immediately after the comma. Only choice D places "the carpet" there, making clear that the carpet, not the weaver, the dyes or a skill, was woven.

A This places "the weaver" after the modifier, illogically saying that the weaver was woven by hand over three years.

B This places "more than forty natural dyes" after the modifier, suggesting that the dyes were woven over three years.

C This makes "the weaver's skill" the noun being described, illogically saying that a skill was woven by hand.

Review: Chapter 12, Form, Structure, and Sense (page 180)

22. Correct answer: C RW-T2M2-22

Transitions · Expression of Ideas · Easier

C *Finally,*

The passage follows the volcano's activity in chronological order: its 1943 beginning, its growth during the first year, several more years of activity and its 1952 end. "Finally" introduces the last event in that sequence.

A "For instance" introduces an example, but the end of the eruption is the last stage of the sequence, not an example of its growth.

B "Similarly" signals a likeness, but the eruption's end is not similar to its growth; it is the next and last event.

D "Instead" signals a replacement, but the eruption's end follows its growth in time rather than replacing it.

Review: Chapter 13, Transitions (page 195)

23. Correct answer: B RW-T2M2-23

Transitions · Expression of Ideas · Medium

B *For instance,*

The first sentence introduces chunking as a general strategy for grouping items into meaningful units. The next sentence gives a specific case of that strategy: remembering a phone number as three groups of digits. "For instance" correctly introduces this example.

A "In contrast" signals a difference, but the phone number illustrates chunking rather than contrasting with it.

C "Nevertheless" signals something unexpected given the previous claim, but the phone number example is exactly what the claim predicts.

D "Meanwhile" signals events happening at the same time, which does not describe how an example relates to a general strategy.

Review: Chapter 13, Transitions (page 195)

24. Correct answer: D RW-T2M2-24

Transitions · Expression of Ideas · Harder

D *Similarly,*

The text first describes bronze sculptors casting large figures in sections and joining them. The final sentence describes ceramic sculptors doing the same thing for a related practical reason. Because it presents a parallel practice in a different medium, "Similarly" is the logical transition.

A Although the materials differ, the ceramic practice matches the bronze practice, so "In contrast" points the wrong way.

B Firing ceramic pieces is not a case of bronze casting, so it cannot serve as an example of the practice just described.

C Bronze casting methods do not cause ceramic sculptors to fire pieces separately; the two practices are parallel, not cause and effect.

Review: Chapter 13, Transitions (page 195)

25. Correct answer: B RW-T2M2-25

Rhetorical Synthesis · Expression of Ideas · Easier

B *Wood frogs often breed in shallow temporary pools filled by snowmelt and rain.*

The goal asks for the kind of place where wood frogs breed. Choice B supplies that habitat detail from the notes: shallow temporary pools filled by snowmelt and rain. The other choices describe timing, geographic range or the male's call.

A This tells when wood frogs breed, not the kind of place where they breed.

C This describes the frog's overall geographic range, not the specific kind of place it uses for breeding.

D This describes the male's call, which says nothing about where wood frogs breed.

Review: Chapter 14, Rhetorical Synthesis (page 208)

26. Correct answer: A RW-T2M2-26

Rhetorical Synthesis · Expression of Ideas · Medium

A *The Elm Falls night school shaped the town's civic life: several of its graduates became the first mill workers ever elected to the town council.*

The goal is to show why the night school mattered for civic life. Choice A states that significance directly and backs it with the most relevant note: several graduates became the first mill workers ever elected to the town council, giving workers a new place in local government.

B High enrollment shows the school was popular, but it says nothing about the school's effect on civic life.

C This describes the school's schedule and subjects without explaining its significance for the town's civic life.

D This gives background about why the school was needed, not what it meant for civic life afterward.

Review: Chapter 14, Rhetorical Synthesis (page 208)

27. Correct answer: C RW-T2M2-27

Rhetorical Synthesis · Expression of Ideas · Medium

C *Ortiz and Reyes both work with discarded materials: Ortiz with worn-out jeans from thrift stores, and Reyes with rope that has washed up on beaches.*

The goal is a similarity in materials. Choice C names what the two artists share, discarded materials, and supports it with each artist's source: Ortiz uses worn-out jeans from

thrift stores, and Reyes uses rope that has washed up on beaches. The word "both" makes the similarity explicit.

A This emphasizes a difference in how the works are displayed, not a similarity in materials.

B This describes only Reyes's materials, so it presents no comparison with Ortiz.

D This describes only Ortiz's work, so it presents no similarity between the two artists.

Review: Chapter 14, Rhetorical Synthesis (page 208)

Explanations: Math, Module 1

1. Correct answer: C M-T2M1-01

Linear equations in one variable · Algebra · Easier

C *5*

Distribute first, then collect the x-terms on one side. Since 6(x − 2) = 6x − 12, the equation becomes 6x − 12 = 2x + 8. Subtract 2x from both sides and add 12 to both sides to get 4x = 20, so x = 5. Check: 6(5 − 2) = 18 and 2(5) + 8 = 18.

1. Distribute: 6(x − 2) = 6x − 12, so 6x − 12 = 2x + 8.
2. Subtract 2x from both sides: 4x − 12 = 8.
3. Add 12 to both sides: 4x = 20.
4. Divide both sides by 4: x = 5.

A Moves −12 to the right side without changing its sign, which gives 4x = 8 − 12 = −4 and x = −1.

B Multiplies only the x by 6 and not the 2, so it solves 6x − 2 = 2x + 8 and gets 4x = 10.

D Stops at 4x = 20 and skips the last step of dividing both sides by 4.

Calculator option: Graph y = 6(x − 2) and y = 2x + 8; the lines intersect where x = 5.

Review: Chapter 16, Linear Equations in One Variable (page 239)

2. Correct answer: B M-T2M1-02

Percentages · Problem-Solving and Data Analysis · Easier

B *360*

If 85% of the seats were filled, then 100% − 85% = 15% were empty. Find 15% of 2,400: 0.15 × 2,400 = 360 empty seats. Check another way: the filled seats number 0.85 × 2,400 = 2,040, and 2,400 − 2,040 = 360.

1. Percent of seats empty = 100% − 85% = 15%.
2. Empty seats = 0.15 × 2,400.
3. 0.15 × 2,400 = 360.

A Gives the percent of seats that were empty, 15%, rather than the number of empty seats.

C Finds 85% of 2,400, which is the number of filled seats, not the number of empty seats.

D Increases 2,400 by 15% (2,400 × 1.15) instead of finding 15% of the 2,400 seats.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

3. Correct answer: B M-T2M1-03

Lines, angles, and triangles · Geometry and Trigonometry · Easier

B *52*

At line ℓ, the 128° angle and the angle just to its right (between ℓ and t, above ℓ) form a straight line, so that angle is 180° − 128° = 52°. It sits in the same position at line ℓ as the angle marked x° sits at line m, so the two are corresponding angles. Because ℓ is parallel to m, they are equal: x = 52.

1. The 128° angle and the angle beside it along line ℓ form a straight angle, so that angle is 180° − 128° = 52°.
2. That 52° angle and the angle marked x° are corresponding angles: each lies above its line and to the right of t.
3. Corresponding angles formed by parallel lines are equal, so x = 52.

A Finds the 52° angle but then takes its complement, 90° − 52° = 38°; nothing in the figure involves a right angle.

C Assumes x equals the marked angle, but the 128° angle is obtuse and x° is acute; they are supplementary, not equal.

D Subtracts 128 from 360 instead of from 180; two angles that form a straight line add to 180°.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: A M-T2M1-04

Linear functions · Algebra · Easier

A *f(x) = 2x − 3*

Each time x increases by 2, f(x) increases by 4, so the slope is 4/2 = 2. The table gives f(0) = −3, which is the y-intercept. So f(x) = 2x − 3. Check another row: f(6) = 2(6) − 3 = 9, which matches the table.

1. Slope = (change in f(x))/(change in x) = (1 − (−3))/(2 − 0) = 4/2 = 2.
2. The y-intercept is f(0) = −3.
3. So f(x) = 2x − 3. Check: f(4) = 8 − 3 = 5, which matches the table.

B Uses 1, the value of f(2), as the y-intercept; the y-intercept is f(0), which the table shows is −3.

C Uses the change in f(x), 4, as the slope without dividing by the matching change in x, which is 2.

D Divides the change in x by the change in f(x); slope is the change in output over the change in input, 4/2.

Calculator option: Type the table into the calculator and run a linear regression; it reports a slope of 2 and an intercept of −3.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

5. Correct answer: 7/8 or .875 or 0.875 M-T2M1-05

Linear equations in one variable · Algebra · Easier

Collect the x-terms on one side. Adding 3x to both sides gives 8x + 2 = 9, and subtracting 2 gives 8x = 7, so x = 7/8. Enter 7/8, .875 or 0.875. A common wrong answer is 7/2, from subtracting 3x instead of adding it (2x = 7). Another is 11/8, from adding 2 to 9 instead of subtracting it.

1. Add 3x to both sides: 8x + 2 = 9.

2. Subtract 2 from both sides: $8x = 7$.

3. Divide both sides by 8: $x = 7/8$, which can also be entered as .875.

Calculator option: Graph $y = 5x + 2$ and $y = 9 - 3x$; they intersect at $x = 0.875$.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: D M-T2M1-06

Equivalent expressions · Advanced Math · Easier

D $3x^2 + 11x - 20$

Multiply each term of the first factor by each term of the second: $(3x)(x) = 3x^2$, $(3x)(5) = 15x$, $(-4)(x) = -4x$ and $(-4)(5) = -20$. Combine the like terms $15x$ and $-4x$ to get $11x$. The product is $3x^2 + 11x - 20$.

1. Multiply every pair of terms: $(3x)(x) = 3x^2$, $(3x)(5) = 15x$, $(-4)(x) = -4x$, $(-4)(5) = -20$.

2. Combine like terms: $15x - 4x = 11x$.

3. Result: $3x^2 + 11x - 20$.

A Multiplies only the first terms and the last terms, leaving out the middle products $15x$ and $-4x$.

B Treats the inner product as $+4x$ instead of $-4x$, so the middle terms are combined as $15x + 4x$.

C Reverses the sign of the middle term; this is the expansion of $(3x + 4)(x - 5)$, not of $(3x - 4)(x + 5)$.

Calculator option: Graph $y = (3x - 4)(x + 5)$ and then each choice; only the graph of choice D lies exactly on top of the original.

Review: Chapter 20, Equivalent Expressions (page 289)

7. Correct answer: C M-T2M1-07

One-variable data: distributions and measures of center and spread · Problem-Solving and Data Analysis · Easier

C *30*

With 15 rides, the median is the 8th value when the lengths are listed in order. Count dots from the left: 3 rides at 20 miles (rides 1 to 3), 4 rides at 25 miles (rides 4 to 7) and 2 rides at 30 miles (rides 8 and 9). The 8th value is 30, so the median is 30 miles.

1. There are 15 values, so the median is the $(15 + 1)/2 = 8$th value in order.

2. Running totals: 3 rides through 20 miles, 7 through 25 miles, 9 through 30 miles.

3. The 8th value falls in the 30-mile column, so the median is 30.

A Gives the position of the median in the ordered list, the 8th value, instead of the value itself.

B Gives the mode, the length with the most dots (4 rides at 25 miles), rather than the middle value.

D Averages the smallest and largest lengths, $(20 + 45)/2$; that is the midrange, not the median.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

8. Correct answer: A M-T2M1-08

Linear equations in two variables · Algebra · Medium

A $y = -2x + 3$

The slope is $(-5 - 7)/(4 - (-2)) = -12/6 = -2$. Substitute the point $(-2, 7)$ into $y = -2x + b$: $7 = -2(-2) + b = 4 + b$, so $b = 3$. The equation is $y = -2x + 3$. Check the other point: $-2(4) + 3 = -5$.

1. Slope $m = (-5 - 7)/(4 - (-2)) = -12/6 = -2$.

2. Write $y = -2x + b$ and substitute $(-2, 7)$: $7 = 4 + b$.

3. Solve: $b = 3$, so $y = -2x + 3$.

B Has the right slope but drops the negative sign of $x = -2$ when solving for b: $7 = -2(2) + b$ gives $b = 11$.

C Makes a sign error in the slope; this line passes through $(-2, 7)$, but at $x = 4$ it gives $y = 19$, not -5.

D Divides the change in x by the change in y; this line passes through $(-2, 7)$ but not through $(4, -5)$.

Calculator option: Plot the points $(-2, 7)$ and $(4, -5)$, then graph each choice; only $y = -2x + 3$ passes through both points.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: 2.5 or 5/2 M-T2M1-09

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Medium

Find the unit rate: $840 \div 6 = 140$ brochures per minute. Then $21{,}000 \div 140 = 150$ minutes. Convert to hours: $150 \div 60 = 2.5$ hours, entered as 2.5 or 5/2. An answer of 150 gives the time in minutes; the question asks for hours.

1. Rate = 840 brochures ÷ 6 minutes = 140 brochures per minute.

2. Time = 21,000 ÷ 140 = 150 minutes.

3. Convert: 150 minutes ÷ 60 minutes per hour = 2.5 hours.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

10. Correct answer: D M-T2M1-10

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

D *7*

Rewrite both sides with base 2. Since $4 = 2^2$ and $8 = 2^3$, the equation becomes $2^{2(x-1)} = 2^{3(x-3)}$, or $2^{2x-2} = 2^{3x-9}$. Equal powers of the same base have equal exponents, so $2x - 2 = 3x - 9$ and $x = 7$. Check: $4^6 = 4{,}096$ and $8^4 = 4{,}096$.

1. Write $4 = 2^2$ and $8 = 2^3$: $2^{2(x-1)} = 2^{3(x-3)}$.

2. Set the exponents equal: $2x - 2 = 3x - 9$.

3. Solve: $x = 7$.

A Distributes $3(x - 3)$ as $3x + 9$, a sign error that leads to $2x - 2 = 3x + 9$ and $x = -11$.

B Writes the new exponents as $2x - 1$ and $3x - 3$, multiplying only the x by the new power instead of the whole exponent.

C Multiplies each base by its exponent and solves $4(x - 1) = 8(x - 3)$; an exponent is not a multiplier of the base.

Calculator option: Graph $y = 4^{x - 1}$ and $y = 8^{x - 3}$; the curves intersect at $x = 7$. Zoom out vertically, because the y-value there is 4,096.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

11. Correct answer: D M-T2M1-11

Linear functions · Algebra · Medium

D *The cyclist finishes the route 135 minutes after starting.*

The t-intercept is the value of t that makes $d(t) = 0$, the moment when no distance remains. Solve $54 - 0.4t = 0$: $0.4t = 54$, so $t = 135$. The graph meets the t-axis at (135, 0), which means the cyclist finishes the route 135 minutes after starting.

1. The t-intercept is the point where $d(t) = 0$.
2. Solve $54 - 0.4t = 0$: $0.4t = 54$, so $t = 135$.
3. At $t = 135$ minutes the remaining distance is 0, so the ride is finished.

A Describes the other intercept, $d(0) = 54$, which is the distance left at the start, not where the graph meets the t-axis.

B Describes the rate of decrease in remaining distance; the graph's slope is −0.4 kilometer per minute, not an intercept.

C Uses the right number, 135, but treats it as a distance; t is measured in minutes, and 54 kilometers remain at the start.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

12. Correct answer: C M-T2M1-12

Linear inequalities in one or two variables · Algebra · Medium

C *(3, 4)*

Test each point in both inequalities. For (3, 4): $4 < 3(3) - 2 = 7$ is true, and $3 + 2(4) = 11 \geqslant 7$ is true. A solution to a system must satisfy every inequality, and (3, 4) is the only choice that does.

1. Test (3, 4) in $y < 3x - 2$: $4 < 7$ is true.
2. Test (3, 4) in $x + 2y \geqslant 7$: $11 \geqslant 7$ is true.
3. Each other choice makes at least one inequality false, so (3, 4) is the only listed point that satisfies the system.

A Satisfies the first inequality, since $1 < 10$, but fails the second: $4 + 2(1) = 6$, which is less than 7.

B Satisfies the second inequality but fails the first: 5 is not less than $3(1) - 2 = 1$.

D Lies on the boundary line of the first inequality: $4 < 3(2) - 2 = 4$ is false, because the inequality is strict.

Calculator option: Graph both inequalities and plot the four points; only (3, 4) lies in the region shaded by both. A point on the dashed boundary of $y < 3x - 2$ does not count.

Review: Chapter 19, Linear Inequalities (page 278)

13. Correct answer: 13 M-T2M1-13

Systems of two linear equations in two variables · Algebra · Medium

Substitute $3x - 7$ for y in the second equation: $5x - 2(3x - 7) = 9$, so $5x - 6x + 14 = 9$ and $-x = -5$, giving $x = 5$. Then $y = 3(5) - 7 = 8$, so $x + y = 13$. Entering 5 or 8 answers for only one variable, and distributing −2 as $-6x - 14$ leads to a sum of −99.

1. Substitute $y = 3x - 7$ into $5x - 2y = 9$: $5x - 2(3x - 7) = 9$.
2. Simplify: $5x - 6x + 14 = 9$, so $-x = -5$ and $x = 5$.
3. Find y: $y = 3(5) - 7 = 8$.
4. Add: $x + y = 5 + 8 = 13$.

Calculator option: Graph both equations; they intersect at (5, 8), so $x + y = 13$.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

14. Correct answer: D M-T2M1-14

Area and volume · Geometry and Trigonometry · Medium

D *36,000π*

The radius is half the diameter: $60 \div 2 = 30$ centimeters. Use the sphere formula from the reference sheet, $V = (4/3)\pi r^3$: $V = (4/3)\pi(30)^3 = (4/3)\pi(27{,}000) = 36{,}000\pi$ cubic centimeters.

1. Radius: $r = 60/2 = 30$ cm.
2. $V = (4/3)\pi r^3 = (4/3)\pi(27{,}000)$.
3. $V = 36{,}000\pi$ cubic centimeters.

A Squares the radius instead of cubing it: $(4/3)\pi(30)^2 = 1{,}200\pi$. A volume needs the radius cubed.

B Computes $4\pi r^2 = 3{,}600\pi$, which is the surface area of the sphere, not the volume inside it.

C Leaves out the factor 4/3 and computes only $\pi r^3 = \pi(30)^3 = 27{,}000\pi$.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

15. Correct answer: A M-T2M1-15

Nonlinear functions · Advanced Math · Medium

A *f(x) = (x + 1)(x − 5)*

The graph crosses the x-axis at $x = -1$ and $x = 5$, so the factors are $(x + 1)$ and $(x - 5)$. The parabola opens upward, so the leading coefficient is positive. Check the y-intercept: $(0 + 1)(0 - 5) = -5$, which matches the point (0, −5) on the graph.

1. Read the x-intercepts, −1 and 5, so f(x) = a(x + 1)(x − 5) for some a.
2. Use the y-intercept (0, −5): a(1)(−5) = −5, so a = 1.
3. f(x) = (x + 1)(x − 5), which opens upward as shown.

B Reverses the signs of the zeros; (x − 1)(x + 5) equals 0 at x = 1 and x = −5, not at −1 and 5.

C Has the correct zeros, but its negative leading coefficient makes the parabola open downward, unlike the graph.

D Has the correct zeros, but its y-intercept is 5(1)(−5) = −25, not −5 as shown on the graph.

Calculator option: Graph each choice and compare it with the figure; only f(x) = (x + 1)(x − 5) passes through (−1, 0), (5, 0) and (0, −5).

Review: Chapter 22, Nonlinear Functions (page 313)

16. Correct answer: B M-T2M1-16

Probability and conditional probability · Problem-Solving and Data Analysis · Harder

B *7/20*

The condition "did not sit on the floor" sets the denominator: 135 + 105 = 240 of the surveyed attendees sat in the two balcony sections. Of those, 54 + 30 = 84 bought merchandise. The probability is 84/240, which simplifies to 7/20.

1. Attendees not on the floor: 135 + 105 = 240.
2. Of these, merchandise buyers: 54 + 30 = 84.
3. Probability = 84/240 = 7/20.

A Divides the 84 balcony buyers by all 400 attendees, ignoring the condition that limits the group to attendees not on the floor.

C Uses only the lower balcony, 54/135; the upper balcony is also not the floor, so its attendees belong in the group.

D Divides by all 180 buyers, which answers a different question: the probability of a balcony seat given a purchase.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

17. Correct answer: -14 M-T2M1-17

Equivalent expressions · Advanced Math · Harder

Expand the left side: $2ax^2 - 5ax + 6x - 15$. Matching the x^2-coefficients gives 2a = 8, so a = 4. The x-coefficient is −5a + 6 = −20 + 6 = −14, so b = −14. An answer of 26 comes from treating −5a as +20, and −20 comes from leaving out the 6x term.

1. Expand: $(ax + 3)(2x - 5) = 2ax^2 - 5ax + 6x - 15$.
2. Match the x^2-terms: 2a = 8, so a = 4.
3. Match the x-terms: b = −5a + 6 = −20 + 6 = −14.

Review: Chapter 20, Equivalent Expressions (page 289)

18. Correct answer: A M-T2M1-18

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

A *−2 only*

First exclude x = 5 and x = −1, which make denominators zero. For all other x, multiply both sides by (x − 5)(x + 1): x(x + 1) − 4(x − 5) = 30, so $x^2 - 3x + 20 = 30$ and $x^2 - 3x - 10 = 0$. Factoring gives (x − 5)(x + 2) = 0, so the candidate values are 5 and −2. Reject 5 because it is outside the original domain. Only x = −2 works.

1. Exclude x = 5 and x = −1 because they make denominators zero.
2. Multiply every term by (x − 5)(x + 1): x(x + 1) − 4(x − 5) = 30.
3. Simplify: $x^2 - 3x + 20 = 30$, so $x^2 - 3x - 10 = 0$.
4. Factor: (x − 5)(x + 2) = 0, so the candidate values are 5 and −2.
5. Reject x = 5 because it makes a denominator zero; the only solution is −2.

B Keeps x = 5 without checking it; substituting 5 makes two denominators zero, so 5 cannot be a solution.

C Factors $x^2 - 3x - 10$ as (x + 5)(x − 2), which expands to $x^2 + 3x - 10$, so both roots have the wrong sign.

D Throws out both roots after spotting that 5 is excluded; x = −2 makes no denominator zero and satisfies the equation.

Calculator option: Graph the left side and the right side as two separate functions; they intersect only at x = −2. Near x = 5 both graphs have a vertical asymptote, not an intersection.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

19. Correct answer: C M-T2M1-19

Systems of two linear equations in two variables · Algebra · Harder

C *510*

Let s be the number of standing sections and t the number of seated sections. Then s + t = 14 and 120s + 75t = 1,410. Substituting s = 14 − t gives 1,680 − 45t = 1,410, so t = 6 and s = 8. Standing sections held 8 × 120 = 960 people and seated sections held 6 × 75 = 450, a difference of 510.

1. Let s = standing sections and t = seated sections: s + t = 14 and 120s + 75t = 1,410.
2. Substitute s = 14 − t: 120(14 − t) + 75t = 1,410, so 1,680 − 45t = 1,410.
3. Solve: t = 6, so s = 8.
4. People: 8 × 120 = 960 standing and 6 × 75 = 450 seated, and 960 − 450 = 510.

A Subtracts the numbers of sections, 8 − 6, rather than the numbers of people in those sections.

B Gives the number of people in seated sections, 6 × 75, one step before the final subtraction.

D Gives the number of people in standing sections, 8 × 120, without subtracting the 450 people who were seated.

Calculator option: Graph x + y = 14 and 120x + 75y = 1410, with x for standing and y for seated sections; the lines intersect at (8, 6).

Review: Chapter 18, Systems of Two Linear Equations (page 264)

20. Correct answer: B M-T2M1-20

Right triangles and trigonometry · Geometry and Trigonometry · Harder

B $6 + 6\sqrt{3}$

Triangle ABD is a 45°-45°-90° triangle, so its legs are equal: AD = BD = 6. Triangle BDC is a 30°-60°-90° triangle in which BD, opposite the 30° angle, is the shorter leg. The longer leg is $\sqrt{3}$ times the shorter leg, so DC = $6\sqrt{3}$. Therefore AC = AD + DC = $6 + 6\sqrt{3}$.

1. In 45°-45°-90° triangle ABD, the legs are equal: AD = BD = 6.
2. In 30°-60°-90° triangle BDC, BD = 6 is opposite the 30° angle, so it is the shorter leg.
3. The longer leg is DC = $6\sqrt{3}$.
4. AC = AD + DC = $6 + 6\sqrt{3}$.

A Treats BD as the longer leg of the 30°-60°-90° triangle, giving DC = $6/\sqrt{3} = 2\sqrt{3}$; BD is opposite the 30° angle.

C Uses BC = 12, the hypotenuse of triangle BDC, as if it were the segment DC along the base.

D Uses AB = $6\sqrt{2}$, the hypotenuse of triangle ABD, in place of the leg AD = 6.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

21. Correct answer: 48 M-T2M1-21

Nonlinear functions · Advanced Math · Harder

First find a: f(0) = a(−2)(6) = −12a = 36, so a = −3 and the parabola opens downward. Its vertex lies halfway between the zeros 2 and −6, at x = −2. The maximum is f(−2) = −3(−4)(4) = 48. Entering 36 gives the y-intercept rather than the maximum, and −48 comes from using a = 3.

1. Use f(0) = 36: a(0 − 2)(0 + 6) = −12a = 36, so a = −3.
2. Because a < 0, the parabola opens downward and has a maximum at its vertex.
3. The vertex x-coordinate is the average of the zeros: (2 + (−6))/2 = −2.
4. f(−2) = −3(−2 − 2)(−2 + 6) = −3(−4)(4) = 48.

Calculator option: Graph y = a(x − 2)(x + 6) with a slider for a, set a = −3 so the graph passes through (0, 36), and read the vertex, (−2, 48).

Review: Chapter 22, Nonlinear Functions (page 313)

22. Correct answer: D M-T2M1-22

Nonlinear functions · Advanced Math · Harder

D $f(x) = x^2 - 3x$

Let u = 2x, so x = u/2. Then $f(u) = 4(u/2)^2 - 6(u/2) = u^2 - 3u$, which means $f(x) = x^2 - 3x$. Check: $f(2x) = (2x)^2 - 3(2x) = 4x^2 - 6x$, which matches the given rule.

1. Let u = 2x, so x = u/2.
2. Substitute: $f(u) = 4(u/2)^2 - 6(u/2) = u^2 - 3u$.
3. Rename the variable: $f(x) = x^2 - 3x$.
4. Check: $f(2x) = 4x^2 - 6x$.

A Substitutes 2x into the given expression a second time; with this f, f(2x) would equal $64x^2 - 24x$.

B Assumes f(x) follows the same rule as f(2x); with this f, f(2x) would equal $16x^2 - 12x$, not $4x^2 - 6x$.

C Divides the whole expression by 2 instead of replacing x with x/2; with this f, f(2x) would equal $8x^2 - 6x$.

Calculator option: Graph $y = 4x^2 - 6x$, then graph each choice with 2x in place of x, such as $y = (2x)^2 - 3(2x)$; only choice D matches the first graph.

Review: Chapter 22, Nonlinear Functions (page 313)

Explanations: Math, Module 2

1. Correct answer: A M-T2M2-01

Linear functions · Algebra · Easier

A $f(x) = -(2/3)x + 4$

The graph passes through (0, 4) and (6, 0). The slope is (0 − 4)/(6 − 0) = −4/6 = −2/3, and the y-intercept is 4. So f(x) = −(2/3)x + 4. Check: f(6) = −(2/3)(6) + 4 = −4 + 4 = 0, which matches the x-intercept.

1. Read two points on the line: (0, 4) and (6, 0).
2. Slope = (0 − 4)/(6 − 0) = −2/3.
3. The y-intercept is 4, so f(x) = −(2/3)x + 4.

B Divides the run by the rise; slope is the change in y over the change in x, −4/6, not −6/4.

C Has a positive slope, but the line falls from left to right, so its slope must be negative.

D Uses the x-intercept, 6, as the y-intercept; the line crosses the y-axis at (0, 4).

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

2. Correct answer: D M-T2M2-02

Equivalent expressions · Advanced Math · Easier

D $2x + 3$

Divide each term of the numerator by 3x: $6x^2/(3x) = 2x$ and $9x/(3x) = 3$. So the expression equals 2x + 3. Check by multiplying back: $3x(2x + 3) = 6x^2 + 9x$, which is the original numerator.

1. Split the fraction: $6x^2/(3x) + 9x/(3x)$.
2. Simplify each part: 2x + 3.
3. Check: $3x(2x + 3) = 6x^2 + 9x$.

A Divides each term by 3 but not by x; the denominator is 3x, so both the 3 and the x must divide out.

B Divides each term by x but not by 3, leaving the coefficients 6 and 9 unchanged.

C Divides $6x^2$ by 3x correctly but divides 9x only by x, keeping 9 instead of 3.

Calculator option: Graph $y = (6x^2 + 9x)/(3x)$ and y = 2x + 3; for x > 0 the two graphs are the same line.

Review: Chapter 20, Equivalent Expressions (page 289)

3. Correct answer: C M-T2M2-03

Lines, angles, and triangles · Geometry and Trigonometry · Easier

C 117

An exterior angle of a triangle equals the sum of the two interior angles that are not next to it. Angle ACD is the exterior angle at C, so x = 41 + 76 = 117. Check another way: the interior angle at C is 180 − 41 − 76 = 63, and 180 − 63 = 117.

1. Interior angle at C: 180 − 41 − 76 = 63.
2. Angles ACB and ACD form a straight line, so x = 180 − 63.
3. x = 117, which also equals 41 + 76.

A Subtracts the two given angles, 76 − 41; the exterior angle equals their sum, not their difference.

B Gives the interior angle at C, 180 − 41 − 76 = 63, rather than the exterior angle marked x° along line BD.

D Subtracts only 41 from 180; the angle marked x° is supplementary to the interior angle at C, which is 63°.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

4. Correct answer: 6 M-T2M2-04

Ratios, rates, proportional relationships, and units · Problem-Solving and Data Analysis · Easier

Divide the volume by the rate to get the time in minutes: 6,480 ÷ 18 = 360 minutes. Then convert minutes to hours: 360 ÷ 60 = 6 hours. Entering 360 gives the time in minutes, but the question asks for hours.

1. Time in minutes = 6,480 gallons ÷ 18 gallons per minute = 360 minutes.
2. Time in hours = 360 ÷ 60 = 6.

Review: Chapter 23, Ratios, Rates, Proportions, Units and Percentages (page 328)

5. Correct answer: C M-T2M2-05

Linear equations in one variable · Algebra · Medium

C -2

Distribute to get 5x − 10 + kx = 3x − 10, or (5 + k)x − 10 = 3x − 10. A linear equation has infinitely many solutions when both sides are identical, so the x-coefficients must match: 5 + k = 3, which gives k = −2. The constants, −10 and −10, already match.

1. Distribute: 5x − 10 + kx = 3x − 10.
2. Group the x-terms: (5 + k)x − 10 = 3x − 10.
3. For infinitely many solutions, the coefficients must match: 5 + k = 3, so k = −2.

A Sets 5 + k equal to −3 instead of 3; with k = −8 the equation simplifies to −6x = 0, which has one solution.

B Chooses k to cancel the 5x on the left; then −10 = 3x − 10, which has exactly one solution, x = 0.

D Solves 5 + k = 3 by computing 5 − 3 = 2; subtracting 5 from both sides gives k = −2.

Calculator option: Graph y = 5(x − 2) + kx and y = 3x − 10 with a slider for k; the two lines coincide only when k = −2.

Review: Chapter 16, Linear Equations in One Variable (page 239)

6. Correct answer: A M-T2M2-06

Two-variable data: models and scatterplots · Problem-Solving and Data Analysis · Medium

A *Each additional page increases the predicted print time by about 0.06 minute.*

The slope is (35 − 5)/(500 − 0) = 30/500 = 0.06. Because y is the time in minutes and x is the number of pages, the slope is a rate of 0.06 minute per page: the line predicts that each additional page adds about 0.06 minute to a job's print time.

1. Slope = (35 − 5)/(500 − 0) = 30/500 = 0.06.
2. The units of the slope are minutes of print time per page.
3. So each additional page adds about 0.06 minute to the predicted time.

B Reverses the variables; the slope is minutes per page, because print time is on the vertical axis and pages are on the horizontal axis.

C Confuses the slope with the y-intercept; the line predicts about 5 minutes, not 0.06 minute, for a job with 0 pages.

D Uses the y-intercept, 5, as if it were the rate of change; 5 minutes is the predicted time when x = 0.

Review: Chapter 24, One-Variable and Two-Variable Data (page 340)

7. Correct answer: B M-T2M2-07

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Medium

B *(−2, 6)*

Set the two expressions for y equal: $x^2 - 3x - 4 = x + 8$, so $x^2 - 4x - 12 = 0$. Factoring gives (x − 6)(x + 2) = 0, so x = 6 or x = −2. When x = −2, y = −2 + 8 = 6, so (−2, 6) is a solution. The other solution, (6, 14), is not listed.

1. Set the right sides equal: $x^2 - 3x - 4 = x + 8$.
2. Rearrange: $x^2 - 4x - 12 = 0$.
3. Factor: (x − 6)(x + 2) = 0, so x = 6 or x = −2.
4. For x = −2: y = −2 + 8 = 6, which gives (−2, 6).

A Makes a sign error evaluating the quadratic at x = −2; −3(−2) is +6, so y = 4 + 6 − 4 = 6, not −6.

C Uses x = 2 from a sign error in factoring; (2, 10) lies on the line but not on the parabola, where y = −6.

D Reverses the coordinates of a solution; at x = 6 both equations give y = 14, not −2.

Calculator option: Graph both equations; they intersect at (−2, 6) and (6, 14). Only (−2, 6) appears among the choices.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

8. Correct answer: -18 M-T2M2-08

Linear functions · Algebra · Medium

The slope is (27 − 7)/(5 − 1) = 20/4 = 5. Using f(1) = 7 gives 7 = 5(1) + b, so b = 2 and f(x) = 5x + 2. Then f(−4) = 5(−4) + 2 = −18. A common wrong answer is 22, from a sign slip that makes 5(−4) equal 20; another is 2, which is f(0).

1. Slope = (27 − 7)/(5 − 1) = 5.
2. Find the intercept: 7 = 5(1) + b, so b = 2.
3. f(x) = 5x + 2, so f(−4) = −20 + 2 = −18.

Calculator option: Enter the points (1, 7) and (5, 27) in a table and run a linear regression to get y = 5x + 2; then evaluate it at x = −4.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

9. Correct answer: C M-T2M2-09

Linear inequalities in one or two variables · Algebra · Medium

C *$4p + 15b \leqslant 480$*

The total printing time in minutes is 4p for posters plus 15b for banners. The 8-hour limit must be written in the same unit: 8 × 60 = 480 minutes. "At most" means less than or equal to, so the inequality is $4p + 15b \leqslant 480$.

1. Time for posters: 4p minutes; time for banners: 15b minutes.
2. Convert the limit: 8 hours × 60 = 480 minutes.
3. "At most" means $\leqslant$, so $4p + 15b \leqslant 480$.

A Reverses the inequality; "at most 8 hours" sets an upper limit, so the total time must be less than or equal to 480 minutes.

B Mixes units: the left side is measured in minutes, but the 8 on the right side is measured in hours.

D Swaps the two printing times, giving each poster 15 minutes and each banner 4 minutes.

Review: Chapter 19, Linear Inequalities (page 278)

10. Correct answer: B M-T2M2-10

Area and volume · Geometry and Trigonometry · Medium

B *540π*

The radius is half the diameter: 18 ÷ 2 = 9 centimeters. Use the cone formula from the reference sheet, $V = (1/3)\pi r^2 h$: $V = (1/3)\pi(9)^2(20) = (1/3)\pi(81)(20) = 540\pi$ cubic centimeters.

1. Radius: r = 18/2 = 9 cm.
2. $V = (1/3)\pi r^2 h = (1/3)\pi(81)(20)$.
3. $V = 540\pi$ cubic centimeters.

A Does not square the radius, computing $(1/3)\pi(9)(20) = 60\pi$; the formula uses r^2, so 9 must be squared.

C Leaves out the factor 1/3 and computes $\pi r^2 h$, which is the volume of a cylinder with the same base and height.

D Uses the diameter, 18, as the radius: $(1/3)\pi(18)^2(20) = 2{,}160\pi$, which is four times the correct volume.

Review: Chapter 26, Area, Volume, Lines, Angles and Triangles (page 372)

11. Correct answer: 2 M-T2M2-11

Systems of two linear equations in two variables · Algebra · Medium

A system has infinitely many solutions when one equation is a multiple of the other. Multiplying the second equation by 3 gives $3kx - 9y = 15$, which matches the first equation's y-term and constant. So the x-terms must match too: $3k = 6$ and $k = 2$. Entering 18 comes from multiplying 6 by 3 instead of dividing.

1. Compare the y-terms and constants: $-9y = 3(-3y)$ and $15 = 3(5)$, so the first equation is 3 times the second.
2. The x-terms must match the same way: $6 = 3k$.
3. Solve: $k = 2$.

Calculator option: Graph both equations with a slider for k; the two lines lie on top of each other only when $k = 2$.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

12. Correct answer: D M-T2M2-12

Equivalent expressions · Advanced Math · Medium

D *$(x + 7)/((x - 2)(x + 1))$*

Rewrite both fractions over the common denominator $(x - 2)(x + 1)$. The numerator becomes $3(x + 1) - 2(x - 2) = 3x + 3 - 2x + 4 = x + 7$, because subtracting $2(x - 2)$ subtracts $2x$ and adds 4. So the expression equals $(x + 7)/((x - 2)(x + 1))$.

1. Common denominator: $(x - 2)(x + 1)$.
2. Numerator: $3(x + 1) - 2(x - 2) = 3x + 3 - 2x + 4$.
3. Simplify: $x + 7$, so the expression equals $(x + 7)/((x - 2)(x + 1))$.

A Subtracts the numerators and adds the denominators; fractions can be combined only after they share a common denominator.

B Distributes -2 as $-2x - 4$ instead of $-2x + 4$, so the constant becomes $3 - 4 = -1$.

C Multiplies only the x by 3, writing $3(x + 1)$ as $3x + 1$, so the constant becomes $1 + 4 = 5$.

Calculator option: Graph the original expression and each choice; for $x > 2$ only choice D lies exactly on top of the original graph.

Review: Chapter 20, Equivalent Expressions (page 289)

13. Correct answer: C M-T2M2-13

Evaluating statistical claims: observational studies and experiments · Problem-Solving and Data Analysis · Medium

C *Sending a text-message reminder likely causes higher attendance among people on this venue's email list.*

Random assignment of the reminder makes this an experiment. Together with the reported statistical evidence, it supports a cause-and-effect conclusion. Random selection from this venue's email list supports generalizing the overall result to that list, but not to other venues. So a reminder likely causes higher attendance among people on this venue's email list. This is a conclusion about attendance rates, not a guarantee that every person will attend.

1. Random assignment, together with the reported statistical evidence, supports a cause-and-effect conclusion.
2. Random selection from the venue's email list supports generalizing the overall result to that list.
3. Together: a reminder likely causes higher attendance for people on this list.

A Generalizes too far; the sample came only from this venue's email list, so the results apply to that list, not to every venue.

B Misidentifies the design; the venue assigned the reminder at random, which makes this an experiment, not an observational study.

D Is too narrow; because the 400 people were selected at random from the list, the results can be generalized to the whole list.

Review: Chapter 25, Probability, Sampling, Inference and Evaluating Claims (page 357)

14. Correct answer: A M-T2M2-14

Nonlinear functions · Advanced Math · Harder

A *It increases by 30% each year.*

Rewrite the exponent: $(1.69)^{t/2} = ((1.69)^{1/2})^t = (1.3)^t$, because $\sqrt{1.69} = 1.3$. So $N(t) = 1{,}600(1.3)^t$, and the number of subscribers is multiplied by 1.3 each year. A growth factor of 1.3 means an increase of 30% per year.

1. Write $(1.69)^{t/2}$ as $((1.69)^{1/2})^t$.
2. Since $\sqrt{1.69} = 1.3$, $N(t) = 1{,}600(1.3)^t$.
3. A yearly factor of $1.3 = 1 + 0.30$ means a 30% increase each year.

B Halves the 69% growth over 2 years; percent growth compounds, so the yearly factor is $\sqrt{1.69} = 1.3$, not 1.345.

C Ignores the $t/2$ in the exponent; the factor 1.69 applies once every 2 years, not every year.

D Treats the growth factor 1.69 as a 169% increase; a factor of 1.69 means a 69% increase every 2 years.

Review: Chapter 22, Nonlinear Functions (page 313)

15. Correct answer: 10 M-T2M2-15

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

Set the right sides equal: $x^2 + 4x + c = -2x + 1$, so $x^2 + 6x + (c - 1) = 0$. Exactly one solution means the line touches the parabola once, so the discriminant is zero: $6^2 - 4(1)(c - 1) = 0$. Then $36 = 4(c - 1)$, $c - 1 = 9$ and $c = 10$. Entering 9 forgets to add 1 back.

1. Set the expressions equal: $x^2 + 4x + c = -2x + 1$.
2. Rearrange: $x^2 + 6x + (c - 1) = 0$.
3. One solution requires a discriminant of 0: $36 - 4(c - 1) = 0$.
4. Solve: $c - 1 = 9$, so $c = 10$.

Calculator option: Graph $y = x^2 + 4x + c$ with a slider for c along with $y = -2x + 1$; the parabola just touches the line, at $(-3, 7)$, when $c = 10$.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

16. Correct answer: B M-T2M2-16

Linear equations in two variables · Algebra · Harder

B *It decreases by 54 minutes.*

Solve for y: $y = 9 - 1.8x$, so each extra hour on flat roads removes 1.8 hours from the hilly roads. Thirty minutes is 0.5 hour, so y decreases by $1.8 \times 0.5 = 0.9$ hour, or 54 minutes. Check: 0.5 hour at 18 miles per hour covers 9 miles, and 9 miles at 10 miles per hour takes 0.9 hour.

1. Solve for y: $y = 9 - 1.8x$.
2. The slope -1.8 means y drops 1.8 hours for each added hour of x.
3. Added flat time: 30 minutes = 0.5 hour, so y changes by $-1.8 \times 0.5 = -0.9$ hour.
4. Convert: 0.9 hour × 60 = 54 minutes less on hilly roads.

A Uses the inverted ratio 10/18 instead of 18/10; 9 extra flat miles mean 9 fewer hilly miles, which take 0.9 hour.

C Applies the change for a full hour, 1.8 hours or 108 minutes, even though the extra flat-road time is only 30 minutes.

D Has the right size but the wrong direction; with a fixed total distance, more time on flat roads means less time on hilly roads.

Review: Chapter 17, Linear Equations in Two Variables and Linear Functions (page 251)

17. Correct answer: D M-T2M2-17

Right triangles and trigonometry · Geometry and Trigonometry · Harder

D *35/12*

Angles A and B are complementary, so $\sin A = \cos B$. Then $2k + 2 = k + 7$, so $k = 5$ and $\sin A = 12/37$. The side opposite A and the hypotenuse are in the ratio 12:37. Scale the triangle to give these sides lengths 12 and 37; its remaining leg is $\sqrt{(1{,}369 - 144)} = 35$. Scaling preserves trigonometric ratios. Angle B is opposite the 35 and adjacent to the 12, so $\tan B = 35/12$.

1. Because $A + B = 90°$, $\sin A = \cos B$, so $2k + 2 = k + 7$ and $k = 5$.
2. Then $\sin A = 12/37$. Scale the triangle so BC = 12 (opposite A) and AB = 37 (hypotenuse); this preserves its trigonometric ratios.
3. $AC = \sqrt{(37^2 - 12^2)} = 35$.
4. $\tan B$ = (side opposite B)/(side adjacent to B) = AC/BC = 35/12.

A Gives $\sin A = 12/37$, the value found after solving for k, rather than the requested $\tan B$.

B Gives $\tan A$, opposite over adjacent for angle A; for angle B the opposite and adjacent sides switch roles.

C Gives $\cos A$, which equals $\sin B$; tangent is opposite over adjacent, not opposite over hypotenuse.

Review: Chapter 27, Right Triangles and Trigonometry (page 389)

18. Correct answer: 12.5 or 25/2 M-T2M2-18

Nonlinear functions · Advanced Math · Harder

Divide the outputs to remove a: $f(5)/f(2) = ab^5/(ab^2) = b^3 = 400/50 = 8$, so $b = 2$. Then $f(2) = a(2)^2 = 4a = 50$, so $a = 12.5$. Since $f(0) = ab^0 = a$, the answer is 12.5, or 25/2. Entering 25 gives f(1), and entering 8 gives b^3.

1. Divide: $f(5)/f(2) = b^3 = 400/50 = 8$, so $b = 2$.
2. Substitute into f(2): $a(2)^2 = 50$, so $a = 12.5$.
3. $f(0) = a(2)^0 = a = 12.5$.

Calculator option: Enter the points (2, 50) and (5, 400) in a table and fit an exponential model of the form $y = ab^x$; it reports $a = 12.5$ and $b = 2$.

Review: Chapter 22, Nonlinear Functions (page 313)

19. Correct answer: D M-T2M2-19

Systems of two linear equations in two variables · Algebra · Harder

D *44*

For infinitely many solutions, the second equation must be a multiple of the first. The y-coefficients show the multiplier: $-8 \div (-2) = 4$. Multiplying $5x - 2y = 6$ by 4 gives $20x - 8y = 24$, so $a = 20$ and $c = 24$. Therefore $a + c = 44$.

1. Find the multiplier from the y-terms: $-8 = 4(-2)$, so the factor is 4.
2. Multiply the first equation by 4: $20x - 8y = 24$.
3. Match terms: $a = 20$ and $c = 24$.
4. $a + c = 20 + 24 = 44$.

A Uses a multiplier of -4 instead of 4; dividing -8 by -2 gives a positive 4, so a and c are both positive.

B Adds the original values 5 and 6 without multiplying them by 4 to match the second equation.

C Multiplies only the x-coefficient by 4 and keeps c = 6; the constant must be scaled too, or the lines are parallel.

Review: Chapter 18, Systems of Two Linear Equations (page 264)

20. Correct answer: A M-T2M2-20

Nonlinear equations in one variable and systems of equations in two variables · Advanced Math · Harder

A *−3*

The equation means 3x − 6 = k or 3x − 6 = −k, so x = (6 + k)/3 or x = (6 − k)/3. Their difference is 2k/3 = 10, so k = 15. The solutions are x = 7 and x = −3, and the smaller one is −3. Check: |3(−3) − 6| = |−15| = 15.

1. Split into two cases: 3x − 6 = k or 3x − 6 = −k.
2. Solve: x = (6 + k)/3 or x = (6 − k)/3.
3. The difference is 2k/3 = 10, so k = 15.
4. The solutions are 7 and −3; the smaller is −3.

B Assumes the solutions differ by k, so it uses k = 10 and gets (6 − 10)/3 = −4/3; the difference is actually 2k/3.

C Gives the larger solution, (6 + 15)/3 = 7, instead of the smaller one.

D Gives the value of k, which is the distance of 3x − 6 from zero, not a value of x.

Calculator option: After finding k = 15, graph y = |3x − 6| and y = 15; they intersect at x = −3 and x = 7, which are 10 apart.

Review: Chapter 21, Nonlinear Equations and Systems (page 300)

21. Correct answer: -29 M-T2M2-21

Circles · Geometry and Trigonometry · Harder

Complete the square in x and in y: $(x^2 + 8x + 16) + (y^2 - 12y + 36) = -c + 16 + 36$, so $(x + 4)^2 + (y - 6)^2 = 52 - c$. The right side equals $r^2 = 81$, so 52 − c = 81 and c = −29. Entering 29 is a sign error, and 43 comes from setting 52 − c equal to 9 instead of 81.

1. Group terms: $(x^2 + 8x) + (y^2 - 12y) = -c$.
2. Add 16 and 36 to both sides to complete the squares: $(x + 4)^2 + (y - 6)^2 = 52 - c$.
3. The radius is 9, so $52 - c = 9^2 = 81$.
4. Solve: c = −29.

Calculator option: Graph the equation with c = −29 to check: the circle is centered at (−4, 6) and reaches from x = −13 to x = 5, so its radius is 9.

Review: Chapter 28, Circles (page 405)

22. Correct answer: B M-T2M2-22

Nonlinear functions · Advanced Math · Harder

B *−10*

Since f(0) = c, c = −6. A parabola is symmetric about its axis, so equal outputs at x = −1 and x = 4 put the axis halfway between them, at x = 1.5. For f, the axis is x = −b/(2a) = −b/4, so −b/4 = 1.5 and b = −6. Then f(1) = 2 − 6 − 6 = −10.

1. f(0) = c, so c = −6.
2. f(−1) = f(4), so the axis of symmetry is x = (−1 + 4)/2 = 1.5.
3. The axis is x = −b/(2a) = −b/4, so −b/4 = 1.5 and b = −6.
4. $f(1) = 2(1)^2 - 6(1) - 6 = -10$.

A Places the axis at x = 3 by adding −1 and 4 without halving; that gives b = −12 and f(1) = −16.

C Gives f(0) = c = −6, the y-intercept, rather than evaluating the function at x = 1.

D Makes a sign error and uses b = 6, which puts the axis of symmetry at x = −1.5 instead of 1.5.

Review: Chapter 22, Nonlinear Functions (page 313)

CHAPTER 33

The Final Two Weeks and Beyond

The last two weeks are not the time to learn whole new topics. They are the time to sharpen what you know, fix the mistakes you keep making, rehearse the test under real conditions and arrive rested and organized. This chapter gives you a day-by-day plan, a last pass through your error log, and a look past test day: getting your scores, sending them and deciding whether to test again.

The 14-day plan

In the table, Day 14 means 14 days before your test and Day 1 is the day before it. If your test is on a Saturday, Day 14 is the Saturday two weeks earlier. Move a day or two if your schedule requires it, but keep the two full-length tests about a week apart, with a review day after each.

Day	Main task	About how long
14	Take a full-length practice test in the College Board's Bluebook app, in one sitting with the 10-minute break, at the time of day you will test.	2.5 hours
13	Review every miss and every guess from Day 14. Log each one in your error log with its miss type.	1.5 to 2 hours
12	Weakest skill 1: reread the lesson, redo the worked examples without looking, then redo the Practice Set timed.	1.5 hours
11	Weakest skill 2: the same routine.	1.5 hours
10	Error-log redo: every entry that is due, timed.	1 to 1.5 hours
9	Weakest skill 3, then review the student-produced response rules and calculator methods in Chapter 15.	1.5 hours
8	Light day: 30 minutes of error-log review, then rest.	30 minutes
7	Second full-length test: another Bluebook practice test, or this book's Practice Test 2 if you have not taken it yet. Same conditions as Day 14.	2.5 hours
6	Review every miss and every guess from Day 7, and log them.	1.5 to 2 hours
5	Redo every error-log entry marked Trap or Misread. For SAT Weekend, complete exam setup in Bluebook and print your admission ticket; for School Day, follow your school's setup instructions.	1 to 1.5 hours
4	Timed practice: one Reading and Writing Practice Set and one Math Practice Set from your weakest domains.	1 hour
3	Final error-log pass: read your "rule or fix" column and copy your ten most useful rules onto one page. Reread the checkpoints in Chapter 3.	45 minutes
2	Logistics: test your device in Bluebook, gather your ID and supplies, and plan your route and arrival time. Read your one-page rule list.	30 to 45 minutes

Day	Main task	About how long
1	Rest. No new material. Pack your bag, charge your device and go to bed at your usual time.	—

Two full-length tests in two weeks is enough. A third test in the final days adds fatigue without adding much information, so spend any extra time on the error log and your weakest skills.

TIP On Days 14 and 7, record which modules felt rushed and where your checkpoints fell. If you finished a module with no time to spare, practice the stuck rule from Chapter 3 on Days 10 and 4.

If you have less than two weeks

Use Days 7 through 1 of the plan, and make Day 7 a full-length Bluebook practice test. On Day 6, review it and choose your two weakest skills. Spend Days 5 and 4 on those skills: one per day, using the Day 12 routine. Keep Days 3, 2 and 1 exactly as written, because a calm, organized final three days is worth more than one more cram session. If you have only a few days, skip new practice entirely: read your error log, review Chapters 3 and 15, and handle the logistics.

Redoing the error log

By now your error log holds the most valuable study material in this book: the questions you have missed. Use it in four steps.

1. **Sort.** Retire every entry you answered correctly on both of its redos. Group the remaining entries by skill.
2. **Redo cold.** Cover the explanation and solve each remaining entry under time. Do not just reread your notes; a question you can explain only while looking at the answer is not yet learned.
3. **Rewrite the rule.** For each entry, write the rule or fix in one sentence you could teach to a friend. For example: "A supplement that opens with a dash must close with a dash." Collect your ten most useful rules on a single page.
4. **Pick one test-day habit.** Count your miss types once more. If Misread is most common, your habit is to name the task word in every question before you solve it. If Careless is most common, your habit is a five-second recheck before moving on. If Time is most common, your habit is the stuck rule.

Rest and logistics

The week before the test, protect your sleep. Go to bed and wake at about the times you will need on test day, so the early start is not a shock. Keep meals and exercise normal.

Use this SAT Weekend list on Day 2 and again on Day 1. For SAT School Day, follow your school's readiness check, school-issued sign-in ticket and supply instructions instead; the Weekend admission-ticket process does not apply. Confirm the current lists on the College Board website, because rules can change.

- **Device.** Fully charged, with Bluebook installed and exam setup completed. The device needs a battery life of at least 3 hours. Bring a charging cable or portable charger. A phone cannot be the testing device. A backup device is recommended if you have one.
- **Admission ticket.** It becomes available in Bluebook after you complete exam setup, starting 5 days before the test. Print it; a printed copy is preferred.

- **Photo ID.** An acceptable photo ID that is a physical document. Check the ID requirements on the College Board website now, not on test morning.
- **Calculator.** Optional, because the Desmos calculator is built into Bluebook. If you bring your own, it must be on the approved list and must not have computer algebra system (CAS) functionality. Put in fresh batteries.
- **Pencils or pens.** For scratch work. Staff provide the scratch paper and collect it at the end.
- **Account details.** Your College Board username and password.
- **Extras.** A watch without an audible alarm, and a drink or snack for the break.

Phones, smartwatches, smart glasses, fitness trackers and other wearables or communication devices may not be used during the test or the breaks, so leave them at home or follow the testing staff's directions for putting them away. Plan your route, allow extra travel time and arrive early.

Test morning

A simple routine keeps small problems from becoming big ones.

1. Wake at the time you practiced during the final week, and eat a breakfast you are used to.
2. Check your bag against the list above one last time.
3. Leave early enough to arrive with time to spare, and follow the arrival and check-in directions the College Board and your test center give you.
4. Before you leave home, read your one-page rule list once. Leave notes and study materials out of the testing room; they are prohibited. Do not start practice questions while waiting.
5. When the test begins, use the habits from Chapter 3: write your checkpoints on your scratch paper, read the first question slowly, use the two-pass method, answer every question and reset after every hard one.

If something goes wrong on test day

Even well-prepared students hit snags. What matters is a calm response.

- **A device or app problem.** Raise your hand and tell the testing staff right away. Do not try to fix it quietly while your time runs.
- **You feel unwell.** Tell the testing staff. They can explain your options.
- **A module goes badly.** It is over, and the next module is a fresh start. Your section score uses both modules, so give the next question your full attention.
- **You are behind at a checkpoint.** Apply the stuck rule more strictly, guess and mark the hardest questions, and make sure every question has an answer before time ends.

If a problem affected your testing in a way the staff could not resolve, ask them how to report it, and check the College Board website for your options after the test.

After the test

Your scores appear in your College Board account when they are released. The College Board posts score release dates, so check them on the College Board website rather than relying on what you hear from friends.

Sending scores

- **Score Choice.** Score Choice lets you choose by test date and has no selection fee, although score-report fees may apply. It does not apply to the free score sends selected for that test administration. Check each college's policy; some require all scores.
- **Superscoring.** Some colleges combine your highest section scores from different test dates. This is a college practice, not a College Board rule, and policies vary from college to college. Look up the policy of each college on your list.
- **Score reports.** Additional score reports cost $15 each as of October 2026. For SAT Weekend, the first four reports are free if ordered by 9 days after the test date. Confirm the current conditions before choosing recipients, including the Score Choice exception above.

The Science Practices and Foundations score

Starting in spring 2027, your score report may include a separate cross-section score called Science Practices and Foundations. It is calculated from questions already on the test, in both Reading and Writing and Math, and adds no testing time. It is sent to your school, your district and sometimes your state, not to colleges. It is separate from your total score and your two section scores.

Deciding whether to retake

A second attempt can make sense, but it is not automatic; it depends on your goals, your results and your calendar. Start by comparing your score with the target you set in Chapter 2, then use the table.

If this describes you	Consider this
Your score is at or above the middle of the published ranges for the colleges you care about most	You may be finished. Put your time into coursework and applications.
You are below your target, and your misses were mostly content gaps in a few skills	Retake after a focused plan of 4 to 8 weeks built around those chapters.
You ran out of time in one or more modules	Retake after practicing the checkpoints and two-pass method on full-length tests.
Your score was well below your recent Bluebook practice test scores	Something may have gone wrong on the day. A retake after a short review may show your usual level.
Your target colleges superscore and one section lags behind	Retake with most of your study time on the weaker section.
No remaining test date leaves enough time before your application deadlines	Focus on your applications and send your best scores, using Score Choice where colleges allow it.

If you decide to retake, do not start over at Chapter 1. Begin with your error log and your most recent test results. Rebuild a plan from the tables in Chapter 2 using your weakest domains, take any Bluebook practice tests you have not used, and use Practice Test 2 if you have not taken it. Register early enough to choose your test center and date; Chapter 2 lists the dates and deadlines that were published as of October 2026.

Changes to watch for

This book's exam facts were checked against the College Board's published pages on 29 October 2026. The following details are the ones most likely to change, so confirm them on the College Board website before you register and again a week before your test.

- Test dates, registration deadlines and change deadlines, including the anticipated 2027–28 dates.
- Registration and other fees, and fee waiver rules.
- Bluebook tools, the reference sheet layout and device requirements, such as the Windows and Chromebook rules introduced for fall 2026.
- The approved calculator list.
- The college and career readiness benchmarks, which the College Board has said it will re-examine.
- The percentile tables, which are updated each year.
- How the Science Practices and Foundations score is reported.
- Accommodation procedures and test-day rules.

If something on the College Board website differs from this book, follow the College Board website.

ONLINE (OPTIONAL) Corrections to this book, and exam changes announced after 29 October 2026, are posted on the website's updates page. Its test-day guide has the checklist and time budget on one page.

Updates and corrections
skillbyskillprep.com/updates

Test-day guide
skillbyskillprep.com/test-day

A last word

You have done the work that matters most: you learned each skill by name, practiced it until the method was yours, and studied your own mistakes until they stopped repeating. On test day, trust that preparation. Read each question carefully, use your checkpoints, answer everything and let each question be the only one in front of you.

Appendices

Appendix A: Grammar and Punctuation Reference

This appendix collects the Standard English Conventions rules from Chapters 11 and 12. Use it to review before a practice test or to look up a rule named in an explanation.

Punctuation at a glance

An independent clause has a subject and a finite verb and can stand alone as a sentence. A dependent clause begins with a word such as because, although, when, which or that, so it cannot stand alone. FANBOYS are the seven coordinating conjunctions: for, and, nor, but, or, yet, so.

Mark	What it joins or sets off	Example	Common wrong use
Period	Ends a statement; separates two independent clauses	The dam held. The river fell.	Ending a dependent clause: "Because the dam held."
Semicolon	Joins two independent clauses; separates series items that contain commas	The dam held; the river fell.	Before a phrase or dependent clause: "The dam held; because of its design."
Colon	Follows a complete clause to introduce a list, explanation or emphatic development	The kit had three parts: a frame, a motor and a battery.	After a verb or preposition: "The kit included: a frame ..."
Comma + FANBOYS	Joins two independent clauses	The dam held, but the road flooded.	Comma after the conjunction: "The dam held but, the road flooded."
Conjunctive adverb opening a second independent clause	Semicolon or period between the clauses; comma after the introductory adverb	The dam held; however, the road flooded.	Only a comma at the clause boundary: "The dam held, however, the road flooded."
Pair of commas	Sets off a nonessential supplement	The dam, finished decades ago, still works.	Dropping one comma or closing with a dash
Pair of dashes	Sets off a supplement, often one that contains commas	The dam—tall, curved and thin—still works.	Closing the supplement with a comma
Parentheses	Set off an aside	The dam (the region's largest) still works.	Leaving out the closing parenthesis
Question mark	Ends a direct question	Will the dam hold?	After a reported question: "She asked whether the dam would hold?"
No punctuation	Between subject and verb, verb and object; around restrictive clauses and essential names	The engineers who built the dam celebrated.	"The engineers who built the dam, celebrated."

These are common patterns, not fixed positions for every connector. A conjunctive adverb can move: "The dam held; the road, however, flooded." The semicolon remains between clauses. Long clauses with internal commas may use a semicolon before FANBOYS, and a colon can introduce an emphatic contrast even when a conjunction follows it.

Three quick tests settle most punctuation questions.

1. **The period test.** Put a period at the blank. If both sides stand alone, common options are a period, a semicolon or a comma plus FANBOYS. A colon can introduce an explanation or emphatic development.
2. **The removal test.** Lift out the words between two marks. If the sentence still works with the same meaning, the words are a supplement and need a matching pair.
3. **The colon test.** Cover everything after the colon. If what remains is not a complete sentence, the colon is wrong.

Agreement rules

Situation	Rule	Example
Phrase between subject and verb	Ignore the phrase	The box of tools is heavy.
Add-on phrase (along with, as well as)	Ignore the add-on	The coach, along with two players, was late.
Distinct subjects joined by "and"	Normally plural	The coach and the captain were late.
Subjects joined by "or" or "nor"	Agree with the nearer subject	Neither the players nor the coach was late.
each, every, one of	Singular	Each of the bridges is inspected yearly.
the number of / a number of	Singular / plural	The number of entries is rising. A number of entries are late.
Collective noun acting as one unit	Singular (standard American usage)	The orchestra tunes its instruments.
Inverted order	Find the subject after the verb	Along the river stand two mills.
there is / there are	Agree with the noun that follows	There are three exits.
Pronoun and antecedent	Match number	The company raised its prices. The companies raised their prices.

For a person, use the appropriate personal pronouns; singular "they" is also standard. A singular organization acting as one unit still takes "it" and "its."

Verb forms: finite and nonfinite

A finite verb has a tense and can be a sentence's main verb; a nonfinite form cannot, on its own.

Form	Example	Can it be the main verb?	Typical use
Present or past tense	builds, built	Yes	The crew built the deck.

Form	Example	Can it be the main verb?	Typical use
Helper + verb	has built, will build, is building	Yes	The crew has built the deck.
Infinitive	to build	No	Purpose or naming a goal: The goal was to build a deck.
-ing form	building	No (alone)	Describing: Building quickly, the crew finished early. Naming an activity: Building takes time.
Past participle as describer	built	No (alone)	Built in a week, the deck held.

A sentence needs at least one finite verb in its main clause. A verb inside a relative or dependent clause belongs to that clause and does not supply the main clause's verb.

Tense and aspect

Tense	Form	Use	Signal words
Simple present	builds	Habits, general facts, the present	every year, usually, today
Simple past	built	A finished past action	in 1990, last year, ago
Simple future	will build	A later action	next year, soon
Present perfect	has built	A past action linked to now	since, so far, for the past decade
Past perfect	had built	The earlier of two past actions	by the time (+ past), before (+ past event)
Future perfect	will have built	An action finished before a future point	by next spring
Present progressive	is building	An action happening now	currently, at the moment
Past progressive	was building	An action in progress at a past moment	while, when (+ past)

Keep tenses consistent with the surrounding sentences unless a time signal calls for a change.

Modifier checklist

1. Find the modifier, often an opening phrase that ends with a comma ("Built in 1885," "Having trained for months,").
2. Ask who or what performs or receives the action in that phrase.
3. Make sure that noun comes right after the comma as the subject of the main clause.
4. Check possessives. In "the runner's time," the subject is "time," not "runner."
5. Check passive voice. In "the samples were tested by the chemist," the subject is "samples."

6. Move a misplaced phrase next to the word it describes. Wrong: "Covered in frosting, the baker set the cake on the counter." Right: "Covered in frosting, the cake sat on the counter."

Possessives and plurals

Form	Meaning	Example
its	belonging to it	The bridge lost its paint.
it's	it is / it has	It's raining. It's been a year.
their	belonging to them	The players packed their bags.
there	in that place; begins "there is/are"	The bags are over there. There are two.
they're	they are	They're leaving at noon.
whose	belonging to whom or which	the composer whose symphony opened the season
who's	who is / who has	the composer who's visiting next week
your	belonging to you	your notes
you're	you are	You're ready.
Singular possessive	add 's	the player's shoe
Plural possessive ending in s	add ' only	the players' shoes
Irregular plural possessive	add 's	the children's books
Plain plural	add s, no apostrophe	three players

The possessive pronouns its, their, whose and your never take apostrophes. If you can expand the word to "it is," "they are," "who is" or "you are," use the apostrophe form.

Ten commonly confused words

1. **affect / effect.** Affect is usually a verb ("The rain affected the game"); effect is usually a noun ("The rain had an effect").
2. **than / then.** Than compares ("taller than the tower"); then marks time or sequence ("first the frame, then the roof").
3. **accept / except.** Accept means to receive or agree to; except means "other than."
4. **lose / loose.** Lose means to misplace or fail to win; loose means not tight.
5. **past / passed.** Passed is the past tense of "pass" ("The storm passed"); past names an earlier time or means "beyond" ("walked past the gate").
6. **lead / led.** Led is the past tense of the verb "lead" ("She led the team"); lead as a noun is also a metal.
7. **principal / principle.** A principal is a person in charge or a main amount; a principle is a rule or belief.
8. **complement / compliment.** A complement completes or pairs well with something; a compliment is praise.
9. **cite / site / sight.** Cite means to quote as evidence; a site is a location; sight is vision or something seen.
10. **precede / proceed.** Precede means to come before; proceed means to go forward.

Appendix B: Transitions by Logical Relationship

Use this appendix with Chapter 13, whose method asks you to name the relationship between two ideas before you look at the choices. Each family below lists common transitions, a short example and the trap that most often leads to the wrong family. The examples describe fictional places.

Transition families

Relationship	Common transitions	Example	Trap to avoid
Contrast	however, by contrast, in contrast, on the other hand, conversely, yet	Most houses in the village face the river. The old mill, however, faces the road.	A change of topic is not a contrast. The two ideas must differ or oppose each other.
Concession	nevertheless, nonetheless, even so, still, that said, regardless	The trail is steep and poorly marked. Even so, hundreds of hikers climb it each summer.	The first idea must be a point that seems to work against the second. If nothing is admitted, use a contrast word.
Cause (the next idea gives a reason)	after all, the reason is that, because (inside a sentence)	The museum stays open late on Fridays. After all, most of its visitors work during the day.	Result words such as *therefore* point the other way. Do not use them when the second sentence explains the first.
Effect (the next idea is a result)	therefore, thus, as a result, consequently, accordingly, hence, for this reason	The bridge closed for repairs. As a result, traffic shifted to the ferry.	Two facts on the same topic are not cause and effect. The first must produce the second.
Addition	moreover, furthermore, in addition, additionally, also, besides	The new park has a playground. In addition, it has a small outdoor stage.	If the second sentence is one case of a general claim, use an example word instead.
Example	for example, for instance, specifically, to illustrate, in particular	Many towns in the valley began as river ports. For instance, Millbrook grew up around a busy ferry landing.	An example needs a general claim before it. A separate point of the same kind is an addition.
Sequence	first, then, next, subsequently, afterward, later, finally, previously, earlier, meanwhile	The crew cleared the site in March. Next, they poured the foundation.	Check the direction of time. *Previously* and *earlier* point back; *subsequently* and *afterward* point forward; *meanwhile* means at the same time.

Relationship	Common transitions	Example	Trap to avoid
Similarity	similarly, likewise, in the same way, by the same token	City residents borrow library books for free. Likewise, they pay nothing to use the city pools.	Name both halves of the comparison. Without two parallel things, there is no similarity.
Emphasis	in fact, indeed, above all	The festival was popular. In fact, every show sold out within an hour.	The second idea must strengthen the first, not simply add a different point.
Clarification	in other words, that is, put simply	The species is nocturnal. That is, it is active mainly at night.	A clarification restates. If the sentence adds new information or a separate case, choose another family.
Summary	in short, overall, in sum, ultimately, to sum up	The plan costs less, takes less time and uses fewer materials. In short, it is the more efficient choice.	Use it only when the sentence wraps up several earlier points. It cannot introduce a new detail.

Two words in the concession family work differently from the rest. *Admittedly* and *granted* introduce the point being admitted ("Admittedly, the trail is steep."), and the sentence after them usually needs *still* or *even so*.

Punctuation with transitions

Transitions questions test logic, but knowing how each kind of connector is punctuated shows you where the blank attaches. Chapter 11 and Appendix A cover the rules in full.

Pattern	Punctuation	Example
Sentence adverb such as "however" at the start of a sentence	comma after it	However, the bridge stayed open.
Sentence adverb such as "however" opening a second independent clause	semicolon at the clause boundary, comma after the introductory adverb	The bridge was old; however, it stayed open.
Parenthetical transition inside a sentence	commas on both sides	The bridge, however, stayed open.
Coordinating conjunction (for, and, nor, but, or, yet, so) joining two independent clauses	usually a comma before it	The bridge was old, but it stayed open.
Subordinating conjunction (although, because, since, while, whereas) starting a dependent clause	no comma directly after the word	Although the bridge was old, it stayed open.

A *coordinating conjunction* joins two equal parts of a sentence. A *subordinating conjunction* starts a clause that cannot stand alone, called a *dependent clause*. Conjunctive adverbs can move: "The bridge was old; it stayed open, however." The clause boundary remains after "old" even when the adverb moves. Commas mark parenthetical uses; an integrated adverb may need none, as in "The bridge closed; traffic therefore shifted to the ferry."

TRAP "The bridge was old, however, it stayed open." is a comma splice: two complete sentences joined only by commas. Words such as *however*, *therefore* and *moreover* cannot join independent clauses on their own. Put a semicolon or a period at the clause boundary, wherever the adverb sits.

NOTE When a blank follows a period and comes right before a comma, the answer must be a transition that can stand alone, such as *however* or *therefore*. *Although* and *because* cannot fill that spot: each must introduce a clause, so "Although, the bridge stayed open." is incorrect.

Pairs that sound alike but differ in logic

The two words in each pair feel interchangeable, but they do different jobs.

- **However versus therefore.** *However* turns against the previous idea; *therefore* continues it to a result.
- **For instance versus in addition.** *For instance* introduces one case of the claim just made; *in addition* adds a separate point at the same level.
- **However versus nevertheless.** *However* marks a plain contrast; *nevertheless* marks something that holds true despite a point just admitted.
- **Consequently versus subsequently.** *Consequently* means "as a result"; *subsequently* means only "later." Events can follow each other without one causing the other.
- **Similarly versus in addition.** *Similarly* needs two parallel situations; *in addition* simply adds another point.
- **In fact versus in other words.** *In fact* strengthens a point with something more; *in other words* restates the same point more plainly.
- **Thus versus that is.** *Thus* introduces a result; *that is* introduces a restatement.
- **Instead versus in contrast.** *Instead* replaces an option that was rejected or did not happen; *in contrast* compares two things that both exist.
- **Meanwhile versus however.** *Meanwhile* places two events at the same time; it does not by itself signal a contrast.
- **Finally versus in short.** *Finally* marks the last step in a sequence; *in short* sums up several points.
- **On the other hand versus on the contrary.** *On the other hand* introduces a contrasting point; *on the contrary* rejects a statement just made, usually a negative one.
- **Accordingly versus according to.** *Accordingly* means "in response to that"; *according to* names a source and is not a transition.

TIP When two choices both seem to fit, replace each with its full meaning ("as a result," "at a later time," "despite that"). The wrong one usually sounds false once spelled out.

Appendix C: Graphing Calculator Quick Reference

A calculator is allowed on every Math question. The Bluebook testing app includes the Desmos graphing calculator, which you can move and resize and can switch between scientific and graphing modes. These directions describe the built-in graphing calculator. Handheld models vary: some require equations solved for y and do not offer sliders. Chapter 15 has worked examples.

Practice these moves while you study. A move you have practiced ten times saves time; a move you try for the first time on test day costs it.

Tasks at a glance

Task	What to do, in words	When it saves time
Solve an equation in one variable	Graph y = (left side) and y = (right side). The x-coordinate of each intersection point is a solution.	Equations with fractions, decimals or long distributions; checking an answer found by hand.
Solve a system of two equations	Enter both equations as written, even in standard form, and select the intersection point to read its coordinates.	Systems with decimals or awkward coefficients; choices given as ordered pairs.
Count solutions	Count intersections, or x-intercepts when one side is 0. Adjust the window and confirm the count algebraically.	"How many solutions" questions; checking a discriminant.
Find the vertex and zeros of a quadratic	Graph the function and select its highest or lowest point and its x-intercepts.	The function is in standard form and the factors are not obvious.
Compare two expressions	Graph y = (first expression) and y = (second expression). A visible mismatch rules out equivalence. Prove a match by simplifying and checking the domain; one matching input is not enough.	"Which expression is equivalent" questions with long expansions.
Fit a model to data (regression)	Enter the data in a two-column table. Use the table's column names and a tilde (~): for a line, enter $y_1 \sim mx_1 + b$. Read the fitted constants.	Finding a line of best fit or an exponential model from a table.
Mean, median and standard deviation	Enter the data as a list inside the built-in mean, median or standard deviation function.	Lists of more than a few values; comparing the spread of two lists.

Task	What to do, in words	When it saves time
Evaluate a function at several inputs	Define the function once, then use its table of values or evaluate it at each input.	Testing answer choices; checking a pattern in a table.
Sliders for unknown constants	Type the equation with the letter, such as k, and turn the letter into a slider. Drag it until the condition in the question happens.	No solution, infinitely many solutions, parallel lines, a line touching a curve.
Inequality regions	Type each inequality. The calculator shades the solution region and draws the boundary solid for ⩽ or ⩾ and dashed for < or >.	Deciding which point lies in the solution set of a system of inequalities.
Check a student-produced response	Substitute your answer into the original equation, or graph and confirm the intersection. Then write the value in a legal entry form.	Any SPR question you have time to check.

Moves in more detail

Graphing both sides. This method turns any one-variable equation into an intersection problem, whether the equation is linear, quadratic, absolute value, radical or exponential. When the question asks for "the positive solution" or "the sum of the solutions," read every intersection before you decide.

Intersections and points of interest. The built-in calculator marks special points, such as intersections, intercepts and turning points, when you select a graph; handheld models find them through a menu. Selecting a point shows its coordinates. If two graphs look as if they touch, zoom in: they may cross twice, touch once or miss.

Regression. A regression finds the constants that make a model fit the data as closely as possible. For a linear model, the calculator reports a slope and an intercept; for an exponential model, a starting value and a growth or decay factor. The fitted constants answer questions such as "Which equation best models the data?" Use the model type the question names.

Statistics functions. Mean and median functions accept unsorted lists, which removes a frequent median error: forgetting to sort. Standard deviation measures how spread out the values are around the mean. When a question asks you to compare the spreads of two data sets, reason from the shapes first and use the function to confirm.

Sliders. Moving a slider until two lines become parallel, or until a line just touches a parabola, gives an approximate value of the constant. Because sliders move in steps, finish with algebra: set slopes equal, match coefficients, or set a discriminant equal to 0.

Tables of values. A table lists many outputs at once, which helps you test answer choices in "Which value of x..." questions and confirm that a table matches a proposed equation.

Cautions

- **Angle mode.** Before any trigonometry, check whether the calculator is in degree or radian mode. An angle written with a degree sign, such as 35°, is in degrees; an angle written with π, such as π/6, is in radians.
- **Viewing window.** A graph can hide an intersection off the screen or squeeze two nearby intersections into what looks like one. Zoom out once, then zoom in on anything close.

- **Rounding versus exact answers.** The calculator shows rounded decimals. An intersection at 0.333 is probably 1/3, and a vertex at −1.667 is probably −5/3. When choices appear as fractions or radicals, convert your decimal or compare decimal forms of the choices. For SPR answers, follow the entry rules in Chapter 15.
- **Typing errors.** Put parentheses around every numerator, every denominator and every negative input: type (x + 1)/(x − 2), not x + 1/x − 2. One missing parenthesis can draw a convincing but wrong graph.
- **Graphs that look identical.** Two expressions can match everywhere except at a single point where one is undefined, such as $(x^2 - 4)/(x - 2)$ and x + 2 at x = 2. If a question involves a denominator, check the excluded values by hand.
- **Structure questions.** A calculator cannot tell you what a coefficient means in a context or why an equation has no solution; it can only confirm your reasoning.

Handheld calculators

You may bring an approved handheld calculator instead of, or in addition to, the built-in one. Calculators with computer algebra system (CAS) functionality are not allowed. Check the current College Board policy before test day, including its requirements to remove stored documents and programs with algebra functionality. If you bring a handheld calculator, practice with it until every move in this appendix is automatic, and know how to switch its angle mode.

A quick decision rule

If you can write the first line of algebra right away and the numbers are clean, solve by hand and use the calculator to check. If the numbers are messy, the question asks how many solutions exist, or you would have to guess at a method, graph first. On harder questions with constants, graph to understand and use algebra to finish.

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Appendix D: Math Formulas and Facts

During the Math section, Bluebook gives you a reference sheet that you can open at any time. Part 1 lists what that sheet contains, so you know what you can look up instead of memorizing it. Part 2 lists the formulas and facts the sheet does not include, which you need to know yourself. Each Part 2 table names the chapters that teach the idea.

Part 1: What the reference sheet gives you

Topic	What the sheet shows
Circle	$A = \pi r^2$; $C = 2\pi r$
Rectangle	$A = \ell w$
Triangle	$A = (1/2)bh$
Pythagorean theorem	$c^2 = a^2 + b^2$
Special right triangles	30°-60°-90° with sides x, $x\sqrt{3}$ and 2x; 45°-45°-90° with sides s, s and $s\sqrt{2}$
Rectangular solid	$V = \ell wh$
Right circular cylinder	$V = \pi r^2 h$
Sphere	$V = (4/3)\pi r^3$
Right circular cone	$V = (1/3)\pi r^2 h$
Rectangular-base pyramid	$V = (1/3)\ell wh$
Arcs	A circle has 360 degrees of arc and 2π radians of arc
Triangles	The angles of a triangle add to 180 degrees

TIP Knowing what is on the sheet saves time in both directions. You do not need to memorize the volume of a cone, but you do need to know that the sheet will not give you the equation of a circle or the quadratic formula.

Part 2: Formulas you must know

Lines and coordinates (Chapters 17, 18 and 27)

Idea	Formula or fact
Slope through (x_1, y_1) and (x_2, y_2)	$m = (y_2 - y_1)/(x_2 - x_1)$, for $x_2 \neq x_1$; vertical lines have undefined slope
Slope-intercept form	$y = mx + b$ for a nonvertical line, where b is the y-intercept

Idea	Formula or fact
Point-slope form	$y - y_1 = m(x - x_1)$ for a nonvertical line
Standard form	$Ax + By = C$ has slope $-A/B$ when $B \neq 0$; if $B = 0$ and $A \neq 0$, the line is vertical
Parallel lines	Distinct nonvertical lines have equal slopes; distinct vertical lines are also parallel
Perpendicular lines	Nonzero finite slopes are negative reciprocals; their product is -1. A horizontal line and a vertical line are perpendicular
Distance	$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$
Midpoint	$((x_1 + x_2)/2, (y_1 + y_2)/2)$

Quadratics and exponents (Chapters 20, 21 and 22)

In the quadratic formulas, $a \neq 0$. For the general exponent identities, use positive real bases; zero and negative bases need the restrictions explained in Chapter 20.

Idea	Formula or fact
Quadratic formula	For $ax^2 + bx + c = 0$, $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$
Discriminant	$b^2 - 4ac$: positive means two real solutions, zero means one, negative means no real solutions
Vertex	The x-coordinate of the vertex of $y = ax^2 + bx + c$ is $x = -b/(2a)$
Vertex form	$y = a(x - h)^2 + k$ has vertex (h, k)
Product and quotient of powers	$x^a \times x^b = x^{a+b}$; $x^a/x^b = x^{a-b}$
Power of a power	$(x^a)^b = x^{ab}$; $(xy)^a = x^a y^a$
Zero and negative exponents	$x^0 = 1$ for $x \neq 0$; $x^{-a} = 1/x^a$ for $x \neq 0$, wherever the powers are real
Fractional exponents	For $x > 0$ and positive integer n, $x^{1/2} = \sqrt{x}$ and $x^{m/n}$ is the nth root of x raised to the power m; $\sqrt{(x^2)}$ equals the absolute value of x for every real x

Percentages and growth (Chapters 22 and 23)

Idea	Formula or fact
Percent change	(new − old)/old × 100% for a positive original value
Increase by p percent	Multiply by $1 + p/100$
Decrease by p percent	Multiply by $1 - p/100$
Exponential growth	$y = a(1 + r)^t$, where a is the starting amount and r is the rate per period as a decimal
Exponential decay	$y = a(1 - r)^t$, with $0 < r < 1$ for decay of a positive quantity

Idea	Formula or fact
Repeated changes	Successive percent changes multiply; they do not add

Circles and angles (Chapters 26 and 28)

Idea	Formula or fact
Circle equation	$(x - h)^2 + (y - k)^2 = r^2$, with center (h, k) and radius $r > 0$
Arc length	$(\theta/360) \times 2\pi r$ for θ in degrees; $r\theta$ for θ in radians
Sector area	$(\theta/360) \times \pi r^2$ for θ in degrees; $(1/2)r^2\theta$ for θ in radians
Degree-radian conversion	π radians = 180°; multiply by $\pi/180$ or by $180/\pi$
Inscribed angle	Half of its intercepted arc
Tangent line	Perpendicular to the radius at the point of tangency
Interior angles of a polygon	For a simple polygon with n sides, the sum is $(n - 2) \times 180°$
Exterior angles of a polygon	For a convex polygon, one exterior angle at each vertex sums to 360°
Parallel lines and a transversal	Corresponding and alternate interior angles are equal; same-side interior angles add to 180°

Right triangles and similarity (Chapters 26 and 27)

Idea	Formula or fact
SOH-CAH-TOA	sin A = opposite/hypotenuse; cos A = adjacent/hypotenuse; tan A = opposite/adjacent
Complementary angles	$\sin x° = \cos(90° - x°)$; if $\sin a° = \cos b°$ for acute angles, $a + b = 90$
Similar triangles	Matching sides are proportional, and matching angles have equal trigonometric ratios
Scaling by k	Lengths scale by k, areas by k^2 and volumes by k^3

Data and probability (Chapters 24 and 25)

Idea	Formula or fact
Mean	The sum of the values divided by the number of values
Median	The middle value of the ordered list, or the mean of the two middle values when the count is even
Probability	For equally likely outcomes, P(A) = favorable outcomes/total outcomes
Conditional probability	For equally likely outcomes, P(A given B) = the number in both A and B divided by the number in B, which must be nonzero
Complement	P(not A) = 1 − P(A)

Margin of error, in words (Chapter 25)

A margin of error describes how far an estimate from a random sample is likely to be from the true value for the whole population. The estimate plus or minus the margin of error gives a range of plausible values for that population value at the stated confidence level. With everything else the same, a larger random sample gives a smaller margin of error. The margin of error does not fix bias: if the sample was not chosen at random, or if it came from a different group than the one in the claim, the margin of error says nothing about that problem. A conclusion also applies only to the population the sample was drawn from.

CHECK Before test day, cover the right-hand column of each Part 2 table and say each formula aloud. Any formula you cannot produce in a few seconds belongs on your error-log review list.

Appendix E: Error Log and Study Tracker

The questions you miss are your best study material, because they show exactly where your points are going. This appendix gives you three ready-to-copy tools: an error log for recording and redoing every miss, a skill tracker that follows each skill from the diagnostic to Practice Test 2, and a weekly study tracker. You may photocopy these pages for your own use; make a fresh copy whenever a page fills up.

How to use the error log

1. **Log within 24 hours.** Record every question you missed, and every question you got right only by guessing. A lucky guess is a miss you have not paid for yet.
2. **Fill in every column.** In "Where", write the question ID printed beside the question number, such as RW-C1104 or M-T1M2-15; for a Bluebook practice test, write the test, module and question number. In "Skill", copy the skill named in the explanation. In "My answer / correct", write both answers, such as C / A.
3. **Name the miss type.** Use one word from the table below: Gap (for a content gap), Trap, Misread, Careless or Time.
4. **Write the rule or fix in your own words.** Write one sentence you could teach to a friend. "Be more careful" is not a fix; "Check which group the claim names before reading the graph" is.
5. **Schedule two redos.** In "Redo date", write the date 3 days from today. Your second redo comes one week after that, 10 days after the miss. Cover the explanation and solve the question fresh each time.
6. **Record the results.** In "Redo correct?", write Y or N for each redo, such as Y, Y. Retire an entry when both redos are correct. If you miss a redo, reread the chapter section named in the explanation and start a new row with two new redo dates.
7. **Count once a week.** Tally your miss types. The most common type tells you what to change next; Chapter 2 (Step 5) lists the change for each type.

The five miss types

Miss type	How to recognize it	Fix
Content gap	I did not know the rule or method.	Relearn it in the chapter named in the explanation.
Trap	I chose a tempting wrong answer.	Name the trap type from the explanation and add it to your error log.
Misread	I answered a different question than the one asked.	Underline the task word before solving.
Careless	I knew it but slipped.	Add a 5-second check step to your method.
Time	I rushed or guessed at the end.	Practice the two-pass method from Chapter 3.

A sample entry

This entry comes from the second worked example in Chapter 4. The student chose a comma to close a supplement that opened with a dash.

Date logged	Where (ID or test)	Skill (official name)	My answer / correct	Miss type (word)	Rule or fix, written in my own words	Redo date	Redo correct?
Mar 3	Ch. 4, example 2	Boundaries	A / D	Trap	Open with a dash, close with a dash.	Mar 6	Y, Y

Error log

Date logged	Where (ID or test)	Skill (official name)	My answer / correct	Miss type (word)	Rule or fix, written in my own words	Redo date	Redo correct?
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

Date logged	Where (ID or test)	Skill (official name)	My answer / correct	Miss type (word)	Rule or fix, written in my own words	Redo date	Redo correct?
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

Skill tracker

After the diagnostic and each practice test, use the answer key, which names the skill for every question, to count your misses for each skill. Write them as misses out of the total, such as 1 of 3.

After each Practice Set, write your score for each skill the set covers, such as 13 of 15. In "Current status", write Weak, Improving or Strong, and update it as your results change. Write over the dashes as you fill in each row.

Skill	Ch.	Diagnostic misses	Practice Set score	Test 1 misses	Test 2 misses	Current status
Words in Context	5	—	—	—	—	—
Text Structure and Purpose	6	—	—	—	—	—
Cross-Text Connections	7	—	—	—	—	—
Central Ideas and Details	8	—	—	—	—	—
Command of Evidence: Textual	9	—	—	—	—	—
Command of Evidence: Quantitative	9	—	—	—	—	—
Inferences	10	—	—	—	—	—
Boundaries	11	—	—	—	—	—
Form, Structure, and Sense	12	—	—	—	—	—
Transitions	13	—	—	—	—	—
Rhetorical Synthesis	14	—	—	—	—	—
Linear equations in one variable	16	—	—	—	—	—
Linear functions	17	—	—	—	—	—
Linear equations in two variables	17	—	—	—	—	—
Systems of two linear equations in two variables	18	—	—	—	—	—
Linear inequalities in one or two variables	19	—	—	—	—	—

Skill	Ch.	Diagnostic misses	Practice Set score	Test 1 misses	Test 2 misses	Current status
Equivalent expressions	20	—	—	—	—	—
Nonlinear equations in one variable and systems of equations in two variables	21	—	—	—	—	—
Nonlinear functions	22	—	—	—	—	—
Ratios, rates, proportional relationships, and units	23	—	—	—	—	—
Percentages	23	—	—	—	—	—
One-variable data: distributions and measures of center and spread	24	—	—	—	—	—
Two-variable data: models and scatterplots	24	—	—	—	—	—
Probability and conditional probability	25	—	—	—	—	—
Inference from sample statistics and margin of error	25	—	—	—	—	—
Evaluating statistical claims: observational studies and experiments	25	—	—	—	—	—
Area and volume	26	—	—	—	—	—
Lines, angles, and triangles	26	—	—	—	—	—
Right triangles and trigonometry	27	—	—	—	—	—
Circles	28	—	—	—	—	—

Weekly study tracker

Fill in one row at the end of each study week and compare it with your Chapter 2 plan. If you fall behind, move a chapter to the next week rather than skipping its Practice Set or your error-log redos.

Week	Chapters and Practice Sets done	Timed sets, full tests	Redos done	Hours studied
1	—	—	—	—
2	—	—	—	—
3	—	—	—	—
4	—	—	—	—
5	—	—	—	—

Week	Chapters and Practice Sets done	Timed sets, full tests	Redos done	Hours studied
6	—	—	—	—
7	—	—	—	—
8	—	—	—	—
9	—	—	—	—
10	—	—	—	—
11	—	—	—	—
12	—	—	—	—

Question-Type Finder

Each official question type, where it is taught, and where it appears in the diagnostic and practice tests.

Skill	Taught	Diagnostic	Test 1	Test 2
Words in Context	Ch. 5, p. 69	5	9	9
Text Structure and Purpose	Ch. 6, p. 82	2	4	4
Cross-Text Connections	Ch. 7, p. 98	1	2	2
Central Ideas and Details	Ch. 8, p. 112	2	3	3
Command of Evidence: Quantitative	Ch. 9, p. 128	2	4	4
Command of Evidence: Textual	Ch. 9, p. 128	1	3	3
Inferences	Ch. 10, p. 150	2	4	4
Boundaries	Ch. 11, p. 164	4	7	7
Form, Structure, and Sense	Ch. 12, p. 180	3	7	7
Transitions	Ch. 13, p. 195	2	5	5
Rhetorical Synthesis	Ch. 14, p. 208	3	6	6
Linear equations in one variable	Ch. 16, p. 239	2	3	3
Linear equations in two variables	Ch. 17, p. 251	1	2	2
Linear functions	Ch. 17, p. 251	2	4	4
Systems of two linear equations in two variables	Ch. 18, p. 264	2	4	4
Linear inequalities in one or two variables	Ch. 19, p. 278	1	2	2
Equivalent expressions	Ch. 20, p. 289	2	4	4
Nonlinear equations in one variable and systems of equations in two variables	Ch. 21, p. 300	2	5	5
Nonlinear functions	Ch. 22, p. 313	3	6	6
Percentages	Ch. 23, p. 328	1	1	1
Ratios, rates, proportional relationships, and units	Ch. 23, p. 328	1	2	2
One-variable data: distributions and measures of center and spread	Ch. 24, p. 340	1	1	1
Two-variable data: models and scatterplots	Ch. 24, p. 340	0	1	1
Evaluating statistical claims: observational studies and experiments	Ch. 25, p. 357	0	0	1

Skill	Taught	Diagnostic	Test 1	Test 2
Inference from sample statistics and margin of error	Ch. 25, p. 357	0	1	0
Probability and conditional probability	Ch. 25, p. 357	1	1	1
Area and volume	Ch. 26, p. 372	1	2	2
Lines, angles, and triangles	Ch. 26, p. 372	1	2	2
Right triangles and trigonometry	Ch. 27, p. 389	1	2	2
Circles	Ch. 28, p. 405	0	1	1

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These references support factual statements in the lessons and passages. They do not imply endorsement by any organization. Source review date: 29 October 2026.

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- College Board, Changes for 2026-27 testing: https://satsuite.collegeboard.org/sat/testing-staff/changes
- College Board, Score Choice: https://satsuite.collegeboard.org/scores/sending-sat-scores/additional/score-choice
- College Board, Which scores will colleges look at: https://satsuite.collegeboard.org/help-center/which-scores-will-colleges-look-if-my-child-took-sat-more-once
- College Board, SAT fees and refunds: https://satsuite.collegeboard.org/sat/registration/fees-refunds

- College Board, How SAT scores are calculated: https://satsuite.collegeboard.org/scores/what-scores-mean/how-scores-calculated
- College Board, Why offer the SAT: https://k12.collegeboard.org/programs/sat-suite/why-offer-sat
- College Board, SAT School Day: https://bluebook.collegeboard.org/students/sat-school-day

Practice Questions

Questions that use real-world facts or public-domain texts, listed by question ID in book order.

RW-D04: https://www.britannica.com/science/standard-time; https://guides.loc.gov/this-month-in-business-history/november/day-of-two-noons

RW-D18: https://www.britannica.com/topic/Ananse

RW-D26: https://www.britannica.com/biography/Zora-Neale-Hurston; https://www.zoranealehurston.com/about/; https://sfonline.barnard.edu/zora-neale-hurston-a-woman-half-in-shadow-reprint/

RW-D27: https://www.sciencehistory.org/education/scientific-biographies/stephanie-l-kwolek/; https://invention.si.edu/invention-stories/stephanie-kwolek-kevlarr-inventor

RW-C0505: https://www.britannica.com/plant/Welwitschia

RW-C0507: https://www.britannica.com/biography/Rachel-Ruysch

RW-C0508: https://www.britannica.com/biography/Ebenezer-Howard

RW-C0605: https://www.britannica.com/plant/dodder; https://ipm.ucanr.edu/home-and-landscape/dodder/; https://www.frontiersin.org/journals/plant-science/articles/10.3389/fpls.2015.00045/full

RW-C0606: https://www.britannica.com/biography/Jacob-Lawrence; https://www.moma.org/collection/works/78545

RW-C0607: https://www.britannica.com/place/Curitiba

RW-C0803: https://www.gutenberg.org/cache/epub/160/pg160.txt; https://www.britannica.com/topic/The-Awakening-novel-by-Chopin

RW-C0807: https://www.gutenberg.org/cache/epub/3146/pg3146.txt; https://www.britannica.com/biography/Thomas-Hardy

RW-C0813: https://www.gutenberg.org/cache/epub/4276/pg4276.txt; https://www.britannica.com/topic/North-and-South-novel-by-Gaskell

RW-C0904: https://www.gutenberg.org/cache/epub/786/pg786.txt; https://www.britannica.com/topic/Hard-Times-novel-by-Dickens

RW-C0909: https://www.gutenberg.org/cache/epub/107/pg107.txt; https://www.britannica.com/topic/Far-from-the-Madding-Crowd-novel-by-Hardy

RW-C0917: https://www.gutenberg.org/cache/epub/2701/pg2701.txt; https://www.britannica.com/topic/Moby-Dick-novel

RW-C1007: https://www.gutenberg.org/cache/epub/2701/pg2701.txt; https://www.britannica.com/topic/Moby-Dick-novel

RW-C1009: https://nssdc.gsfc.nasa.gov/planetary/mission/near/near_science.html

RW-C1012: https://www.gutenberg.org/cache/epub/3146/pg3146.txt; https://www.britannica.com/biography/Thomas-Hardy

RW-C1103: https://www.invent.org/inductees/mary-anderson

RW-C1109: https://www.invent.org/inductees/hedy-lamarr

RW-C1113: https://www.britannica.com/biography/Inge-Lehmann; https://www.amnh.org/learn-teach/curriculum-collections/earth-inside-and-out/inge-lehmann-discoverer-of-the-earth-s-inner-core

RW-C1117: https://www.britannica.com/biography/Florence-Price

RW-C1209: https://www.britannica.com/biography/Clarence-Birdseye

RW-C1213: https://www.britannica.com/biography/Bartolomeo-Cristofori; https://www.metmuseum.org/art/collection/search/501788

RW-C1218: https://www.britannica.com/biography/Marie-Tharp; https://news.climate.columbia.edu/2020/07/24/marie-tharp-connecting-dots/

RW-C1303: https://www.nobelprize.org/prizes/medicine/1973/frisch/facts/; https://www.nobelprize.org/prizes/medicine/1973/summary/

RW-C1304: https://www.cdc.gov/typhoid-fever/about/index.html

RW-C1305: https://www.fs.usda.gov/wildflowers/pollinators/Monarch_Butterfly/migration/index.shtml; https://www.nps.gov/subjects/pollinators/migratingmonarchs.htm

RW-C1306: https://www.britannica.com/biography/John-Snow-British-physician

RW-C1401: https://www.britannica.com/topic/Fallingwater

RW-C1406: https://www.britannica.com/biography/Sequoyah

RW-C1410: https://www.fs.usda.gov/wildflowers/pollinators/animals/moths.shtml; https://www.fs.usda.gov/wildflowers/pollinators/animals/birds.shtml

RW-C1411: https://www.britannica.com/biography/Florence-Nightingale

RW-T1M1-02: https://www.fws.gov/species/southern-sea-otter-enhydra-lutris-nereis

RW-T1M1-03: https://ich.unesco.org/en/RL/flamenco-00363

RW-T1M1-05: https://www.gutenberg.org/cache/epub/550/pg550.txt

RW-T1M1-06: https://www.gutenberg.org/cache/epub/76703/pg76703.txt; https://www.britannica.com/biography/Christina-Rossetti

RW-T1M1-07: https://nsidc.org/learn/parts-cryosphere/sea-ice/science-sea-ice

RW-T1M1-12: https://www.gutenberg.org/cache/epub/367/pg367.txt; https://www.britannica.com/topic/The-Country-of-the-Pointed-Firs

RW-T1M1-15: https://smarthistory.org/daguerre-paris-boulevard/

RW-T1M1-16: https://oceanservice.noaa.gov/education/tutorial_currents/05conveyor2.html

RW-T1M1-18: https://guides.lib.uoguelph.ca/c.php?g=696372&p=4999535

RW-T1M1-21: https://guides.loc.gov/africa-in-the-indian-ocean-world

RW-T1M1-24: https://oceanexplorer.noaa.gov/ocean-fact/omz/

RW-T1M1-25: https://www.britannica.com/biography/Musa-I-of-Mali

RW-T1M1-26: https://www.britannica.com/biography/Anna-Atkins; https://www.metmuseum.org/art/collection/search/286656

RW-T1M2-01: https://oceanservice.noaa.gov/facts/kelp.html

RW-T1M2-02: https://www.gutenberg.org/cache/epub/394/pg394.txt; https://www.britannica.com/topic/Cranford-novel-by-Gaskell

RW-T1M2-03: https://www.psychologyib.com/uploads/1/1/7/5/11758934/the_spotlight_effect_-_ib_psychology.pdf

RW-T1M2-04: https://www.gutenberg.org/cache/epub/77/pg77.txt; https://www.britannica.com/topic/The-House-of-the-Seven-Gables

RW-T1M2-10: https://www.gutenberg.org/cache/epub/767/pg767.txt; https://www.britannica.com/topic/Agnes-Grey

RW-T1M2-13: https://oceanservice.noaa.gov/facts/upwelling.html

RW-T1M2-15: https://www.nikonusa.com/learn-and-explore/c/tips-and-techniques/understanding-maximum-aperture

RW-T1M2-19: https://oceanservice.noaa.gov/education/tutorial_currents/05conveyor2.html

RW-T1M2-20: https://smarthistory.org/byzantine-seals/

RW-T1M2-27: https://www.britannica.com/art/Labanotation; https://findingaids.loc.gov/repositories/15/archival_objects/861459; https://library.leeds.ac.uk/special-collections/view/895; https://www.dancenotation.org/theory-bulletin-board/resources-and-riches-dance-notation-bureau/

RW-T2M1-01: https://www.gutenberg.org/cache/epub/13417/pg13417.txt; https://en.wikipedia.org/wiki/Kashtanka

RW-T2M1-05: https://www.gutenberg.org/cache/epub/166/pg166.txt

RW-T2M1-06: https://www.gutenberg.org/cache/epub/136/pg136.txt

RW-T2M1-09: https://www.gutenberg.org/cache/epub/6520/pg6520.txt

RW-T2M1-10: https://www.usgs.gov/programs/VHP/volcanoes-can-affect-climate

RW-T2M1-12: https://www.gutenberg.org/cache/epub/45/pg45.txt

RW-T2M1-19: https://pubmed.ncbi.nlm.nih.gov/30736927/; https://pmc.ncbi.nlm.nih.gov/articles/PMC3711810/

RW-T2M1-20: https://www.aucklandmuseum.com/discover/collections/topics/indigo

RW-T2M1-24: https://pmc.ncbi.nlm.nih.gov/articles/PMC7885098/

RW-T2M1-25: https://www.dol.gov/general/aboutdol/history/flsa1938; https://francesperkinscenter.org/wp-content/uploads/2024/07/2024_FrancesPerkins_FactSheet.pdf; https://nclnet.org/early-days-investigate-agitate-and-legislate/

RW-T2M2-01: https://www.nationalzoo.si.edu/animals/eastern-box-turtle

RW-T2M2-02: https://www.gutenberg.org/cache/epub/10935/pg10935.txt; https://www.nobelprize.org/prizes/literature/1909/lagerlof/biographical/

RW-T2M2-04: https://www.gutenberg.org/cache/epub/1429/pg1429.txt; https://en.wikipedia.org/wiki/The_Garden_Party_and_Other_Stories

RW-T2M2-05: https://www.gutenberg.org/cache/epub/591/pg591.txt

RW-T2M2-06: https://www.noaa.gov/jetstream/atmosphere/precipitation; https://pubmed.ncbi.nlm.nih.gov/18309078/; https://pmc.ncbi.nlm.nih.gov/articles/PMC2596265/

RW-T2M2-07: https://www.canada.ca/en/conservation-institute/services/preventive-conservation/guidelines-collections/outdoor-objects.html; https://www.canada.ca/en/conservation-institute/services/preventive-conservation/guidelines-collections/metal-objects.html

RW-T2M2-08: https://www.gutenberg.org/cache/epub/1448/pg1448.txt; https://en.wikisource.org/wiki/Heidi_(1922)/Chapter_2; https://www.tandfonline.com/doi/full/10.1080/13614541.2010.495568

RW-T2M2-10: https://www.gutenberg.org/cache/epub/10376/pg10376.txt; https://www.britannica.com/biography/Zitkala-Sa

RW-T2M2-19: https://www.noaa.gov/jetstream/hail

RW-T2M2-22: https://pubs.usgs.gov/bul/0965a/report.pdf; https://www.esa.int/Applications/Observing_the_Earth/Proba-1/Earth_from_Space_A_volcano_s_life

RW-T2M2-25: https://www.adfg.alaska.gov/static/species/speciesinfo/_aknhp/Wood_Frog.pdf; https://www.adfg.alaska.gov/static/education/wns/frogs_and_toads.pdf